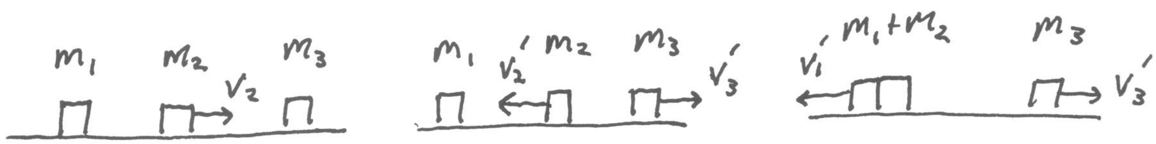
SMU Physics 1307 : Fall 2011

Final Exam

Problem 1: The figure below shows three blocks on a frictionless plane. The middle block has mass $m_2 = 1\,\mathrm{kg}$ and is initially moving to the right with velocity $v_2 = 3\,\mathrm{m/s}$. It then collides elastically with the right block of mass $m_3 = 5\,\mathrm{kg}$ which is initially at rest. Find the velocities v_2' and v_3' following this collision. The middle block then collides inelastically with the left block of mass m_1 which is initially at rest. If the final velocity v_1' of the combined mass $m_1 + m_2$ is equal in magnitude but opposite in direction from the final velocity of the right block, so that $v_1' = -v_3'$, find the mass m_1.



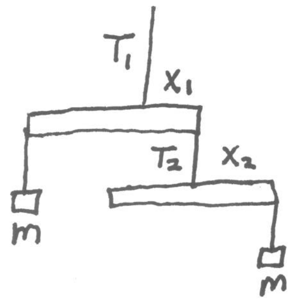
$$V_2' = \frac{m_2 - m_3}{m_2 + m_3} V_2 = -2\,m/s$$

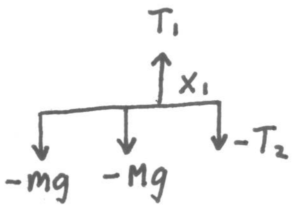
$$V_3' = \frac{2 m_2}{m_2 + m_3} V_2 = 1\,m/s$$

$$V_1' = \frac{m_2 V_2'}{m_1 + m_2} = -V_3'$$

$$m_1 = -m_2 - m_2 V_2'/V_3' = 1\,kg$$

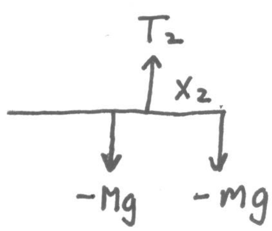
Problem 2: The figure below shows a mobile constructed from two identical bars of length $L = 1\,\mathrm{m}$ and mass $M = 3\,\mathrm{kg}$. Attached to the end of each bar as shown is an identical mass $m = 2\,\mathrm{kg}$. Find the lengths x_1 and x_2 shown in the figure which are required to maintain the mobile in equilibrium. Also find the tensions T_1 and T_2 which support the bars and masses.





