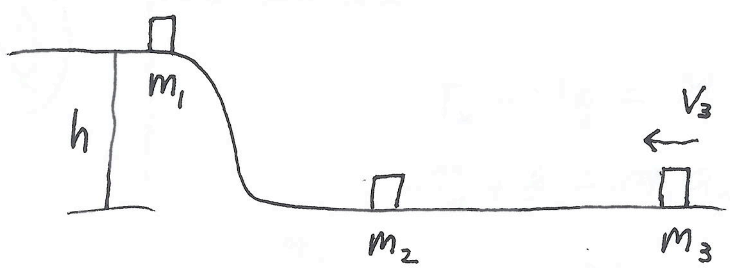
SMU Physics 1307 : Summer 2009

Exam 3

Problem 1 : In the figure below the mass $m_1 = 1\,\text{kg}$ slides down a frictionless hill of height $h = 4\,\text{m}$ and collides inelastically with the mass $m_2 = 3\,\text{kg}$ which is initially at rest. Find the final velocity v_c of the combined mass $m_c = m_1 + m_2$. The combined mass then collides elastically with the mass $m_3 = 5\,\text{kg}$ which is initially moving to the left with velocity $v_3 = -4\,\text{m/s}$. Find the final velocities v_3' of m_3 and v_c' of m_c after this collision. Does the combined mass make it up to the top of the hill? If so, what is its velocity when it gets there? If not, how high does it get up the hill?



$V_1 = \sqrt{2gh}$

$\underline{V_1 = 8.85\,\text{m/s}}$

$$m_1 V_1 = m_c V_c$$

$$V_c = \frac{m_1}{m_1 + m_2}\sqrt{2gh} = \underline{2.21\,\text{m/s}}$$

$$V_c' = \frac{m_c - m_3}{m_c + m_3} V_c + \frac{2m_3}{m_c + m_3} V_3 = \underline{-4.69\,\text{m/s}}$$

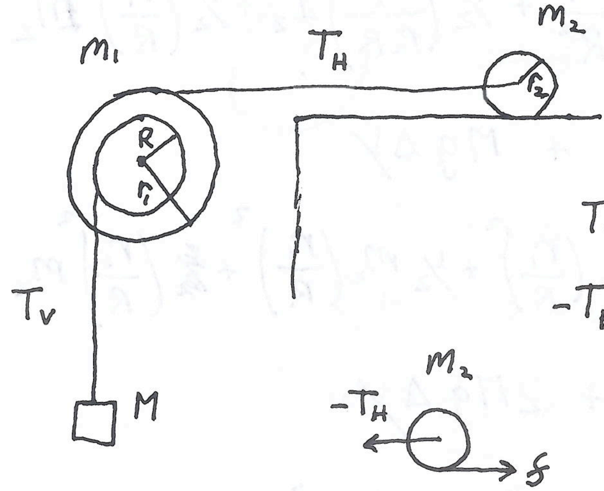
$$V_3' = \frac{m_3 - m_c}{m_c + m_3} V_3 + \frac{2m_c}{m_c + m_3} V_c = \underline{1.52\,\text{m/s}}$$

$|V_c'| < |V_1|$: cant make it up hill.

$$2gh' = V_c'^2$$

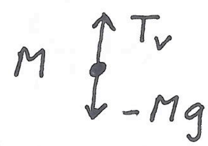
$$\underline{h' = 1.12\,\text{m}}$$

Problem 2 : The figure below shows a uniform disk of mass $m_1 = 5\,\mathrm{kg}$ and radius $r_1 = 0.25\,\mathrm{m}$ ($I_1 = \frac{1}{2}m_1r_1^2$) mounted on a frictionless horizontal axle. There is a string wrapped around the disk (at radius r_1) which is attached (at the axis of rotation) to another uniform disk of mass $m_2 = 2\,\mathrm{kg}$ and radius $r_2 = 0.1\,\mathrm{m}$ ($I_2 = \frac{1}{2}m_2r_2^2$) which rolls without slipping on a horizontal surface. Another string is wrapped around a ring, which can be assumed to be massless, of radius $R = 0.15\,\mathrm{m}$ which is attached to the disk. This string is attached to a vertically hanging mass $M = 5\,\mathrm{kg}$. Find the acceleration a of M, taking an upward acceleration to be positive. Also find the linear velocity v of M after it has dropped by $\Delta y = -1.5\,\mathrm{m}$.



$$T_v - Mg = Ma$$

$$-T_H + f = m_2 a_2$$



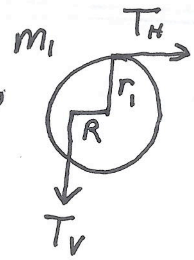
$$T_v = M(g+a)$$

torque on m_2:
(about com)

$$f r_2 = I_2\alpha_2 \qquad a_2 = -r_2\alpha_2$$

$$f r_2^2 = (\tfrac{1}{2}m_2r_2^2)\,r_2\alpha_2 = -\tfrac{1}{2}m_2r_2^2a_2$$

$$f = -\tfrac{1}{2}m_2a_2 \quad \text{so} \quad T_H = -\tfrac{3}{2}m_2a_2$$



torque on m_1: $T_vR - T_Hr_1 = I_1\alpha_1$

$$M(g+a)R + \tfrac{3}{2}m_2a_2r_1 = \tfrac{1}{2}m_1r_1^2\alpha_1$$

$a_2 = -r_1\alpha_1$

$$M(g+a)R + \tfrac{3}{2}m_2\tfrac{r_1^2}{R}a = -\tfrac{1}{2}m_1\tfrac{r_1^2}{R}a$$

$a = -R\alpha_1$

$$a\left(MR^2 + \tfrac{3}{2}m_2r_1^2 + \tfrac{1}{2}m_1r_1^2\right) = -MgR^2$$

$a_2 = \frac{r_1}{R}a$

$$a = -2.42\ m/s^2$$

(V calculation on back)

$$E = \tfrac{1}{2} I_1 w_1^2 + \tfrac{1}{2} I_2 w_2^2 + \tfrac{1}{2} m_2 V_2^2 + \tfrac{1}{2} M V^2 + M g y$$

$E_0 = 0$ and energy is conserved.

