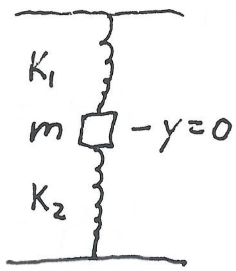
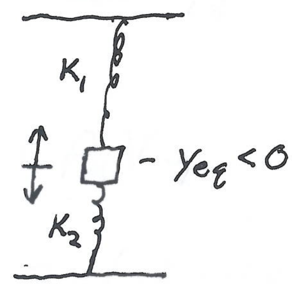
SMU Physics 1307 : Fall 2009

Final Exam

Problem 1 : The figures below show a mass $m = 2\,\text{kg}$ attached to two springs, one with $k_1 = 50\,\text{N/m}$ extending from the ceiling, and another with $k_2 = 100\,\text{N/m}$ extending from the floor. In the left figure the mass is at a point, which we take to be $y = 0$, in which both springs are in their uncompressed positions. In general, as shown in the figure at right, the object will oscillate up and down about some equilibrium position $y_e < 0$. Find the equilibrium position y_e of the object, and find the period T of oscillation.





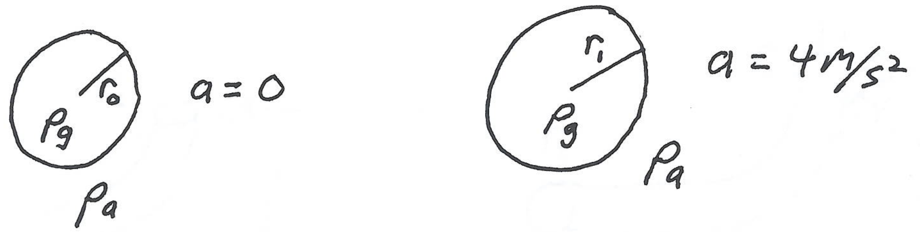
$$ma = -(K_1 + K_2)y - mg$$

$$\frac{d^2y}{dt^2} = -\omega^2 y - g \qquad \omega^2 = (K_1 + K_2)/m = \underline{\underline{75\,s^{-2}}}$$

$$y_{eq} = -g/\omega^2 = \underline{\underline{-.13\,m}}$$

$$T = \frac{2\pi}{\omega} = \underline{\underline{0.726\,s}}$$

Problem 2 : A balloon is to be constructed from a material of mass density per unit area $s = 0.1\,\text{kg/m}^2$, and filled with a gas of density $\rho_g = 0.25\,\rho_a$, where $\rho_a = 1.2\,\text{kg/m}^3$. Assume the balloon is spherical, so that the volume is $V = \frac{4}{3}\pi r^3$ and the area is $A = 4\pi r^2$. Find the radius r_0 such that the balloon will barely lift off, as well as the radius r_1 such that the acceleration of the balloon is $a = 4\,\text{m/s}^2$ upward.



$$M = sA \qquad \hat{M}a = -\hat{M}g + \rho_a V g$$

$$\hat{M} = M + \rho_g V \qquad (sA + \rho_g V)a = -(sA + \rho_g V)g + \rho_a V g$$

$$(s\,4\pi r^2 + \rho_g \tfrac{4}{3}\pi r^3)a = -(s\,4\pi r^2 + \rho_g \tfrac{4}{3}\pi r^3)g + \rho_a \tfrac{4}{3}\pi r^3 g$$

divide by $\tfrac{4}{3}\pi r^2$:

$$(3s + \rho_g r)a = -(3s + \rho_g r - \rho_a r)g$$

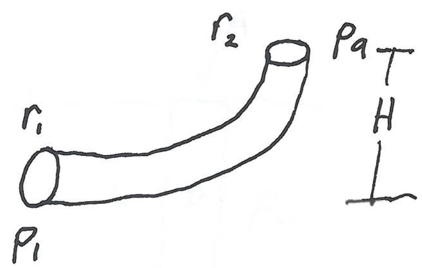
$$3s(a+g) = r(\rho_a g - \rho_g g - \rho_g a)$$

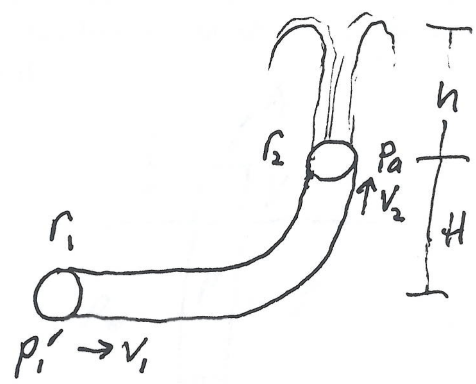
$$r = 3s(a+g)/(\rho_a g - \rho_g(g+a))$$

