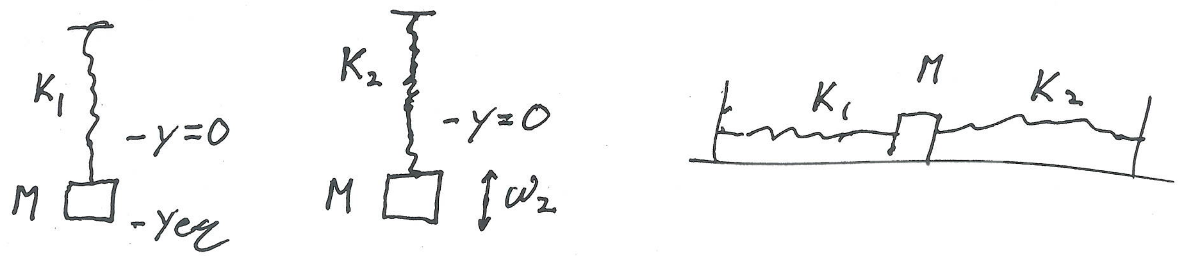
SMU Physics 1307 : Spring2010

Final Exam

Problem 1 : The figure at left below shows a spring with spring constant k_1 attached to a mass M which hangs a distance $y_{eq} = -0.4\,\mathrm{m}$ below the equilibrium position of the spring. The figure in the middle shows a spring with spring constant k_2 attached to the same mass M which oscillates with frequency $\omega_2 = 0.4\,\mathrm{s}^{-1}$. Both springs are now connected to the mass M as shown in the figure at right. Find the oscillation frequency ω of this system. You will need $g = 9.8\,\mathrm{m/s}^2$.

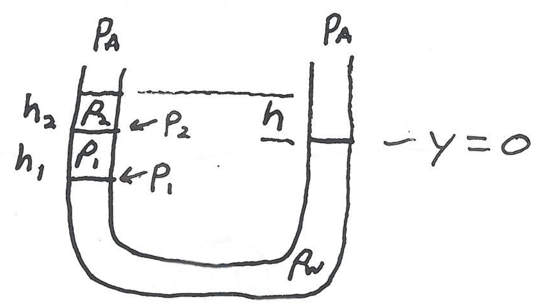


$y_{eq} = -Mg/K_1$ $\qquad \omega_2^2 = K_2/M$ $\qquad \omega^2 = \omega_1^2 + \omega_2^2$

$\omega_1^2 = K_1/M$

$\omega_1^2 = -g/y_{eq}$

Problem 2 : The figure below shows a U-shaped tube with water $\rho_w = 10^3\,\mathrm{kg/m^3}$ in the right half. The left half has water with two segments of fluid above it. The top fluid is of height $h_2 = 0.01\,\mathrm{m}$ and density $\rho_2 = 0.6\rho_w$. The bottom fluid is of height $h_1 = 0.05\,\mathrm{m}$ and density $\rho_2 = 0.8\rho_w$. Find the height h.



$$P_A = P_1 + \rho_w g (h - h_1 - h_2)$$

$$P_1 + \rho_1 g (h - h_1 - h_2) = P_2 + \rho_1 g (h - h_2)$$

$$P_A + \rho_2 g h = P_2 + \rho_2 g (h - h_2)$$

$$P_1 = P_2 + \rho_1 g h_1$$

$$P_2 = P_A + \rho_2 g h_2$$

Thus,

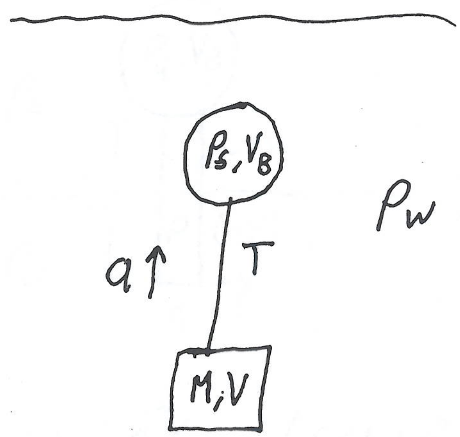
$$P_1 = P_A + \rho_1 h_1 g + \rho_2 h_2 g$$

