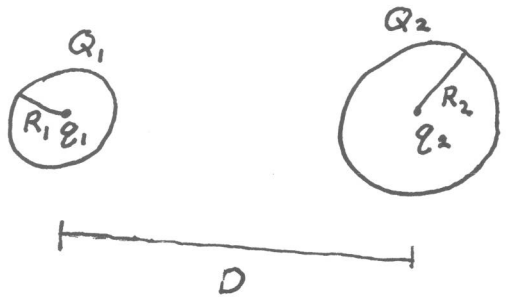
SMU Physics 1308 : Spring 2012

Exam 1

Problem 1 : The figure below shows two spherical shells of charge $Q_1 = 1 \times 10^{-6}$ C and $Q_2 = 2Q_1$ with radii $R_1 = 1 \times 10^{-2}$ m and $R_2 = 2R_1$, respectively. The spheres have point charges $q_1 = 4Q_1$ and $q_2 = -3Q_1$ at their respective centers which are separated by distance $D = 5R_1$. If $x = 0$ at q_1 and $x = D$ at q_2, find the electric field vector at $x = R_1/2$, $x = 2R_1$ and $x = 4R_1$, all for $y = 0$. Also find the electric field vector at $x = D/2$ and $y = D/2$.



$x = R_1/2$: $\vec{E} = \frac{Kq_1}{x^2}\hat{x} - \frac{K(q_2+Q_2)}{(D-x)^2}\hat{x}$

$x = 2R_1$: $\vec{E} = \frac{K(q_1+Q_1)}{x^2}\hat{x} - \frac{K(q_2+Q_2)}{(D-x)^2}\hat{x}$

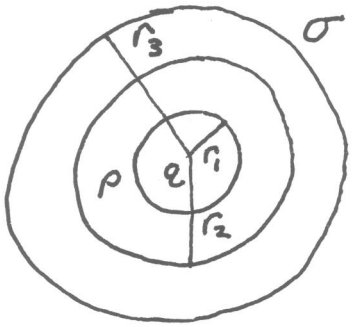
$x = 4R_1$: $\vec{E} = \frac{K(q_1+Q_1)}{x^2}\hat{x} - \frac{Kq_2}{(D-x)^2}\hat{x}$

$x = D/2$, $y = D/2$

$r = \sqrt{x^2+y^2} = \sqrt{D^2/2}$

$\vec{E} = \frac{K(q_1+Q_1)}{r^3}(D/2\,\hat{x} + D/2\,\hat{y}) + \frac{K(q_2+Q_2)}{r^3}(-D/2\,\hat{x} + D/2\,\hat{y})$

Problem 2 : The figure below shows a point charge $q = 1 \times 10^{-6}\,\mathrm{C}$ at the center of a spherical cavity of radius $r_1 = 1 \times 10^{-2}\,\mathrm{m}$. This cavity is surrounded by a constant charge density per unit volume $\rho = 5 \times 10^{-8}\,\mathrm{C/m^3}$ of inner radius r_1 and outer radius $r_2 = 2 \times 10^{-2}\,\mathrm{m}$. Finally, there is a spherical shell of constant charge density per unit area $\sigma = 2 \times 10^{-8}\,\mathrm{C/m^2}$ of radius $r_3 = 3 \times 10^{-2}\,\mathrm{m}$. Find the radial component of the electric field $E(r)$ for $r < r_1$, $r_1 < r < r_2$, $r_2 < r < r_3$, and $r > r_3$. Since r is a variable and not a fixed value, it may be simplest to do this problem entirely algebraically.



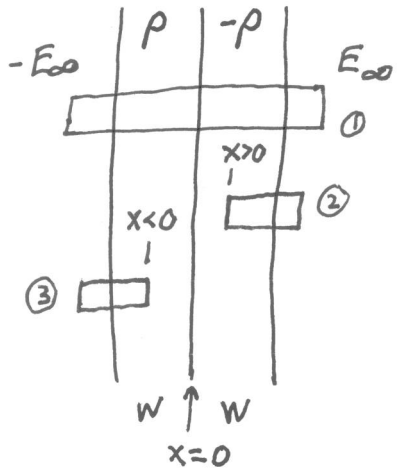
$r < r_1$ $\quad 4\pi r^2 E(r) = 4\pi k q$

$r_1 < r < r_2$ $\quad 4\pi r^2 E(r) = 4\pi k \left(q + \tfrac{4}{3}\pi\rho(r^3 - r_1^3)\right)$

$r_2 < r < r_3$ $\quad 4\pi r^2 E(r) = 4\pi k \left(q + \tfrac{4}{3}\pi\rho(r_2^3 - r_1^3)\right)$

$r > r_3$ $\quad 4\pi r^2 E(r) = 4\pi k \left(q + \tfrac{4}{3}\pi\rho(r_2^3 - r_1^3) + 4\pi\sigma r_3^2\right)$

Problem 3 : The figure below shows two adjacent slabs of thickness $w = 1 \times 10^{-2}\,\mathrm{m}$ and constant charge density per unit volume $\rho = 1 \times 10^{-8}\,\mathrm{C/m^3}$ and $-\rho$, respectively. If $x = 0$ between the slabs, find the electric field for $x < -w$, $-w < x < 0$, $0 < x < w$, and $x > w$. Since x is a variable and not a fixed value, it may be simplest to do this problem entirely algebraically.



① $2AE_\infty = 4\pi K(\rho Aw - \rho Aw) = 0$ $\quad E_\infty = 0$ for $x > w$ and $x < -w$

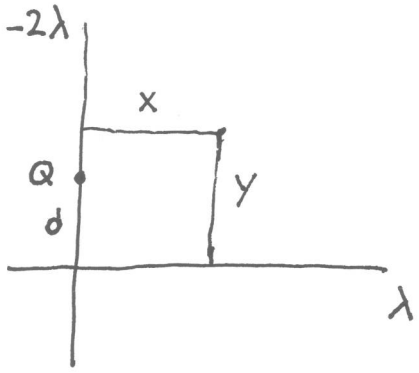
② $AE_\infty - AE(x) = 4\pi K(-\rho A(w-x))$

$w > x > 0 \qquad E(x) = 4\pi K\rho(w-x)$

③ $AE(x) + AE_\infty = 4\pi K(\rho A(w+x))$

$-w < x < 0 \qquad E(x) = 4\pi K\rho(w+x)$

Problem 4 : The figure below shows horizontal and vertical line charges of constant linear charge density $\lambda = 1 \times 10^{-6}\,\mathrm{C/m}$ and -2λ respectively. In addition, with $x = 0$ and $y = 0$ at the crossing point of the line charges, there is a point charge $Q = 1 \times 10^{-6}\,\mathrm{C}$ at $x = 0$ and $y = d = 1 \times 10^{-2}\,\mathrm{m}$. Find the electric field vector at a general point (x, y) for $x > 0$ and $y > 0$. Since x and y are variables and not fixed values, it may be simplest to do this problem entirely algebraically.



$$\vec{E} = \frac{2K\lambda}{y}\hat{y} + \frac{2K(-2\lambda)}{x}\hat{x} + \frac{KQ\,(x\hat{x} + (y-d)\hat{y})}{(x^2 + (y-d)^2)^{3/2}}$$