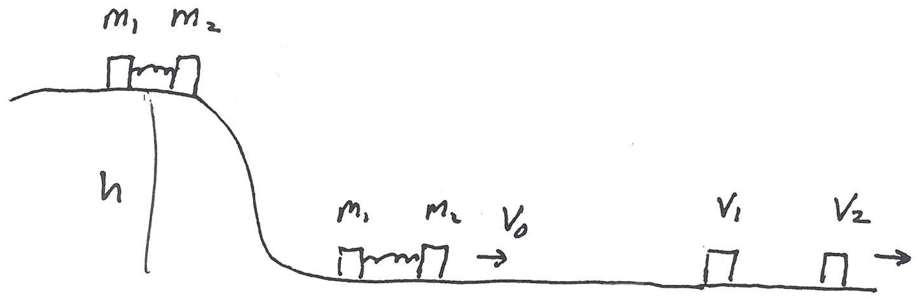
SMU Physics 1307 : Summer 2010

Exam 2

Problem 1 : The figure below shows two masses $m_1 = 5\,\mathrm{kg}$ and $m_2 = 3\,\mathrm{kg}$ which are initially at the top of a frictionless hill of height $h = 8\,\mathrm{m}$ and connected by a spring of spring constant $k = 10^4\,\mathrm{N/m}$ which has been compressed by $x = 0.3\,\mathrm{m}$. The masses then slide down the hill, acquiring a velocity v_0 when they reach the bottom. Following this the spring is released and the masses separate, resulting in velocities v_1 and v_2. Find the three velocities v_0, v_1, and v_2.



$(m_1+m_2)gh = \frac{1}{2}(m_1+m_2)V_0^2 \qquad V_0 = \sqrt{2gh} = \underline{12.5\,\mathrm{m/s}}$

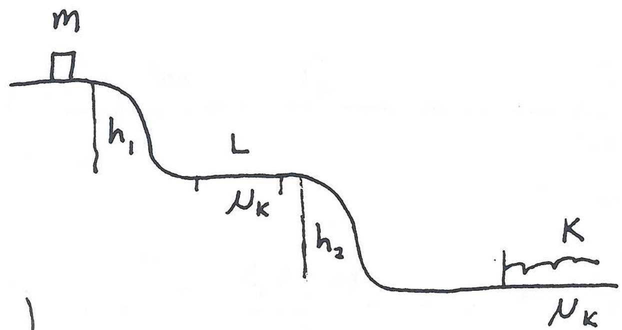
$(m_1+m_2)V_0 = m_1V_1 + m_2V_2$

$\frac{1}{2}KX^2 + \frac{1}{2}(m_1+m_2)V_0^2 = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2$

solve for V_1, V_2 : $\quad V_2 = \underline{26.2\,\mathrm{m/s}}$

$V_1 = \underline{4.3\,\mathrm{m/s}}$

Problem 2 : The figure below shows an object of mass $m = 2\,\mathrm{kg}$ on top of a hill with a plateau which is $h_1 = 10\,\mathrm{m}$ from the top and $h_2 = 15\,\mathrm{m}$ from the bottom of the hill. The plateau has a section of length $L = 6\,\mathrm{m}$ which has coefficient of kinetic friction $\mu_k = 0.6$. At the bottom of the hill is a spring of spring constant $k = 10^2\,\mathrm{N/m}$. To the right of the equilibrium position of the spring is a section which also has coefficient of kinetic friction $\mu_k = 0.6$. Find the maximum distance x_{max} by which the spring is compressed when the object comes to rest for the first time. Assuming the object has coefficient of static friction $\mu_s = 0.7$, describe the final state of the object. That is, where exactly does the object finally come to rest?



$E_1 = mg(h_1+h_2)$

$E_2 = \frac{1}{2} K x^2$

$$E_2 - E_1 = W_{nc} = -mg\mu_K(L+x)$$

$$\tfrac{1}{2}Kx^2 + mg\mu_K x + mg\mu_K L - mg(h_1+h_2) = 0$$

quadratic $\quad x = 2.78\,m$

does it stop: $\quad Kx \le \mu_s mg \quad$ no, slides

does it get off spring: $\quad \frac{1}{2}mV_1^2 - \frac{1}{2}Kx^2 = -mg\mu_K x \quad$ $V_1 = 18.8\,m/s$ (yes)

does it get up hill: $\quad \frac{1}{2}mV_2^2 + mgh_2 = \frac{1}{2}mV_1^2 \quad$ $V_2 = 7.7\,m/s$ (yes)

does it get past plateau: $\quad \frac{1}{2}mV_3^2 - \frac{1}{2}mV_2^2 = -mg\mu_K L$