So when $y = \Delta y$:

$V_2 = -r_2 w_2$

$V_2 = -r_1 w_1$

$$\boxed{\begin{aligned} V &= -R\, w_1 \\ V &= R/r_1\, V_2 \\ V &= -\frac{R r_2}{r_1} w_2 \end{aligned}}$$

$$0 = \left(\tfrac{1}{2} \frac{I_1}{R^2} + \tfrac{1}{2} \left(\frac{r_1}{r_2 R}\right)^2 I_2 + \tfrac{1}{2} \left(\frac{r_1}{R}\right)^2 m_2 + \tfrac{1}{2} M \right) V^2$$

$$+ M g \Delta y$$

$$0 = \left(\tfrac{1}{2} m_1 \left(\frac{r_1}{R}\right)^2 + \tfrac{1}{2} m_2 \left(\frac{r_1}{R}\right)^2 + \left(\frac{r_1}{R}\right)^2 m_2 + M \right) V^2$$

$$+ 2 M g \Delta y$$

$$0 = \left(\tfrac{1}{2}(3 m_2 + m_1) \left(\frac{r_1}{R}\right)^2 + M \right) V^2 + 2 M g \Delta y$$

~~V = -2.90 m/s~~ $V = -2.69$ m/s

or, since a is constant:

$V_0 = 0$ $\qquad V^2 = V_0^2 + 2 a \Delta y$

$$V = -(2 a \Delta y)^{1/2} = -2.69 \text{ m/s}$$

Problem 3 : A disk of pizza dough of mass $M = 1\,\mathrm{kg}$ and initial radius $r_1 = 0.45\,\mathrm{m}$ is thrown into the air while spinning at an initial angular velocity $\omega_1 = 4.3\,s^{-1}$. Since it cant provide the centripetal acceleration necessary to maintain its shape, the disk of dough stretches to a radius $r_2 = 0.75\,\mathrm{m}$. Find the final angular velocity ω_2 of the disk. Also find the initial and final kinetic energies, K_1 and K_2, and the initial and final angular momenta, L_1 and L_2. All external forces, including gravity, may be ignored in this problem. The moment of inertia of a uniform disk of mass M and radius r is $I = \frac{1}{2}Mr^2$.

$I_1 = \frac{1}{2} M r_1^2 = 0.10\ \mathrm{Kg\,m^2}$

$I_2 = \frac{1}{2} M r_2^2 = 0.28\ \mathrm{Kg\,m^2}$

$L_1 = L_2$

$I_1 \omega_1 = I_2 \omega_2$

$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{r_1^2}{r_2^2}\omega_1 = 1.55\ s^{-1}$

$L_1 = I_1 \omega_1 = 0.435\ \mathrm{Kg\,m^2 s^{-1}}$

$L_2 = I_2 \omega_2 = 0.435\ \mathrm{Kg\,m^2 s^{-1}}$ (same)

$K_1 = \frac{1}{2} I_1 \omega_1^2 = 0.936\ \mathrm{J}$

$K_2 = \frac{1}{2} I_2 \omega_2^2 = 0.337\ \mathrm{J}$

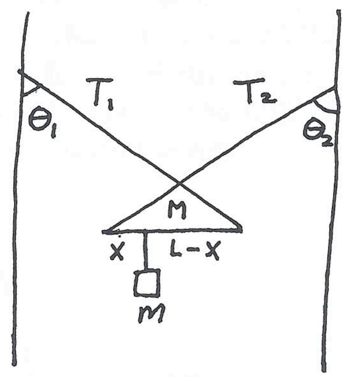
$\Delta K = K_2 - K_1 = -0.599\ \mathrm{J}$

(Kinetic energy lost)

(heats pizza)

Problem 4 :

The figure below shows a horizontal uniform beam of length $L = 3\,\mathrm{m}$ and mass $M = 70\,\mathrm{kg}$ which is attached by wires to two parallel walls as shown. A mass $m = 50\,\mathrm{kg}$ is attached to the beam at a distance x from the left end. The angles shown are $\theta_1 = 45°$ and $\theta_2 = 40°$. The system will remain in equilibrium only for a particular value of x. Find x and the tensions T_1 and T_2 in the wires.



wires do not touch each other

F_x : $\quad T_2 \sin\theta_2 - T_1 \sin\theta_1 = 0$

F_y : $\quad T_1 \cos\theta_1 + T_2 \cos\theta_2 - (M+m)g = 0$

τ : $\quad L T_1 \cos\theta_1 - mgx - Mg\,L/2 = 0$

(left end)

Numerically: $\quad T_2 = 1.10\, T_1$

$$T_1(0.707) + T_2(0.766) = 1176\,\mathrm{N}$$

so, $\quad T_1 = 1176/(1.55) = \underline{759\,N}$

$\quad T_2 = \underline{835\,N}$

also, $\quad mg\,x/L = T_1 \cos\theta_1 - Mg/2 = 193.7\,N$

$x/L = 0.395 \qquad \underline{x = 1.186\,m}$