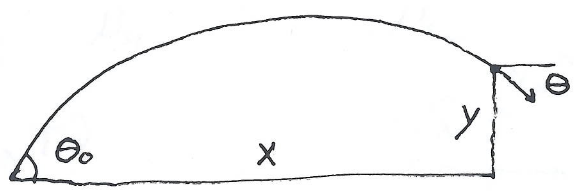
SMU Physics 1307 : Summer 2010

Exam 1

Problem 1 : The figure below shows a ball that has been hit from ground level at angle $\theta_0 = 40°$ and observed a time $t = 2.5\,\mathrm{s}$ later to have gone a horizontal distance $x = 100\,\mathrm{m}$. Find the magnitude $v_0 = |\vec{v}_0|$ of the initial velocity vector. Also find the values at time t of the vertical coordinate y of the ball, its vertical velocity component v_y, and the angle θ that the velocity vector makes with the horizontal.



$$x = V_0 \cos\theta_0 t$$

$$\Rightarrow V_0 = \frac{x}{\cos\theta_0 t} = \underline{52.2\ m/s}$$

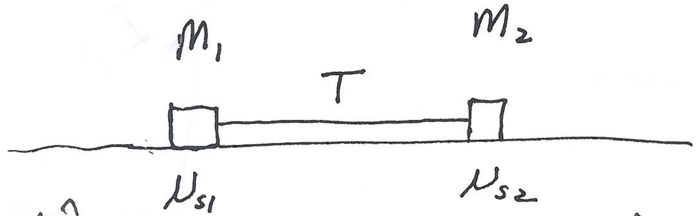
$$y = V_0 \sin\theta_0 t - \tfrac{1}{2} g t^2 = \underline{53.3\,m}$$

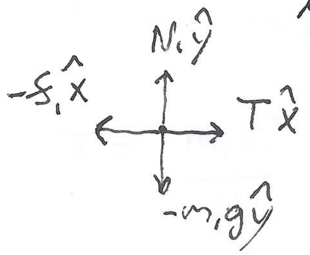
$$V_y = V_0 \sin\theta - g t = \underline{9.06\ m/s^2}$$

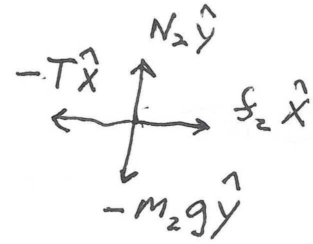
$$\tan\theta = \frac{V_y}{V_x} = \frac{V_y}{V_0 \cos\theta_0}$$

$$\underline{\theta = 12.77°}$$

Problem 2 : As shown in the figure below, two teams are involved in a tug of war. The first team has mass $m_1 = 500\,\mathrm{kg}$ and coefficient of static friction $\mu_{s1} = 0.7$. The second team has mass $m_2 = 600\,\mathrm{kg}$ and coefficient of static friction $\mu_{s2} = 0.6$. The rope that connects the two teams can withstand a tension of $T_c = 5000\,\mathrm{N}$. Does the rope break before either team slides on the surface? If the rope does not break, which team slides first, and what is the tension in the rope at that moment?







$N_1 - m_1 g = 0$ $\qquad$ $N_2 - m_2 g = 0$

$T - f_1 = 0$ $\qquad$ $f_2 - T = 0$

$$f_1 = f_2 = T$$

$f_1 \le \mu_{s1} N_1$ $\qquad$ $f_2 \le \mu_{s2} N$

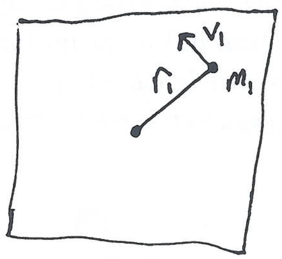
$T \le \mu_{s1} m_1 g = 350\cdot(9.8)\,N$ $\qquad$ $T \le \mu_{s2} m_2 g = 360\cdot(9.8)\,N$

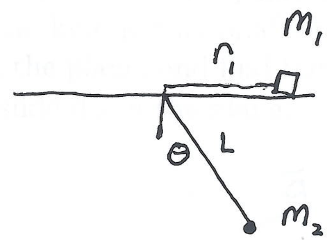
$T_{c1} = \mu_{s1} m_1 g = \underline{3430\,N}$ $\qquad$ $T_{c2} = \mu_{s2} m_2 g = \underline{3528\,N}$

$$T_{c1} < T_{c2} < T_c = 5000\,N$$

m_1 slips first with $\underline{T = T_{c1}}$

Problem 3 : The figure at left below shows a mass $m_1 = 2\,\mathrm{kg}$ revolving on a horizontal table in a circle of radius $r_1 = 0.5\,\mathrm{m}$ with rotational period $t_1 = 0.4\,\mathrm{s}$. The mass m_1 is attached to a string which passes through hole in the table and attaches to a mass $m_2 = 6\,\mathrm{kg}$ which revolves as a conical pendulum of length $L = 0.6\,\mathrm{m}$ and angle θ as shown in the figure at right. Find the rotational period t_2 of m_2, the angle θ, and the tension T in the string.