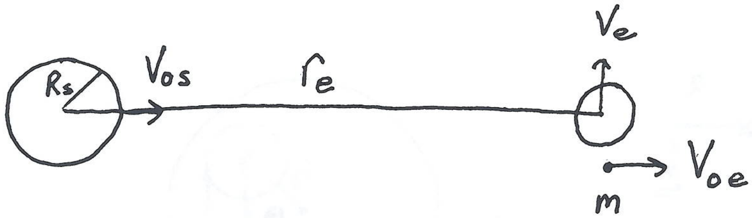
$\frac{1}{2}mV_3^2 < 0 \quad$ so stops.

where: set $V_3 = 0$ $\quad \frac{1}{2}mV_2^2 = mg\mu_K d$

$d = 5.1\,m$

from right of plateau

Problem 3 : An object of mass $m = 1000\,\text{kg}$ is projected radially outwards from the surface of the sun ($M_s = 1.99 \times 10^{30}\,\text{kg}$, $R_s = 6.96 \times 10^8\,\text{m}$) in the general direction of the earth. When it gets to the orbital radius of the earth $r_e = 1.5 \times 10^{11}\,\text{m}$ its radial velocity V_{oe} is twice that of the orbital velocity v_e of the earth around the sun. Find the velocity V_{os} of the object when it left the surface of the sun. Does the object escape the gravitational pull of the sun? If it does, find its velocity V_∞ when it escapes to infinity. If the object does not escape, find it maximum radius r_{max} .



$$\tfrac{1}{2} m V_{os}^2 - \frac{G M_s m}{R_s} = \tfrac{1}{2} m V_{oe}^2 - \frac{G M_s m}{r_e}$$

$$\frac{G M_s M_e}{r_e^2} = \frac{M_e V_e^2}{r_e} \qquad V_e^2 = G M_s / r_e$$

$$\underline{V_{oe} = 2 V_e}$$

$$\tfrac{1}{2} V_{os}^2 = \tfrac{1}{2}(4)\frac{G M_s}{r_e} + \frac{G M_s}{R_s} - \frac{G M_s}{r_e}$$

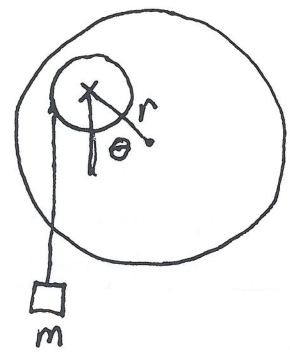
$$V_{os}^2 = \frac{2 G M_s}{r_e} + \frac{2 G M_s}{R_s} \qquad V_{os} = 6.19 \times 10^5\ \text{m/s}$$

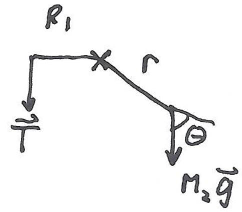
$$E = \tfrac{1}{2} m V_{os}^2 - \frac{G M_s m}{R_s} > 0 \qquad \text{escapes}$$

$$\tfrac{1}{2} m V_\infty^2 = E$$

$$V_\infty^2 = V_{os}^2 - \frac{2 G M_s}{R_s} \qquad \underline{V_\infty = 4.21 \times 10^4\ \text{m/s}}$$

Problem 4 : The figure below shows two disks of radii $R_1 = 0.2\,\text{m}$ and $R_2 = 0.6\,\text{m}$ and masses $M_1 = 3\,\text{kg}$ and $M_2 = 7\,\text{kg}$ which form a single rigid object. The axis of rotation goes through the center of the smaller disk, but is offset from the center of the larger disk by $r = 0.3\,\text{m}$. A string is wrapped around the smaller disk and is attached on the left side of the disk to a vertically hanging mass $m = 11\,\text{kg}$. If the object is at the angle θ shown in the figure, find its angular acceleration α, the vertical acceleration a of the mass, and the tension T in the string. The moment of inertia of a disk of radius R and mass M about its center of mass is $I_{cm} = \frac{1}{2}MR^2$.





$$T - mg = ma \qquad \underline{a = -R_1\alpha}$$

$$I = \tfrac{1}{2}M_1R_1^2 + \tfrac{1}{2}M_2R_2^2 + M_2r^2$$

$$-rM_2 g\sin\theta + TR_1 = I\alpha$$

$$I\alpha = m(g - R_1\alpha)R_1 - M_2 g r\sin\theta$$

$$\underline{(I + mR_1^2)\alpha = mgR_1 - M_2 g r\sin\theta}$$

$$\underline{T = m(g - R_1\alpha)}$$