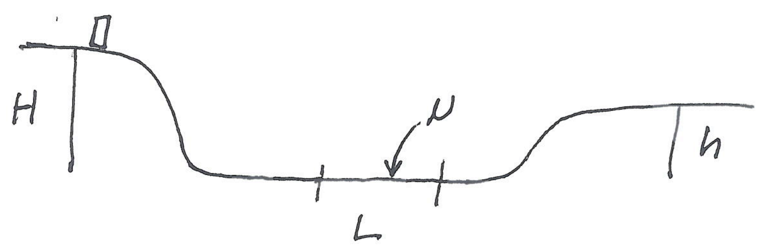
SMU Physics 1307 : Summer 2009

Exam 2

Problem 1 : The block depicted in the figure below begins at a height $H = 10\,\mathrm{m}$ with an initial velocity v_0. It then slides down a frictionless hill and encounters a flat section which is of length $L = 20\,\mathrm{m}$ and coefficient of friction $\mu = 0.6$. Find the minimum value of v_0 required to cross over this frictional section. There is a hill of height $h = 5\,\mathrm{m}$ on the right side of the figure. Find the minimum value of v_0 required to cross over the frictional section and make it to the top of this hill.



(1) $E_0 = \frac{1}{2} m V_0^2 + mgH$ $\qquad \Delta E = E_1 - E_0 = W_{nc} = -\mu m g L$

$E_1 = 0$

$\frac{1}{2} m V_0^2 + mgH = \mu m g L$

$\underline{V_0^2 = 2g(\mu L - H)}$ $\qquad \underline{V_0 = 6.26 \text{ m/s}}$

(2) $E_0 = \frac{1}{2} m V_0^2 + mgH$ $\qquad \Delta E = E_2 - E_0 = -\mu m g L$

$E_2 = mgh$

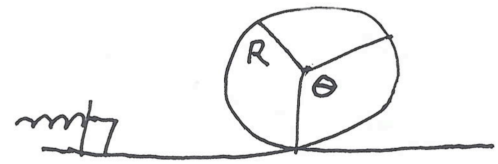
$\frac{1}{2} m V_0^2 + mgH - mgh = \mu m g L$

$\underline{V_0^2 = 2g(\mu L + h - H)}$

$V_0 = \underline{11.71 \text{ m/s}}$

Problem 2 : In the figure below a spring with spring constant $k = 2\,\mathrm{N/m}$ is used to project a mass around a loop of radius $R = 2\,\mathrm{m}$. To what minimum distance x_{min} must the spring be pulled back for the object to go around the loop without falling off? If it is pulled back to $x = 2x_{min}$, what will its velocity be at the bottom, halfway up, and at the top of the loop? If it is pulled back to $x = x_{min}/2$, at what angle θ_c, with $\theta = 0$ chosen as the bottom of the loop, will the object fall off the loop?

$m = 2\,kg$



① $\frac{1}{2} K X_{min}^2 = \frac{1}{2} m V_{top}^2 + mg2R$

$\downarrow -mg = -m V_{top}^2/R$

$V_{top}^2 = gR$

$K X_{min}^2 = 5mgR$

$X_{min} = (5mgR/K)^{1/2} = \underline{9.90\,m}$

② $X = 2X_{min} = \underline{19.8\,m}$

bottom : $\frac{1}{2} K X^2 = \frac{1}{2} m V_{bot}^2$

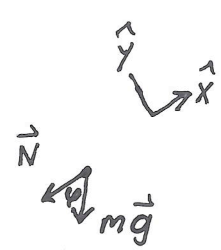
half : $\frac{1}{2} K X^2 = \frac{1}{2} m V_{half}^2 + mgR$

top : $\frac{1}{2} K X^2 = \frac{1}{2} m V_{top}^2 + mg2R$

$V_{bot} = (KX^2/m)^{1/2} = \underline{19.8\,m/s}$

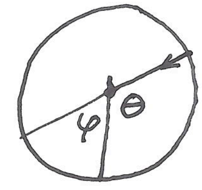
$V_{half} = (KX^2/m - 2gR)^{1/2} = \underline{18.8\,m/s}$

$V_{top} = (KX^2/m - 4gR)^{1/2} = \underline{17.7\,m/s}$



③ $X = X_{min}/2 = \underline{4.95\,m}$

doesn't fall
doesn't get to 90°



$\vec{N} = 0$

$\frac{1}{2} K x^2 = mgy + \frac{1}{2} m v^2$

$mg\cos\varphi = \frac{mv^2}{R}$

$\frac{1}{2} K x^2 = mgR\left((1-\cos\theta) - \frac{1}{2}\cos\theta\right)$

$\underline{y = R(1-\cos\theta)}$

$v^2 = gR\cos\varphi$

$\frac{\frac{1}{2} K x^2}{mgR} = 1 - \frac{3}{2}\cos\theta$

$\underline{v^2 = -gR\cos\theta}$

$\cos\theta = \frac{2}{3}\left(1 - \frac{\frac{1}{2} K x^2}{mgR}\right)$

$\underline{\theta = 67^\circ}$
(slides back down)

Problem 3 : The moon has a mass of $M_m = 7.35 \times 10^{22}$ kg and a radius of $R_m = 1.74 \times 10^6$ m. Find the acceleration due to gravity g_m at the surface of the moon. It turns out that the moon has a period of rotation about its axis which is exactly equal to its orbital period around the earth $T_m = 2.36 \times 10^6$ s; that is why we always see only one side of the moon. Ignoring the gravitational attraction of the earth, suppose that a satellite follows a geosynchronous orbit around the moon; that is, its orbital period is equal to T_m. Find the radius r_{gs} and the velocity v_{gs} of this geosynchronous orbit. Use the more general formula for gravitational force.

$$g_m = \frac{G M_m}{R_m^2} = \underline{1.62 \text{ m/s}^2}$$

$$\frac{G M_m}{r_{gs}^2} = \frac{V_{gs}^2}{r_{gs}} \qquad V_{gs} = \frac{2\pi r_{gs}}{T_m}$$

$$\left(\frac{T_m}{2\pi}\right)^2 G M_m = r_{gs}^3 \qquad r_{gs} = \underline{8.84 \times 10^7 \text{ m}}$$

$$V_{gs} = \underline{235.4 \text{ m/s}}$$

Problem 4 : A neutron star has a mass of $M_n = 1.35 \times 10^{23}$ kg and a radius of $R_n = 1.25 \times 10^4$ m. Find the acceleration due to gravity g_n at the surface of the neutron star. Suppose an object is dropped from a radius $r_1 = 2R_n$, what is its velocity v_s as it strikes the surface of the neutron star? What is the velocity v_{orb} and the period T_{orb} of an orbit of radius $r_2 = 3R_n$ around the neutron star? Use the more general formulas for gravitational force and potential energy.

$$g_n = \frac{GM_n}{R_n^2} = 5.76 \times 10^4 \ m/s^2$$

$$-\frac{GM_n}{2R_n} = \frac{1}{2}V_s^2 - \frac{GM_n}{R_n}$$

$$V_s^2 = \frac{GM_n}{R_n} \qquad V_s = 2.68 \times 10^4 \ m/s$$

$$\frac{GM_n}{r_2^2} = \frac{V_{orb}^2}{r_2} \qquad r_2 = 3R_n$$

$$V_{orb}^2 = \frac{GM_n}{3R_n} \qquad V_{orb} = 1.55 \times 10^4 \ m/s$$

$$T_{orb} = \frac{2\pi r_2}{V_{orb}} = 2\pi \left(\frac{27R_n^3}{GM_n}\right)^{1/2} = 15.2 \ s$$