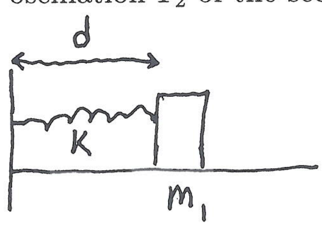
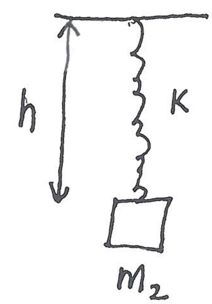
SMU Physics 1307 : Summer 2009

Final Exam

Problem 1 : In the first figure below an unknown mass m_1 is connected to a spring with unknown constant k and oscillates with period $T_1 = 1s$ on a frictionless horizontal surface about an equilibrium point $d = 0.4\,\mathrm{m}$ from the wall on the left. In the second figure below the spring is then oriented vertically, the mass m_1 is removed, and a mass $m_2 = 1.6\,\mathrm{kg}$ is connected to the spring. The mass m_2 experiences no acceleration when placed at a distance $h = 0.5\,\mathrm{m}$ below the ceiling where the spring is attached. Find k and m_1, and also find the period of oscillation T_2 of the second mass m_2 when the spring is vertical.



m_1 shown at equilibrium position



m_2 shown at equilibrium position

$$\omega_1^2 = \left(\frac{2\pi}{T_1}\right)^2 = K/m_1$$

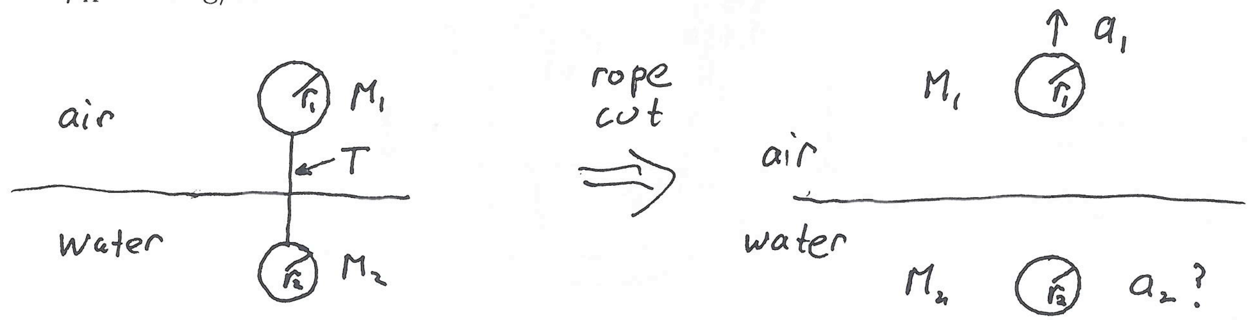
$$m_1 = K\left(\frac{T_1}{2\pi}\right)^2 = \underline{3.97\,Kg}$$

$$y_{eq} = -m_2 g/K = d - h$$

$$K = \frac{m_2 g}{h-d} = \underline{156.8\ N/m}$$

$$T_2 = 2\pi\sqrt{\frac{m_2}{K}} = \underline{0.635\,s}$$

Problem 2 : As shown in the figure below, a spherical balloon of radius $r_1 = 5\,\mathrm{m}$ is tethered by a massless rope to a sphere of radius $r_2 = 3\,\mathrm{m}$ which is completely submerged in water ($V = \frac{4}{3}\pi r^3$ for a sphere). The respective masses of the objects, M_1 and M_2, are unknown, but the entire system has zero acceleration. At some point the rope is cut and M_1 accelerates upward at $a_1 = 3\,\mathrm{m/s}$. What is the mass M_1? What was the tension T in the rope before it was cut? What is the mass M_2, and what is its acceleration a_2 (with sign) after the rope is cut? Take the density of water to be $\rho_w = 10^3\,\mathrm{kg/m^3}$, and the density of air to be $\rho_A = 1.2\,\mathrm{kg/m^3}$.



