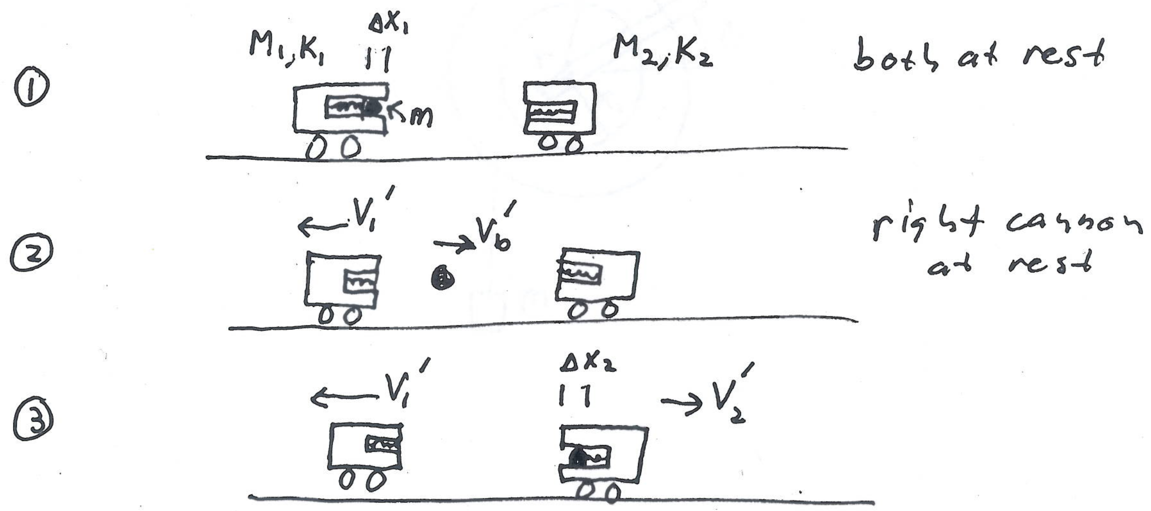
SMU Physics 1307 : Spring 2010

Exam 3

Problem 1 : The figure below shows two spring cannons of masses $M_1 = 250\,\text{kg}$ and $M_2 = 350\,\text{kg}$, with spring constants $k_1 = 1250\,\text{N/m}$ and $k_2 = 1600\,\text{N/m}$, respectively. Both cannons are initially at rest on a frictionless surface, with a cannonball of mass $m = 20\,\text{kg}$ lodged in the left cannon, compressing the spring by $\Delta x_1 = 0.3\,\text{m}$. The spring inside the cannon at left is then released, and the cannonball flies directly into cannon at right. Find the final velocity v_1' of the left cannon and the velocity v_b' of the cannonball while it is in flight. Also find the final velocity v_2' of the right cannon after the cannonball lodges inside it, as well as the distance Δx_2 that the right spring is compressed.



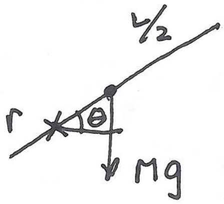
$$0 = M_1 V_1' + m V_b'$$

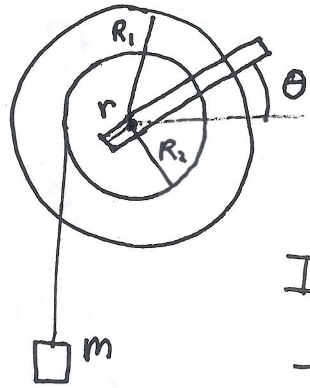
$$\tfrac{1}{2} K_1 X_1^2 = \tfrac{1}{2} M_1 V_1'^2 + \tfrac{1}{2} m V_b'^2 \quad \Rightarrow \underline{V_1', V_b'}$$

$$0 = M_1 V_1' + (M_2 + m) V_2' \Rightarrow \underline{V_2'}$$

$$\tfrac{1}{2} m V_b'^2 = \tfrac{1}{2} K X_2^2 + \tfrac{1}{2}(M_2 + m) V_2'^2 \Rightarrow \underline{X_2}$$

Problem 2 : The figure below shows two uniform disks of masses $M_1 = 5\,\text{kg}$ and $M_2 = 3\,\text{kg}$ and radii $R_1 = 0.3\,\text{m}$ and $R_2 = 0.2\,\text{m}$ which are stuck together and rotate about a common center. Also attached to the disks is a uniform beam of mass $M = 4\,\text{kg}$ and length $L = 0.5\,\text{m}$. The axis of rotation is at a distance $r = 0.1\,\text{m}$ from one end of the beam. As shown in the figure, there is a string wrapped around the smaller disk which is attached to a mass $m = 10\,\text{kg}$. Find the angular acceleration $\alpha(\theta)$ of the system as a function of the angle θ which appears in the figure. Also find the tension $T(\theta)$ in the string as a function of θ. The moment of inertia of a disk of mass M and radius R about its center of mass is $I = \frac{1}{2}MR^2$. The moment of inertia of a uniform beam of mass M and length L about its center of mass is $I = \frac{1}{12}ML^2$.





