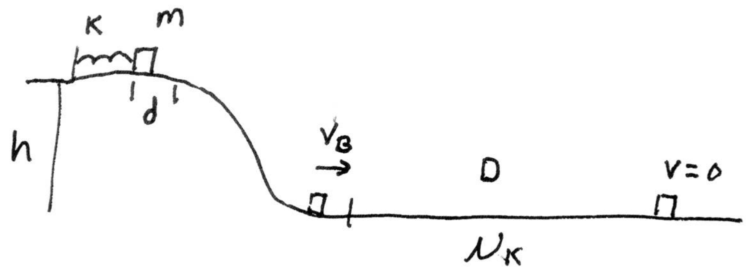
SMU Physics 1307 : Fall 2011

Exam 2

Problem 1: The figure below shows a frictionless hill of height $h = 10\,\mathrm{m}$ on which a spring with $k = 2000\,\mathrm{N/m}$ is mounted. The spring is compressed a distance $d = 0.4\,\mathrm{m}$ and a mass $m = 2\,\mathrm{kg}$ is placed in front of it. When the spring is released the mass slides to the bottom of the hill and enters a flat section on which the coefficient of kinetic friction is $\mu_k = 0.8$. It then slides a distance D from the beginning of the frictional section before coming to rest. Find the velocity v_B of the mass when it reaches the bottom of the hill but before it enters the frictional section. Also find the distance D that it travels over the frictional section before coming to rest.



$$E_1 = \tfrac{1}{2}Kd^2 + mgh = \tfrac{1}{2}mV_B^2 \qquad E_2 = 0$$

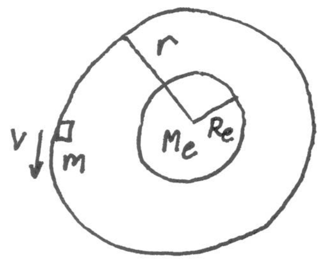
$$\Delta E = E_2 - E_1 = W_f = -mg\mu_K D$$

$$\tfrac{1}{2}Kd^2 + mgh = mg\mu_K D$$

$V_B = 18.9\,\mathrm{m/s}$

$D = 22.7\,\mathrm{m}$

Problem 2: The figure below shows a giant circular ring which has been constructed at a radius $r = 2R_e$ above the surface of the earth. The ring rotates with a period T such that the inhabitants, who live on the interior surface of the ring, feel a normal force N of the same magnitude, though oppositely directed, as that which they would feel on the surface of the earth. Given $G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$, $R_e = 6.37 \times 10^6\,\mathrm{m}$, and $M_e = 5.98 \times 10^{24}\,\mathrm{kg}$, find the period of rotation T of the ring



m

$$\bullet\longrightarrow \qquad N + \frac{GM_e m}{r^2} = m\frac{V^2}{r}$$

$$N = mg = \frac{GM_e m}{R_e^2} \qquad \frac{GM_e}{R_e^2} + \frac{GM_e}{r^2} = V^2/r = \left(\frac{2\pi}{T}\right)^2 r$$

$$r = 2R_e \qquad GM_e\left(\frac{1}{R_e^2} + \frac{1}{(2R_e)^2}\right) = \left(\frac{2\pi}{T}\right)^2 (2R_e)$$

$$\frac{GM_e}{R_e^2}(5/4) = (2R_e)\left(\frac{2\pi}{T}\right)^2$$

$$T^2 = (2\pi)^2 (8/5)\, R_e^3/(GM_e)$$

$$\underline{T = 106.6 \text{ minutes}}$$

Problem 3: You are stranded on a spherical asteroid. After walking around the equator of the asteroid you find that it has a radius of $R_a = 10^4\,\text{m}$. Furthermore you find that you are able to jump to a height $h_a = 200\,\text{m}$ above the surface of the asteroid, while on earth you can only jump to a height $h_e = 0.6\,\text{m}$. Find the mass M_a of the asteroid. Suppose that you are able to throw a rock to a height $H_e = 50\,\text{m}$ on earth. Will a rock thrown at the same speed escape from the asteroid? What is the corresponding escape velocity v_e? If it cannot escape, find the maximum height H_a that the rock attains. If the rock does escape the asteroid, find the speed at infinity v_∞ that the rock attains after escaping the asteroid.

