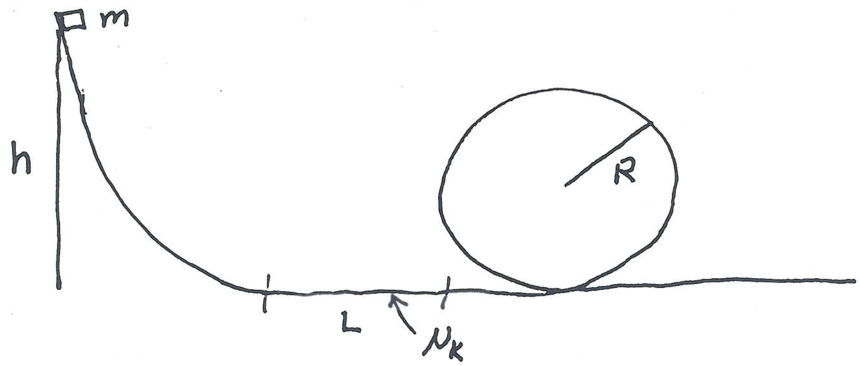
SMU Physics 1307 : Spring 2010

Exam 2

Problem 1 : The figure below shows a hill of height $h = 10\,\text{m}$ with a loop of radius $R = 3\,\text{m}$ at the bottom. Just before the loop there is a section of length $L = 5\,\text{m}$ with a coefficient of kinetic friction μ_k. Find the highest value of μ_k such that an object of mass m makes it around the loop. What is the corresponding velocity v at the top of the loop?



$$\tfrac{1}{2} m v^2 + mg2R - mgh = -\mu_k mgL$$

$$mg = mv^2/R$$

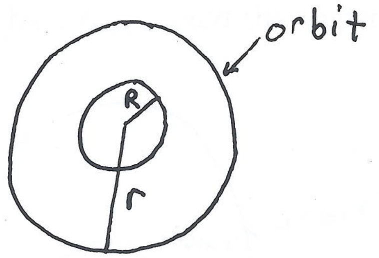
$$\underline{\underline{v^2 = gR}}$$

drop mg

$$\tfrac{1}{2} R + 2R - h = -\mu_k L$$

$$\mu_k = \frac{h - \frac{5}{2}R}{L}$$

Problem 2 : You are observing a distant planet through a telescope and see a small moon orbiting the planet at an orbital radius $r = 2.77\,R$, where R is the unknown radius of the planet. The period of the orbit is observed to be $T = 2.76 \times 10^4\,\mathrm{s}$. You then descend onto the planet surface and measure the acceleration due to gravity to be $g = 3.73\,\mathrm{m/s^2}$. Find the mass M of the planet, and find its radius R. You will need to use $G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$.



$$\left(\frac{T}{2\pi}\right)^2 = \frac{r^3}{GM}$$

$$g = \frac{GM}{R^2}$$

so,

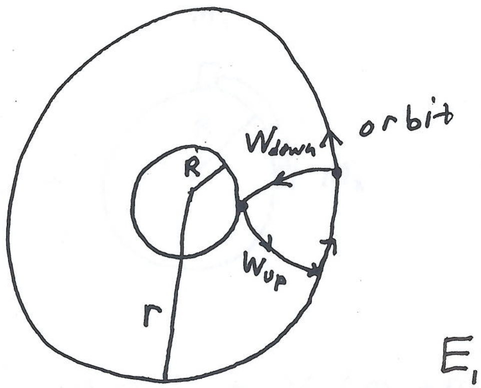
$$\left(\frac{T}{2\pi}\right)^2 = \frac{r^3}{gR^2} = \left(\frac{r}{R}\right)^2 \frac{r}{g}$$