$\underline{a = 0}$ $\quad r_0 = \underline{0.33\,m}$

$\underline{a = 4\,m/s^2}$ $\quad r_1 = \underline{0.543\,m}$

Problem 3 : The figure below shows a pipe of radius $r_1 = 0.05\,\mathrm{m}$ which flows uphill a height $H = 10\,\mathrm{m}$ to the base of a fountain, where it narrows to a radius $r_2 = 0.02\,\mathrm{m}$. What is the pressure p_1 when the water is stationary in the pipe with $p_2 = p_a = 10^5\,\mathrm{N/m^2}$? Now suppose that the water emerges from the fountain and shoots to a height $h = 15\,\mathrm{m}$. What pressure p_1' is necessary (again $p_2' = p_a$) for this to occur, and what are the velocities v_1 and v_2 in this case? Use $\rho_w = 10^3\,\mathrm{kg/m^3}$.





$V = 0$

$$P_1 = P_a + \rho_w g H = \underline{1.98 \times 10^5\ N/m^2}$$

$V \neq 0$

$$P_1' + \tfrac{1}{2}\rho_w V_1^2 = P_a + \tfrac{1}{2}\rho_w V_2^2 + \rho_w g H = P_a + \rho_w g (H + h)$$

$$\boxed{\begin{array}{l} A_1 = \pi r_1^2 = 7.85 \times 10^{-3}\,m^2 \\ A_2 = \pi r_2^2 = 1.26 \times 10^{-3}\,m^2 \end{array}}$$

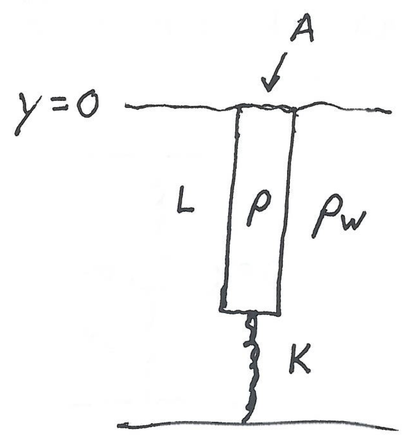
$$\tfrac{1}{2}\rho_w V_2^2 = \rho_w g h$$

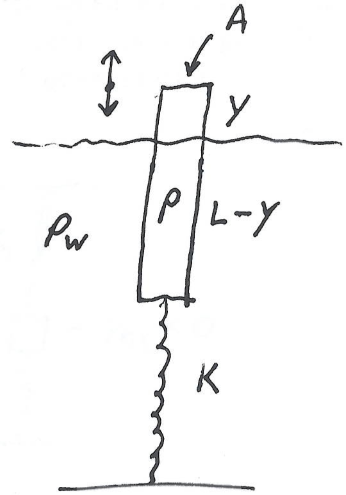
$$V_2 = (2gh)^{1/2} = \underline{17.15\ m/s}$$

$$A_1 V_1 = A_2 V_2 \qquad V_1 = V_2 \, A_2/A_1 = \underline{2.743\ m/s}$$

$$P_1' = P_a + \tfrac{1}{2}\rho_w (V_2^2 - V_1^2) + \rho_w g H = \underline{3.41 \times 10^5\ N/m^2}$$

Problem 4 : The figures below show a cylindrical object of cross-sectional area $A = 0.1\,\mathrm{m}^2$ and length $L = 1\,\mathrm{m}$ which is attached to a spring with $k = 400\,\mathrm{N/m}$. In the figure at left, the top of the object is exactly at the surface of the water, which we take to be $y = 0$, and the spring is in its uncompressed position. In general, as shown in the figure at right, the object will sit above the surface of the water and bob up and down about some equilibrium position. If the object has a density $\rho = 0.6\,\rho_w$, where $\rho_w = 10^3\,\mathrm{kg/m}^3$, find the equilibrium position y_e of the object, and find the period T of oscillations of the object in the water. Neglect the buoyancy force due to the air above the water.





$$\rho L A a = -Ky - \rho L A g + \rho_w (L-y) A g$$

$$\rho L A a = -(K + \rho_w A g) y + (\rho_w - \rho) L A g$$

$a = 0$

$$y_{eq} = \left(\frac{(K + \rho_w A g)}{(\rho_w - \rho) L A g}\right)^{-1} = \underline{\underline{0.284\,m}}$$

$$\omega^2 = (K + \rho_w A g)/(\rho L A) = \underline{\underline{23.0\,s^{-2}}}$$

$$T = 2\pi/\omega = \underline{\underline{1.31\,s}}$$