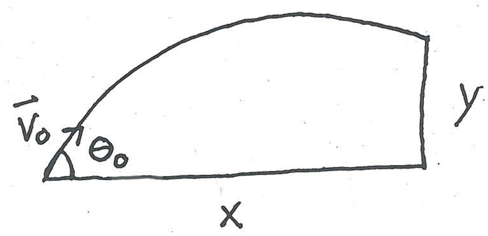
SMU Physics 1307 : Spring 2010

Exam 1

Problem 1 : The figure below shows a baseball that is struck at ground level at an initial angle $\theta_0 = 35°$, with an unknown magnitude of velocity $v_o = |\vec{v}_o|$. It travels an unknown horizontal distance x before striking a foul pole at a height $y = 30\,\mathrm{m}$. The total time of flight of the baseball is $t_f = 3.4\,\mathrm{s}$. Find the distance x, the magnitude of velocity v_o, and the vertical component of the velocity v_y at the moment when the ball impacts the foul pole. Use $g = 9.8\,\mathrm{m/s^2}$.



$$x = V_o \cos\theta_o t_f$$

$$y = V_o \sin\theta_o t_f - \tfrac{1}{2} g t_f^2$$

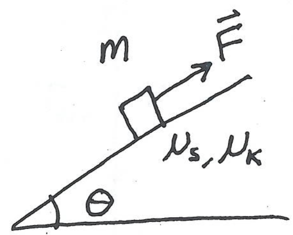
so,

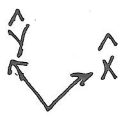
$$V_o = (y + \tfrac{1}{2} g t_f^2)/(\sin\theta_o t_f) = \underline{\underline{44.43\ \mathrm{m/s}}}$$

$$\Rightarrow \quad x = \underline{\underline{123.74\ \mathrm{m}}}$$

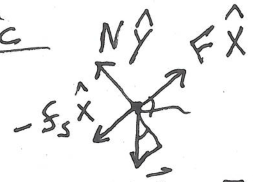
$$V_y = V_o \sin\theta_o - gt = \underline{\underline{-7.84\ \mathrm{m/s}}}$$

Problem 2 : The figure below shows a block of mass $m = 3\,\text{kg}$ on a inclined plane of angle $\theta = 25°$. The interface between the block and plane has coefficients of static and kinetic friction given by $\mu_s = 0.8$ and $\mu_k = 0.6$, respectively. An external force of magnitude F acts up the plane. Find the critical value of the force $F = F_c$ at which the block starts to slide. Now suppose that a force of $F = 2F_c$ is applied. Find the acceleration a of the block up the plane.





static



$m\vec{g} = -mg\sin\theta\,\hat{x} - mg\cos\theta\,\hat{y}$

$F - f_s - mg\sin\theta = 0$

$N - mg\cos\theta = 0$

$f_s \leq \mu_s N$

slip when: $(F = F_c)$ $\quad F_c - mg\sin\theta = \mu_s mg\cos\theta$

$$F_c = mg(\sin\theta + \mu_s\cos\theta) = \underline{\underline{33.74\,N}}$$

after slippage : $F = 2F_c$

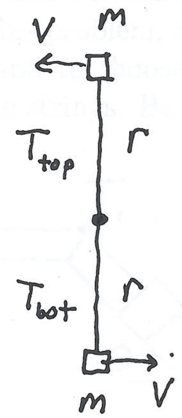
$F - \mu_k N - mg\sin\theta = ma$

$N - mg\cos\theta = 0$

$2F_c - mg(\sin\theta + \mu_k\cos\theta) = ma$

$$a = \frac{2F_c}{m} - g(\sin\theta + \mu_k\cos\theta) = \underline{\underline{13.02\,m/s^2}}$$

Problem 3 : The figure below shows a mass $m = 5\,\mathrm{kg}$ which is attached to a string and whirls in a vertical circle (thus under the influence of gravity) of radius $r = 0.75\,\mathrm{m}$. At the top of the circle the tension in the string is $T_{top} = 85\,\mathrm{N}$. Find the velocity v of the mass, and find the tension T_{bot} in the string at the bottom of the circle.



top:

$-mg\,\hat{y}$

$-T_{top}\,\hat{y}$

$\vec{a} = -\frac{v^2}{r}\,\hat{y}$

$$-mg - T_{top} = -m\frac{v^2}{r}$$

$$v^2 = \frac{r}{m}(T_{top} + mg)$$

$$v = \underline{\underline{4.48\ \mathrm{m/s}}}$$

bot:

$T_{bot}\,\hat{y}$

$-mg\,\hat{y}$

$\vec{a} = \frac{v^2}{r}\,\hat{y}$

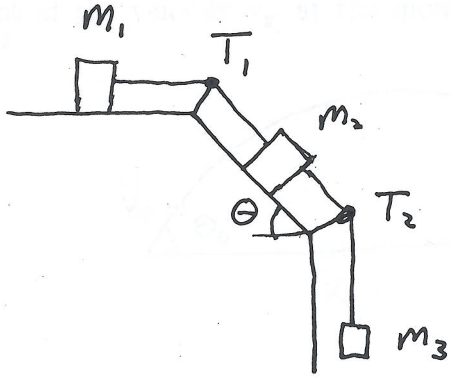
$$T_{bot} - mg = \frac{v^2}{r}\cdot m$$

$$T_{bot} = mg + mv^2/r = \underline{\underline{183\ N}}$$

Problem 4 : The figure below shows three masses which are connected together with two strings. The mass $m_1 = 2\,\text{kg}$ rests on a horizontal frictionless surface. The mass $m_2 = 3\,\text{kg}$ rests on an inclined frictionless surface with angle $\theta = 30°$ as shown. The mass $m_3 = 5\,\text{kg}$ hangs vertically. Find the acceleration a of the system, with a taken to be positive when m_1 moves to the right. Also find the tensions T_1 and T_2 in the strings as shown in the diagram. It will be helpful in this problem, since there is no friction, to ignore the directions perpendicular to the strings, and to choose to work with separate coordinate systems for each object which are along the strings. Be careful about the signs of the accelerations.

$\hat{x} \rightarrow$

$\vec{a}_1 = a\hat{x}$



ignore $\hat{y}$ eqs

m_1 :

$\vec{a}_1 = a\hat{x}$

$N\hat{y}$, $T_1\hat{x}$, $-mg\hat{y}$

$T_1 = m_1 a$

m_2 :

$\vec{a}_2 = a\hat{x}$

$-T_1\hat{x}$, $N\hat{y}$, $T_2\hat{x}$, $-m_2 g\cos\theta\,\hat{y}$, $+m_2 g\sin\theta\,\hat{x}$

$$-T_1 + T_2 + m_2 g\sin\theta = m_2 a$$

m_3 :

$\vec{a}_3 = -a\hat{y}$

$T_2\hat{y}$, $-m_3 g\hat{y}$

$$T_2 - m_3 g = -m_3 a$$

plug in for T_1 and T_2 : $-m_1 a + m_3 g - m_3 a + m_2 g\sin\theta = m_2 a$

$$m_3 g + m_2 g\sin\theta = (m_1 + m_2 + m_3)a$$

$$a = 6.37\,m/s^2$$

$T_1 = m_1 a = 12.74\,N$

$T_2 = m_3 g - m_3 a = 17.15\,N$