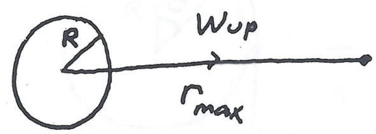
$$r = g\,(R/r)^2 \left(\frac{T}{2\pi}\right)^2$$

$$R = r/2.77$$

$$M = \frac{gR^2}{G}$$

Problem 3 : The mass of the moon is $M = 7.35 \times 10^{22}$ kg , and its radius is $R = 1.74 \times 10^{6}$ m . Assume that there is a lunar lander of mass $m = 5 \times 10^{4}$ kg which is in a circular orbit at $r = 3R$, and which must touch down (with zero velocity) on the lunar surface. Neglecting the rotation of the moon, find how much work W_{down} must be done by the thrusters on the lander to touch down on the surface. Also, find how much work W_{up} must be done by the thrusters on the lander to return to precisely the same orbit. Starting from rest on the moon surface, if the work W_{up} was applied entirely to radial, rather than circular, motion of the lander, how high r_{max} would the lander get before falling back onto the moon.





$$E_1 = K_1 + U_1 = -\frac{GMm}{2r} = -\frac{GMm}{6R}$$

$\underline{K_2 = 0}$

$$E_2 = U_2 = -\frac{GMm}{R} \qquad W_{down} = E_2 - E_1 = -\frac{5}{6}\frac{GMm}{R}$$

$$W_{up} = E_1 - E_2 = -W_{down} = \frac{5}{6}\frac{GMm}{R}$$

$\underline{K_3 = 0}$

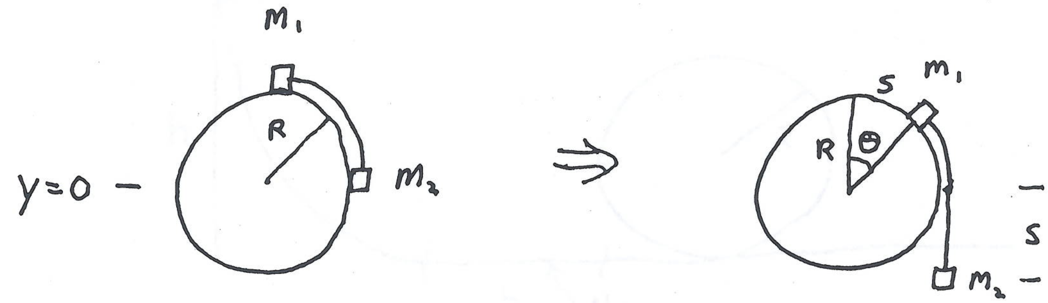
$$E_3 = U_3 = -\frac{GMm}{r_{max}}$$

$$E_3 - E_2 = W_{up} = E_1 - E_2$$

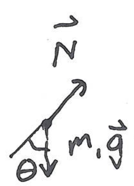
$$\underline{E_3 = E_1} \qquad -\frac{GMm}{r_{max}} = -\frac{GMm}{6R}$$

$$\underline{\underline{r_{max} = 6R}}$$

Problem 4 : The figure at left below shows a frictionless surface of circular curvature of radius $R = 1\,\mathrm{m}$. A mass $m_1 = 2\,\mathrm{kg}$ is placed at the top of the surface and is connected by a string to another mass m_2 which is initially at a position ($y = 0$) at the same height as the horizontal diameter of the circle. As shown in the figure at right, the objects move together until m_1 leaves the surface at $\theta = 41°$. During this time m_2 will have fallen a distance equal to the arclength $s = R\theta * \pi/180$ (or $s = R\theta$ if θ is expressed in radians) that m_1 will have moved along the circle. Find the mass m_2 and the velocity v of the objects when m_1 leaves the surface. You will need to use $g = 9.8\,\mathrm{m/s^2}$.



$$m_1 g R = \tfrac{1}{2}(m_1+m_2)v^2 + m_1 g R\cos\theta - m_2 g R\theta$$



$$N - m_1 g\cos\theta = -m_1 v^2/R$$

$$N=0 \Rightarrow \underline{\underline{v^2 = gR\cos\theta}}$$

drop gR:

$$m_1 = \tfrac{1}{2}(m_1+m_2)\cos\theta + m_1\cos\theta - m_2\theta$$

$$m_2(\theta - \tfrac{1}{2}\cos\theta) = m_1(\tfrac{3}{2}\cos\theta - 1)$$

$$m_2 = m_1\frac{(\tfrac{3}{2}\cos\theta - 1)}{(\theta - \tfrac{1}{2}\cos\theta)}$$