$$\frac{1}{2} m V_1^2 = m g h_e = G M_a m \left(\frac{1}{R_a} - \frac{1}{R_a + h_a} \right) = \frac{G M_a m h_a}{R_a (R_a + h_a)}$$

$$M_a = \frac{g h_e R_a (R_a + h_a)}{G h_a} = \underline{4.50 \times 10^{16}\ \text{Kg}}$$

$$\frac{1}{2} m V_2^2 = m g H_e \qquad V_2 = (2 g H_e)^{1/2} = 31.3\ \text{m/s}$$

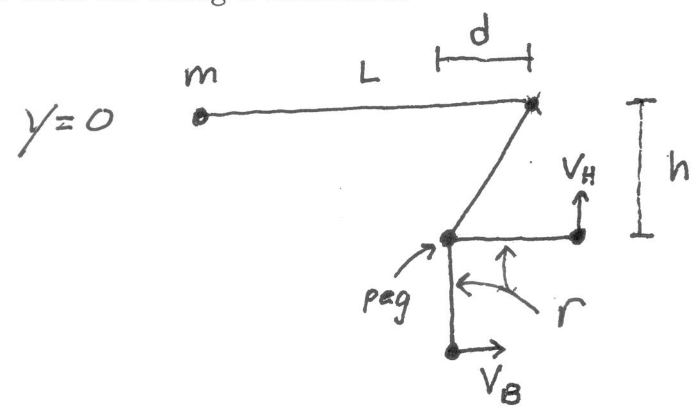
$$\frac{1}{2} m V_e^2 = \frac{G M_a m}{R_a} \qquad V_e = \left(\frac{2 G M_a}{R_a} \right)^{1/2} = \underline{24.5\ \text{m/s}}$$

It escapes :

$$\frac{1}{2} m V_2^2 - \frac{G M_a m}{R_a} = \frac{1}{2} m V_\infty^2$$

$$V_\infty^2 = V_2^2 - \frac{2 G M_a}{R_a} \qquad \underline{V_\infty = 19.50\ \text{m/s}}$$

Problem 4: The figure below shows a mass $m = 3\,\mathrm{kg}$ at the end of a string of length $L = 1\,\mathrm{m}$. The mass is released when the string is in a horizontal position as shown, and swings down until it reaches a peg a vertical distance $h = 0.5\,\mathrm{m}$ below and horizontal distance $d = 0.25\,\mathrm{m}$ to the left of the point where the string is attached. Find the velocity of the mass v_B when it reaches the bottom of its circular arc around the peg, and find the tension T_B in the string at this moment. Also find the velocity of the mass v_H and tension T_H at the point along this arc when the string is horizontal.



$W_T = 0$

at bottom:

$y_B = -h - (L - \sqrt{h^2+d^2}) = -h - r$

$\Delta E = 0$

$r = L - \sqrt{h^2+d^2}$

$0 = \frac{1}{2} m V_B^2 + m g y_B$

$\uparrow T_B$

$\downarrow -mg$

$V_B^2 = 2g(h+r)$ $\quad V_B = 4.29\,m/s$

$T_B - mg = m V_B^2 / r$ $\quad T_B = 154.9\,N$

at 90°:

$y_H = -h$

$0 = \frac{1}{2} m V_H^2 + m g y_H$

$-T_H$

$-mg$

$V_H^2 = 2gh$ $\quad V_H = 3.13\,m/s$

$+T_H = +m V_H^2 / r$ $\quad T_H = 66.7\,N$