$$0 = \rho_A V_1 g - M_1 g - T$$
$$0 = \rho_W V_2 g - M_2 g + T$$

$$M_1 a_1 = \rho_A V_1 g - M_1 g$$
$$M_2 a_2 = \rho_W V_2 g - M_2 g$$

$$M_1 = \frac{\rho_A V_1 g}{a_1 + g} = \underline{481\ \mathrm{kg}}$$

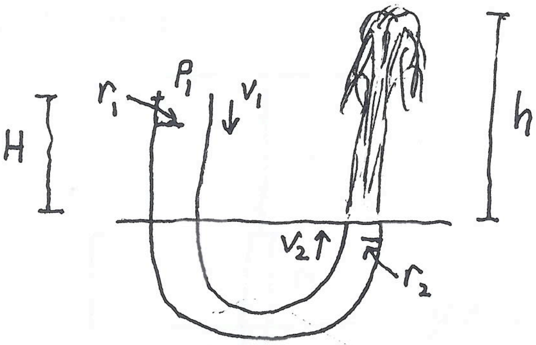
$$T = \rho_A V_1 g - M_1 g = \underline{1443\ \mathrm{N}}$$

$$M_2 = \rho_W V_2 + T/g = \underline{1.13 \times 10^5\ \mathrm{kg}}$$

$$a_2 = -g + \rho_W V_2 g / M_2 = \underline{-1.27 \times 10^{-2}\ \mathrm{m/s^2}}$$

BTW: $M_2/V_2 = 1.0013\,\rho_W$

Problem 3 : Water in the pipe in the figure below descends from a height $H = 3\,\mathrm{m}$, where the pipe is of radius $r_1 = 0.1\,\mathrm{m}$, into a semicircular section which is below ground, until finally exiting the pipe to form a fountain at ground level, where its radius is $r_2 = 0.04\,\mathrm{m}$ ($A = \pi r^2$ for a circle). If the water from the fountain flies to a height $h = 9\,\mathrm{m}$, find the velocity v_1 and pressure p_1 in the pipe at height H, and find the velocity v_2 when it exits the pipe at ground level.



$P_A = 1.01 \times 10^5\ N/m^2$

$\rho_W = 10^3\ Kg/m^3$

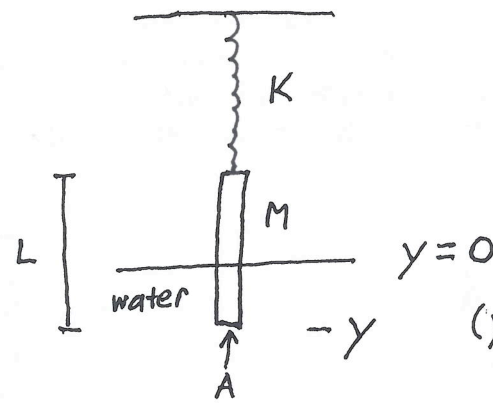
$$P_1 + \tfrac{1}{2}\rho_W V_1^2 + \rho_W g H = P_A + \tfrac{1}{2}\rho_W V_2^2 = P_A + \rho_W g h$$

$$V_2^2 = 2gh \qquad V_2 = \underline{13.28\ m/s}$$

$$A_1 V_1 = A_2 V_2 \qquad V_1 = \left(\tfrac{r_2}{r_1}\right)^2 V_2 = \underline{2.13\ m/s}$$

$$P_1 = \tfrac{1}{2}\rho_W (V_2^2 - V_1^2) - \rho_W g H + P_A = \underline{1.58 \times 10^5\ N/m^2}$$

Problem 4 : The figure below shows a massless spring with $k = 3\,\mathrm{N/m}$ which is attached to a rod of length $L = 1\,\mathrm{m}$, cross sectional area $A = 10^{-4}\,\mathrm{m}^2$, and mass $M = 0.2\,\mathrm{kg}$, which has one end submerged in water. If the equilibrium position of the spring when it is not attached to the mass is 1 m above the water, find the equilibrium position y_{eq} of the bottom of the rod (careful with the sign) when it is attached to the spring. Also find the period T of oscillation of the system. Take the density of water to be $\rho_w = 10^3\,\mathrm{kg/m^3}$, and ignore the effect of buoyancy due to the air.



(y denotes bottom of rod)
(when y = 0 spring unstretched)

$$M\frac{d^2y}{dt^2} = -Mg - Ky + \rho_w A(-y)g = -(K+\rho_w A g)y - Mg$$

$$\frac{d^2y}{dt^2} = -\omega^2 y - g \qquad \omega^2 = (K+\rho_w A g)/M$$

at $y = y_{eq}$: $\dfrac{d^2y}{dt^2} = -\omega^2 y_{eq} - g = 0$

$$y_{eq} = -g/\omega^2 = \underline{-0.492\,\mathrm{m}}$$

$$T = 2\pi/\omega = \underline{1.41\,\mathrm{s}}$$