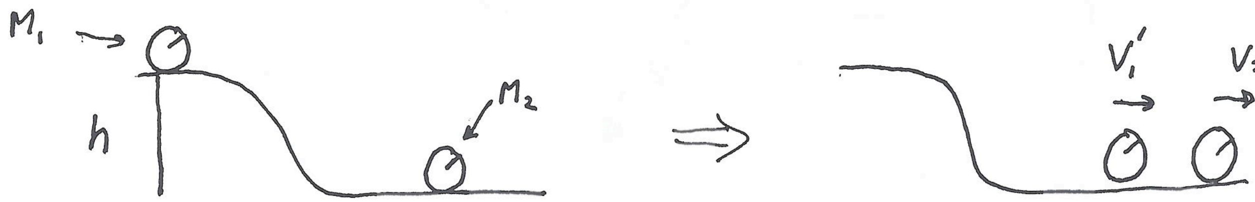
SMU Physics 1307 : Spring 2009

Exam 3

Problem 1 : A ball ($I = \frac{2}{5}MR^2$) of radius $R_1 = 0.1\,\mathrm{m}$ and mass $M_1 = 1\,\mathrm{kg}$ begins at rest and rolls down a hill of height $h = 10\,\mathrm{m}$. It then slides along a horizontal frictionless surface and collides elastically with another ball of mass $M_2 = 0.5\,\mathrm{kg}$ which is initially at rest. During the collision neither object changes angular momentum since the surface is frictionless. Find the velocity v_1 of the first mass just prior to the collision, and find the velocities v_1' and v_2' of both balls following the collision. Show that total energy is conserved in this process by comparing the initial potential energy with the final kinetic energy.



$$M_1 g h = \tfrac{1}{2} M_1 V_1^2 + \tfrac{1}{2} I_1 \omega_1^2 = \tfrac{1}{2} M_1 V_1^2 (1 + \tfrac{2}{5})$$

$$I_1 = \tfrac{2}{5} M_1 R_1^2 \qquad V_1^2 = \omega_1^2 R_1^2$$

$$\underline{V_1^2 = \tfrac{10}{7} g h} \qquad \underline{V_1 = 11.83\ \mathrm{m/s}}$$

$$V_2 = 0 \qquad \omega_2 = 0$$

$$L_1 = L_1' \qquad L_2 = L_2' = 0$$

$$\omega_1 = \omega_1' \qquad \omega_2 = \omega_2' = 0$$

(ang mom conserved in collision)

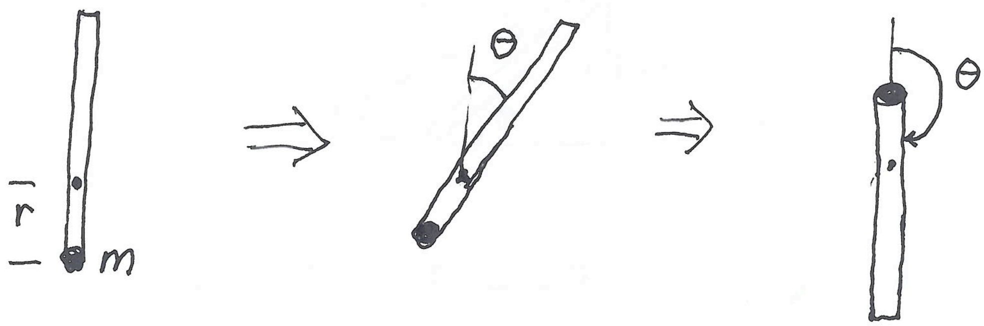
$$V_1' = \frac{M_1 - M_2}{M_1 + M_2} V_1 = \underline{3.94\ \mathrm{m/s}} \qquad V_2' = \frac{2M_1}{M_1 + M_2} V_1 = \underline{15.78\ \mathrm{m/s}}$$

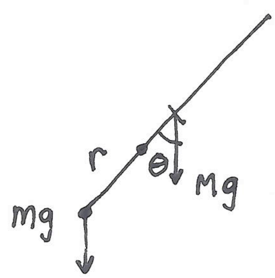
$$E = M_1 g h = \tfrac{1}{2} M_1 V_1^2 + \tfrac{1}{2} I_1 \omega_1^2 = \underline{98\ \mathrm{J}}$$

$$E' = \tfrac{1}{2} M_1 V_1'^2 + \tfrac{1}{2} I_1 \omega_1'^2 + \tfrac{1}{2} M_2 V_2'^2 = \underline{98\ \mathrm{J}}$$

Energy is conserved.

Problem 2 : The figure below show a uniform beam of length $L = 3\,\mathrm{m}$ and mass $M = 5\,\mathrm{kg}$. It rotates around an axis $r = 1\,\mathrm{m}$ from one end. This end is initially downward and is attached to a mass $m = 2\,\mathrm{kg}$. If, starting from rest, the beam is allowed to freely rotate, find the angular acceleration $\alpha(\theta)$ and the angular velocity $\omega(\theta)$ as a function of the angle. You will need the parallel axis theorem, and the momentum of inertia $I_{cm} = \frac{1}{12}ML^2$ about the center of mass of a uniform beam. You will also need the moment of inertia $I_m = mr^2$ of a mass m at a distance r from the rotation axis.





$$I = \tfrac{1}{12}ML^2 + (L/2 - r)^2 M$$

$$\tau = mgr\sin\theta - Mg(L/2 - r)\sin\theta$$
$$= (I + mr^2)\alpha$$

$$\alpha(\theta) = \frac{(mr - M(L/2 - r))g\sin\theta}{(mr^2 + \frac{1}{12}ML^2 + M(L/2 - r)^2)}$$

