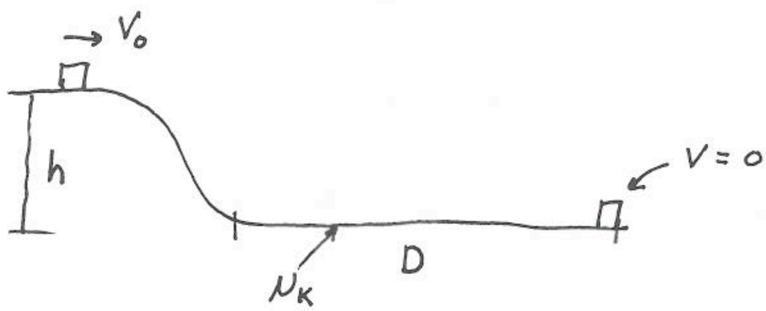
SMU Physics 1307 : Spring 2009

Exam 2

Problem 1 : A bobsled with an initial velocity $v_0 = 5\,\mathrm{m/s}$ slides down a frictionless hill of height $h = 15\,\mathrm{m}$. It then slides on a horizontal surface of coefficient of kinetic friction $\mu_k = 0.6$. Find the distance D it travels along the horizontal surface before stopping.



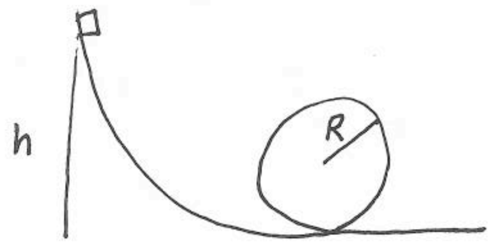
$$E_0 = \frac{1}{2} m V_0^2 + mgh \qquad E_f = 0$$

$$\Delta E = E_f - E_0 = W_{nc} = -\mu_K m g D$$

$$\frac{1}{2} m V_0^2 + mgh = \mu_K m g D$$

$$\underline{D = (\frac{1}{2} V_0^2 + gh)/(\mu_K g)}$$

Problem 2 : A block slides down a frictionless hill of height $h = 10\,\mathrm{m}$, and enters a loop of radius $R = 2\,\mathrm{m}$. Find the velocity of the block at the bottom, half way up, and at the top of the loop. If the car weighs $1000\,\mathrm{kg}$, also find the normal force at each of these points.



$\Delta E = 0$

$E = mgh$

bottom:

$$mgh = \tfrac{1}{2} m V_b^2$$

$$V_b^2 = 2gh$$

$$N - mg = \frac{m V_b^2}{R} \qquad N = mg + 2mg\, h/R$$

$$N = mg(2h/R + 1)$$

90°

$$mgh = mgR + \tfrac{1}{2} m V_{90}^2$$

$$V_{90}^2 = 2g(h - R)$$

$$N = m\frac{V_{90}^2}{R} = 2mg(h/R - 1)$$

$$N = mg\left(\frac{2h}{R} - 2\right)$$

top

$$mgh = 2mgR + \tfrac{1}{2} m V_t^2$$

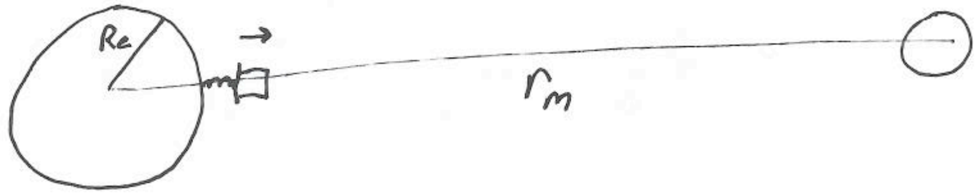
$$V_t^2 = 2g(h - 2R)$$

$$N + mg = \frac{m V_t^2}{R}$$

$$N = -mg + 2mg(h/R - 2)$$

$$N = mg(2h/R - 5)$$

Problem 3 : A scientist devises a very straightforward way to get to the moon by constructing a spring loaded launch vehicle. If the spring is pulled back $d = 1\,\mathrm{m}$, and the spacecraft weighs $m = 1000\,\mathrm{kg}$, what must be the spring constant k? Assume the spring is released at the surface of the earth, $r_1 = R_e = 6.37 \times 10^6\,\mathrm{m}$, and that the spacecraft rises to the radius of the moons' orbit $r_2 = r_m = 3.85 \times 10^8\,\mathrm{m}$ before falling back to earth. Neglect the gravitational pull of the moon, and use the correct potential for an object of mass m in the non-uniform gravitational field of the earth. You will need the mass of the earth $M_e = 5.97 \times 10^{24}\,\mathrm{kg}$, and Newton's constant $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}$.



$K_o = 0$ $\qquad E_o = E_f$

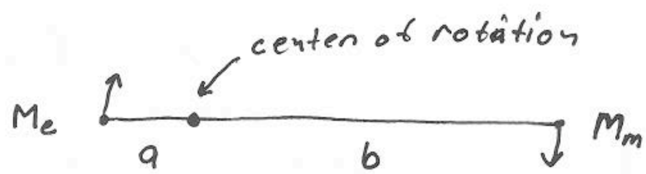
$K_f = 0 \quad \Rightarrow \quad U_o = U_f$

$$\tfrac{1}{2} K d^2 - \frac{G m M_e}{R_e} = -\frac{G m M_e}{r_m}$$

$$\tfrac{1}{2} K d^2 = G m M_e \left(\tfrac{1}{R_e} - \tfrac{1}{r_m} \right)$$

$$K = \frac{2 G m M_e}{d^2} \left(\tfrac{1}{R_e} - \tfrac{1}{r_m} \right)$$

Problem 4 : The earth and moon actually orbit around a common center under their gravitational attraction. The earth has mass $M_e = 5.97 \times 10^{24}\,\text{kg}$ and orbits at $a = 5.0 \times 10^6\,\text{m}$ around the common center. Take the moon to have unknown mass M_m and to orbit at $b = 3.8 \times 10^8\,\text{m}$ around the common center. Find the mass M_m and the period T of the orbit. Note that the period T will be the same for both objects, each of which will have an equation relating force to centripetal acceleration. Once solved these will give the mass and orbital period (about 27 days) of the moon. Note that the common center is inside the earth, since $R_e = 6.37 \times 10^6\,\text{m}$, although this is not relevant for solving the problem.



$$\underline{T_e = T_m = T}$$

$$M_e \frac{V_e^2}{a} = \frac{G M_e M_m}{(a+b)^2} \qquad\qquad M_m \frac{V_m^2}{b} = \frac{G M_e M_m}{(a+b)^2}$$

$$V_e = 2\pi a / T \qquad\qquad V_m = 2\pi b / T$$

$$\left(\frac{2\pi}{T}\right)^2 a = \frac{G M_m}{(a+b)^2} \qquad\qquad \left(\frac{2\pi}{T}\right)^2 b = \frac{G M_e}{(a+b)^2}$$

$$\underline{T = 2\pi \left(\frac{b(a+b)^2}{G M_e} \right)^{1/2}}$$

$$\underline{M_m = \left(\frac{2\pi}{T}\right)^2 \frac{a(a+b)^2}{G}}$$