$$T_1 - T_2 - (m+M)g = 0$$

$$-T_1 X_1 + Mg\,L/2 + mgL = 0$$



$$T_2 - (m+M)g = 0$$

$$-T_2 X_2 + Mg\,L/2 = 0$$

$$T_1 = 2(m+M)g = 98\,N$$

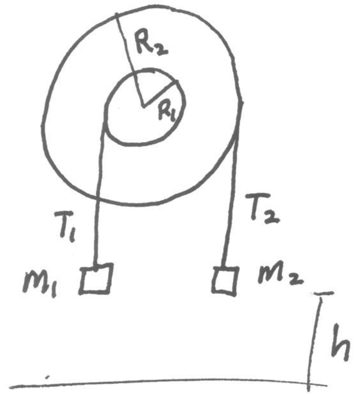
$$T_2 = (m+M)g = 49\,N$$

$$X_1 = \frac{(M/2 + m)}{2(M+m)} L = 0.35\,m$$

$$X_2 = \frac{(M/2)}{(M+m)} L = 0.3\,m$$

Problem 3: The figure below shows two disks of radii $R_1 = 0.05\,\mathrm{m}$ and $R_2 = 0.1\,\mathrm{m}$ and masses $M_1 = 1\,\mathrm{kg}$ and $M_2 = 3\,\mathrm{kg}$ with respective moments of inertia $I_1 = \frac{1}{2}M_1R_1^2$ and $I_2 = \frac{1}{2}M_2R_2^2$. Two masses $m_1 = 3\,\mathrm{kg}$ and $m_2 = 2\,\mathrm{kg}$ are attached to the disks as shown. Find the angular acceleration α of the disks and find the respective linear accelerations (taking upward to be positive) a_1 and a_2 of the masses. Also find the tensions T_1 and T_2 in the respective strings. If the masses begin a distance $h = 0.15\,\mathrm{m}$ above the ground, which mass (either m_1 or m_2) hits the ground first and what is its velocity (either v_1 or v_2)? (Hint: What is the sign of α?)

$I = I_1 + I_2$



$T_1 - m_1 g = m_1 a_1$

$T_2 - m_2 g = m_2 a_2$

$T_1 R_1 - T_2 R_2 = I\alpha$

$a_1 = -R_1\alpha$

$a_2 = R_2\alpha$

$$m_1 g R_1 - m_1 R_1^2\alpha - m_2 g R_2 - m_2 R_2^2 \alpha = I\alpha$$

$$\alpha = \frac{(m_1R_1 - m_2R_2)g}{(I + m_1R_1^2 + m_2R_2^2)} = -11.2\ s^{-1} \quad \left(\begin{matrix} m_2 \text{ hits} \\ \text{ground}\end{matrix}\right)$$

$a_1 = -R_1\alpha = 0.56\ m/s^2$ $\quad T_1 = m_1 g - m_1 R_1 \alpha = 31.1\ N$

$a_2 = R_2\alpha = -1.12\ m/s^2$ $\quad T_2 = m_2 g + m_2 R_2 \alpha = 17.4\ N$

$\theta(t=0) = 0$

$y_1 = -R_1\theta + h$

$y_2 = R_2\theta + h$

$(y_1 - h)R_2 = -(y_2 - h)R_1$

if $y_2 = 0$

$y_1 = h + h\,R_1/R_2$

$v_1 = -R_1\omega$

$v_2 = R_2\omega$

$\cancel{K} + U = K' + U'$

$\cancel{m_1 g h} + m_2 g h = \cancel{m_1 g h} + m_1 g h\,R_1/R_2$
$+ \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I\omega^2$

$(m_2 - m_1 R_1/R_2) g h = \frac{1}{2} v_2^2 \left(m_1 R_1^2/R_2^2 + m_2 + I/R_2^2\right)$

$$v_2^2 = \frac{2gh(m_2 - m_1R_1/R_2)}{(m_1R_1^2/R_2^2 + m_2 + I/R_2^2)}$$

$v_2 = -0.580\ m/s$

Problem 4: The figure at left below shows a disk of radius $R = 5\,\mathrm{m}$ and mass $M = 200\,\mathrm{kg}$ with moment of inertia $I = \frac{1}{2}MR^2$ which is initially spinning clockwise with angular velocity $\omega = -2\,\mathrm{rad/s}$ as shown. A person of mass $m = 60\,\mathrm{kg}$ rides on the perimeter of the disk and initially spins with the same angular frequency ω. As shown in the figure at right, suppose that the person begins to run on the disk until they reach a speed with respect to the surface of the disk so that they are at rest with respect to the ground. What is the new angular velocity ω' of the disk? What is the change in kinetic energy $\Delta K = K' - K$ of the combined system? Assume that there are no external torques on the system.

$$L = L'$$

$$(I + mR^2)\,\omega = I\omega'$$

$$\omega' = (1 + mR^2/I)\,\omega = -3.2\,s^{-1}$$

$$K = \tfrac{1}{2}(I + mR^2)\,\omega^2 = 8 \times 10^3\,\mathrm{J}$$

$$K' = \tfrac{1}{2} I \omega'^2 = \tfrac{1}{2}(I + mR^2)^2\,\omega^2/I$$

$$K' = (1 + mR^2/I)\,K = 1.28 \times 10^4\,\mathrm{J}$$

$$\Delta K = K' - K = 4.8 \times 10^3\,\mathrm{J}$$