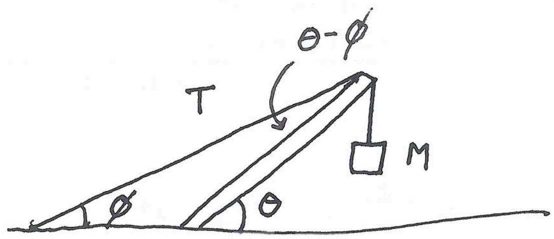
$$K = \tfrac{1}{2}(I + mr^2)\omega^2 \qquad U = Mg(L/2 - r)\cos\theta$$
$$- mgr\cos\theta = U(\theta)$$

$$K_0 + U_0 = K_f + U_f$$

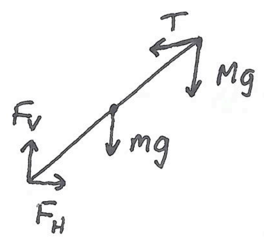
$$\cancel{\omega^2(\theta) = U(\theta)} \qquad K_0 = 0 \quad U_0 = U(0)$$

$$\tfrac{1}{2}(I + mr^2)\omega^2(\theta) = U(0) - U(\theta)$$

Problem 3 : The figure below shows a block of mass $M = 200\,\mathrm{kg}$ hanging from a uniform beam of length $L = 10\,\mathrm{m}$ and mass $m = 80\,\mathrm{kg}$. The cable attached to the top of the beam, which is not connected to the cable which holds the mass, makes an angle of $\phi = 25°$ with the horizontal. The beam makes an angle of $\theta = 40°$ with the horizontal. Find the tension T in cable attached to the top of the beam, and the horizontal F_H and vertical F_V components of the force (indicate directions) exerted by the horizontal surface on the bottom of the beam.



beam: uniform of mass m



$$F_x = -T\cos\phi + F_H = 0$$

$$F_y = -T\sin\phi - Mg - mg + F_V = 0$$

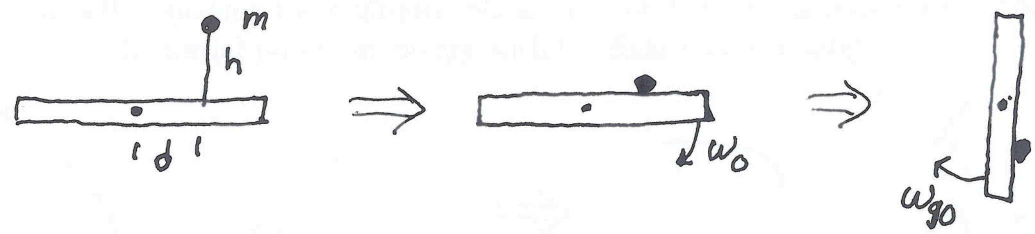
$$\tau = -mg\,L/2\,\cos\theta - MgL\cos\theta + T\sin(\theta-\phi)L = 0$$

$$T = \frac{(Mg + mg/2)\cos\theta}{\sin(\theta-\phi)} = \underline{6961N}$$

$$F_H = T\cos\phi = \underline{6309\,N}$$

$$F_V = Mg + mg + T\sin\phi = \underline{5686\,N}$$

Problem 4 : The figure below shows a uniform rod of length $L = 2\,\mathrm{m}$ and mass $M = 2\,\mathrm{kg}$ which can rotate around about its center of mass ($I = \frac{1}{12}ML^2$). It is initially horizontal and a mass $m = 0.5\,\mathrm{kg}$ is dropped from a height $h = 1\,\mathrm{m}$ onto the beam at a distance $d = 0.5\,\mathrm{m}$ from the center of rotation. The mass sticks to the beam, and thus then has $I_m = md^2$, and both begin to rotate downward. Find the angular velocity ω_0 of the system immediately after the collision when the rod is still horizontal, as well as the angular velocity ω_{90} when the rod becomes vertical. Assume that, despite the gravitational forces acting, during the time of the collision any external torques on the system can be ignored. Find how much energy $(K + U)$ is lost in this process.

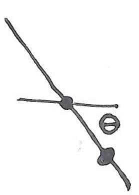


during collision:

$\Delta L = 0$ $\qquad L_m = -V_m m d \qquad V_m = \sqrt{2gh}$

$$-\sqrt{2gh}\, m d = (I + md^2)\omega_0$$

$$\omega_0 = \frac{-\sqrt{2gh}\, m d}{(\frac{1}{12}ML^2 + md^2)} = \underline{-1.40\,s^{-1}}$$



$K_0 + U_0 = K + U \qquad K = \frac{1}{2}(I + md^2)\omega^2$

$K_0 = \frac{1}{2}(I + md^2)\omega_0^2 \qquad U = -mgd\sin\theta$

$U_0 = 0$

$$\frac{1}{2}(I + md^2)\omega_0^2 = \frac{1}{2}(I + md^2)\omega_{90}^2 - mgd$$

$$\omega_{90}^2 = \omega_0^2 + \frac{2mgd}{(I + md^2)} \qquad \underline{\omega_{90} = -2.85\,s^{-1}}$$

Initial energy : mgh

final energy : $\frac{1}{2}(I + md^2)\omega_0^2$ $\qquad \Delta E = \frac{1}{2}(I + md^2)\omega_0^2 - mgh$

$$\underline{\underline{\Delta E = -4.13\ J}}$$