$$I = \tfrac{1}{2}M_1R_1^2 + \tfrac{1}{2}M_2R_2^2 + \frac{1}{12}ML^2 + M(L/2 - r)^2$$

$$I\alpha = TR_2 - Mg(L/2 - r)\cos\theta$$

$$T - mg = ma = -m\alpha R_2$$

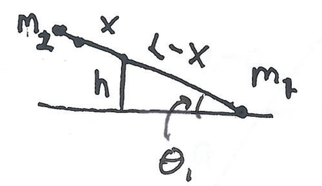
$$a = -\alpha R_2$$

$$I\alpha = mgR_2 - m\alpha R_2^2 - Mg(L/2 - r)\cos\theta$$

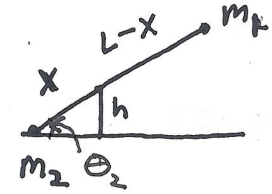
$$\alpha = \frac{mgR_2 - Mg(L/2 - r)\cos\theta}{I + mR_2^2}$$

$$T = mg - mR_2\alpha$$

Problem 3 : The figure below shows a (uniform) see-saw of length $L = 7\,\mathrm{m}$ and mass $M = 60\,\mathrm{kg}$, with moment of inertia about the center of mass given by $I_{\mathrm{com}} = \frac{1}{12}ML^2$. The axis of rotation is located at $x = 3\,\mathrm{m}$ from the left end of the see-saw and is elevated by $h = 1.5\,\mathrm{m}$. Initially the right end of the see-saw is on the ground and has a person of mass $m_1 = 100\,\mathrm{kg}$ sitting on it, while the left end of the see-saw has a person of mass $m_2 = 150\,\mathrm{kg}$ sitting on it. Find the angular velocity ω of the see-saw when the left end hits the ground, and find the velocities v_1 and v_2 of the two masses at this time. In computing the potential energies it may be helpful to define the ground to be $y = 0$.



$\Rightarrow$



M: mass of beam

L: length of beam

$$E_1 = m_2 g L \sin\theta_1 = m_2 g L h/(L-x)$$

$$E' = m_1 g L \sin\theta_2 + I\omega^2 + \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

$$v_1^2 = (L-x)^2\omega^2 \qquad v_2^2 = x^2\omega^2$$

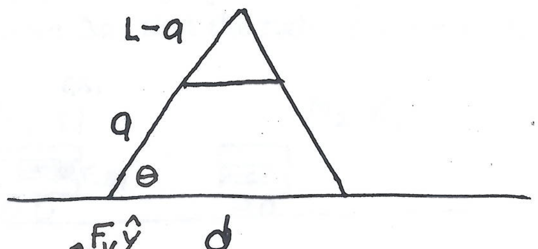
$$I = \frac{1}{12} ML^2 + M(L/2 - x)^2$$

$$E' = m_1 g L h/x + \tfrac{1}{2}\underbrace{(I + m_1(L-x)^2 + m_2 x^2)}_{I_{total}}\omega^2$$

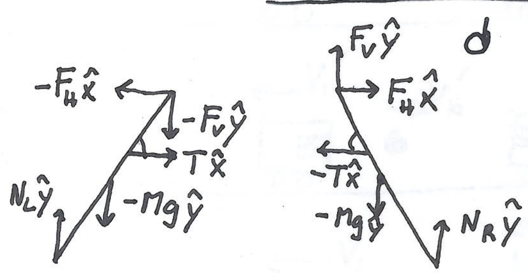
$$\Delta E = 0$$

$$\tfrac{1}{2} I_{total}\,\omega^2 = gLh\left(\frac{m_2}{(L-x)} - \frac{m_1}{x}\right) = \frac{gLh}{x(L-x)}\left(m_2 x - m_1(L-x)\right)$$

Problem 4 : The figure below shows a ladder standing on a frictionless surface (no horizontal forces from the floor) consisting of two identical sides of length $L = 3\,\mathrm{m}$ and mass $M = 30\,\mathrm{kg}$. The bottoms of the ladder are separated by $d = 2.5\,\mathrm{m}$, and there is a horizontal wire connecting the two sides which is at a distance $a = 2.0\,\mathrm{m}$ from the bottom of each side of the ladder. Find the (identical) normal forces N from the ground on each side, the tension T in the wire, and the horizontal F_H and vertical F_V components of the force on the right side of the ladder due to contact between the sides where they touch at the top. Note that on the left side the corresponding contact forces will be $-F_H$ and $-F_V$. It will be helpful to draw a force/torque diagram, and write down the corresponding force/torque equations, for both sides of the ladder.



$$\cos\theta = \frac{d/2}{L}$$



$T - F_H = 0$ $\qquad$ $F_H - T = 0$

$N_L - Mg - F_V = 0$ $\qquad$ $N_R - Mg + F_V = 0$

torques about top:

$$-N_L L\cos\theta + Mg\tfrac{L}{2}\cos\theta + T(L-a)\sin\theta = 0$$

$$N_R L\cos\theta - Mg\tfrac{L}{2}\cos\theta - T(L-a)\sin\theta = 0$$

Thus: $N_L = N_R = N$ (given)

$$N - Mg = F_V = -F_V$$

Thus: $\underline{F_V = 0}$ and $\underline{N = Mg}$

$$\underline{T = \frac{Mg\frac{L}{2}\cos\theta}{(L-a)\sin\theta}}$$

$$\underline{F_H = T}$$