$$P_A = P_A + \rho_1 h_1 g + \rho_2 h_2 g + \rho_w g (h - h_1 - h_2)$$

$$\rho_w (h_1 + h_2) - \rho_w h = \rho_1 h_1 + \rho_2 h_2$$

$$h = h_1 + h_2 - \frac{\rho_1}{\rho_w} h_1 - \frac{\rho_2}{\rho_w} h_2$$

Problem 3 : The figure below shows a object of volume $V = 2\,\mathrm{m}^3$ and unknown mass M which is entirely submerged in water $\rho_w = 10^3\,\mathrm{kg/m}^3$. The object is being lifted via a rope by a balloon of volume $V_B = 6\,\mathrm{m}^3$ filled with a fluid of density $\rho_f = 0.7\rho_w$. If the acceleration of the system is $a = 1\,\mathrm{m/s}^2$ upward, find the mass M and the tension T of the rope. Neglect the mass of the rope and the material of the balloon, but not the fluid inside. Also do not neglect either V or V_B.



object : $Ma = -Mg + T + \rho_w V g$

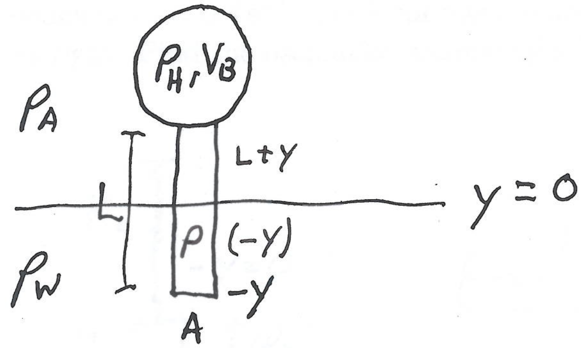
balloon : $\rho_f V_B a = -\rho_f V_B g - T + \rho_w V_B g$

$$(M + \rho_f V_B)(a+g) = \rho_w (V_B + V) g$$

$$M = \frac{\rho_w (V_B + V) g}{(a+g)} - \rho_f V_B$$

$$T = \rho_w V_B g - \rho_f V_B (a+g)$$

Problem 4 : The figure below shows a balloon of volume $V_B = 6\times10^3\,\mathrm{m}^3$ filled with helium $\rho_H = \frac{1}{7}\rho_A$ which is attached to an object of density $\rho = 0.2\rho_w$, length $L = 10\,\mathrm{m}$, and uniform cross-sectional area $A = 7\,\mathrm{m}^2$ which is partially submerged in water. You will need $\rho_A = 1.2\,\mathrm{kg/m^3}$ and $\rho_w = 10^3\,\mathrm{kg/m^3}$. Taking the surface of the water to be $y = 0$, with the position y coinciding with the bottom of the object, find the equilibrium position y_{eq} and oscillation frequency ω of the system.



$$(\rho AL + \rho_H V_B)\frac{d^2y}{dt^2} = -(\rho AL + \rho_H V_B)g + \rho_A V_B g + \rho_A (L+y)Ag \mp \rho_w y g A$$

$$\frac{d^2y}{dt^2}(\rho AL + \rho_H V_B) = -(\rho_w - \rho_A)ygA + \underbrace{\rho_A g(V_B + LA) - (\rho AL + \rho_H V_B)g}_{X}$$

$$w^2 = \frac{(\rho_w - \rho_A)gA}{\rho AL + \rho_H V_B}$$

$$C = \frac{X}{\rho AL + \rho_H V_B} \qquad y_{eq} = C/\omega^2$$