$$T = \frac{m_1 V_1^2}{r_1}$$

$$V_1 = \frac{2\pi r_1}{t_1}$$

$$T = \left(\frac{2\pi}{t_1}\right)^2 m_1 r_1$$

$$= \underline{246.7\,N}$$

$$\cos\theta = m_2 g / T$$

$$\underline{\theta = 76.2^\circ}$$

$$T\sin\theta = \frac{m_2 V_2^2}{r_2}$$

$$T\cos\theta = m_2 g$$

$$V_2 = \frac{2\pi r_2}{t_2}$$

$$T\sin\theta = \left(\frac{2\pi}{t_2}\right)^2 m_2 r_2$$

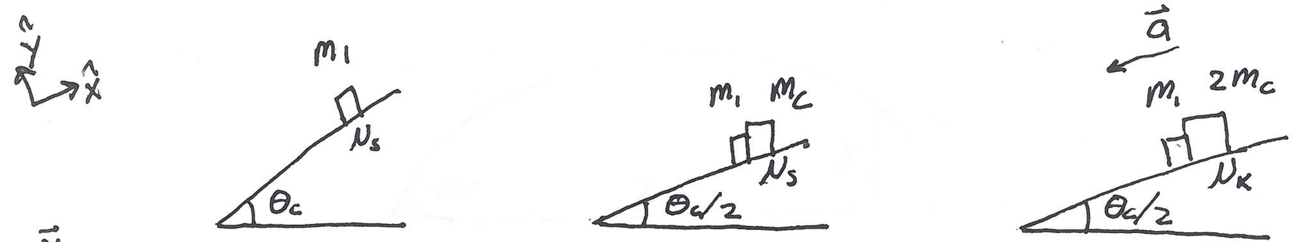
$$r_2 = L\sin\theta$$

$$T = \left(\frac{2\pi}{t_2}\right)^2 m_2 L$$

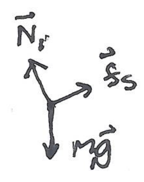
$$t_2 = 2\pi\left(m_2 L / T\right)^{1/2}$$

$$\underline{t_2 = .759\,s}$$

Problem 4 : The figure at left below shows a mass $m_1 = 3\,\text{kg}$ on an incline plane of angle θ. If the mass has a coefficient of static friction $\mu_s = 0.7$, find the critical angle $\theta = \theta_c$ above which the mass will slide down the plane. The figure in the middle below shows the mass m_1 with the same μ_s on an incline plane of angle $\theta_c/2$. Behind it is a mass m_2 which experiences no static frictional force, but exerts a normal force on the mass m_1. Find the critical mass $m_2 = m_c$ above which both objects will slide down the plane. Also find the magnitude $|\vec{N}|$ of the normal force between the two masses just as slippage occurs. The figure at right replaces the critical mass with $m_2 = 2m_c$. If the mass m_1 has a coefficient of kinetic friction $\mu_k = 0.6$ and the mass m_2 experiences no kinetic frictional force, find the magnitude $|\vec{a}|$ of the acceleration of the two masses down the plane, and find the magnitude $|\vec{N}|$ of the normal force between the two masses as they slide down the plane.



(1)



$N_1 - m_1 g\cos\theta = 0$

$f_s - m_1 g\sin\theta = 0$

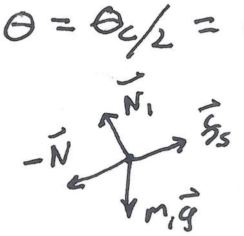
$f_s \le \mu_s N_1$

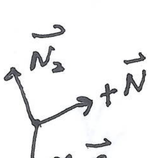
$m_1 g\sin\theta \le \mu_s m_1 g\cos\theta$

$\tan\theta \le \mu_s = \tan\theta_c$

$\theta_c = 35^\circ$

(2) $\theta = \theta_c/2 = 17.5^\circ$





$N_1 - m_1 g\cos\theta = 0$ $\quad$ $N_2 - m_2 g\cos\theta = 0$

$f_s - N - m_1 g\sin\theta = 0$ $\quad$ $N - m_2 g\sin\theta = 0$

$f_s = (m_1+m_2) g\sin\theta \le \mu_s N_1$

slips when $m_2 = m_c$ $\quad$ $f_s = \mu_s N_1$

$(m_1+m_c) g\sin\theta = \mu_s m_1 g\cos\theta$

$m_c = \frac{\mu_s m_1}{\tan\theta} - m_1 = 3.66\text{ Kg}$

$N = m_c g\sin\theta = 10.79\text{N}$

$f_s \to f_k = \mu_k N_1$ $\quad$ $m_2 = 2m_c$

$N_1 - m_1 g\cos\theta = 0$ $\quad$ $N_2 - m_2 g\cos\theta = 0$

(3) $f_k - N - m_1 g\sin\theta = m_1 a$ $\quad$ $a_1 = a_2 = a$

$N - m_2 g\sin\theta = m_2 a$

$\mu_k m_1 g\cos\theta - (m_1+m_2) g\sin\theta = (m_1+m_2)a$

$a = -g\sin\theta + \frac{\mu_k m_1 g\cos\theta}{(m_1+2m_c)} = -2.87\,m/s^2$

$N = 2m_c(a + g\sin\theta) = 0.572\text{N}$