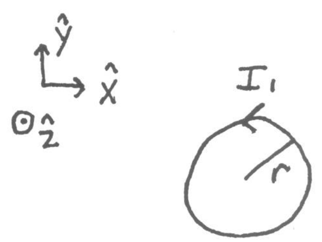
SMU Physics 1308 : Spring 2012

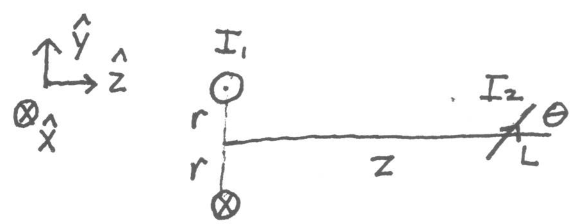
Exam 2

Problem 1 : The figure at left below shows a current loop of radius r carrying a current I_1 in the counter-clockwise direction in the x-y plane. The magnetic field along the z axis due to this loop is given by

$$\vec{B} = \frac{\mu_0 I_1}{2} \frac{r^2}{(r^2 + z^2)^{3/2}} \hat{z}$$

The figure at right shows the same loop in the z-y plane with a segment of wire carrying current I_2 placed at $z\hat{z}$ along the z-axis. The segment of wire is of length L and makes an angle θ with the z axis as shown. Assume that L is small enough so that the $\vec{B}$ field along it is uniform. Find the force $\vec{F}$ on the segment of wire.

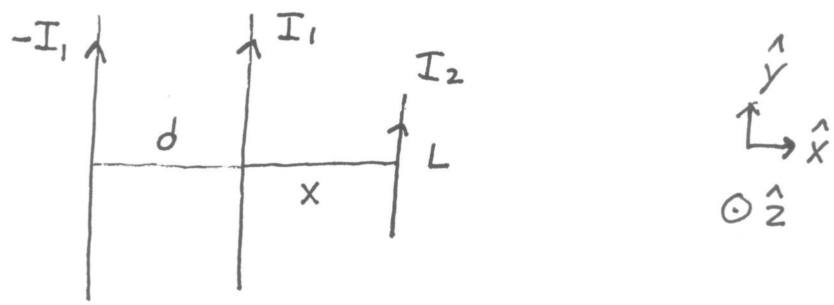




$$\vec{F} = I_2 L (\hat{z} \cos\theta + \hat{y} \sin\theta) \times \left(\frac{\mu_0 I_1}{2}\right) \hat{z} \frac{r^2}{(r^2+z^2)^{3/2}}$$

$$= \frac{\mu_0 I_1 I_2}{2} \hat{x} \sin\theta \frac{L r^2}{(r^2+z^2)^{3/2}}$$

Problem 2 : The figure below shows two wires with oppositely flowing currents I_1 and $-I_1$ which are separated by a distance d as shown. A segment of wire carrying current I_2 and of length L is placed a distance x from the right wire. Find the force $\vec{F}$ on the segment of wire. By finding a common denominator show that for $x >> d$ the field falls as $1/x^2$.



$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi x}(-\hat{z}) + \frac{\mu_0(-I_1)}{2\pi(x+d)}(-\hat{z})$$

$$= \frac{\mu_0 I_1}{2\pi}(-\hat{z})\left(\frac{1}{x} - \frac{1}{x+d}\right) = \frac{\mu_0 I_1}{2\pi}(-\hat{z})\frac{d}{x(x+d)}$$

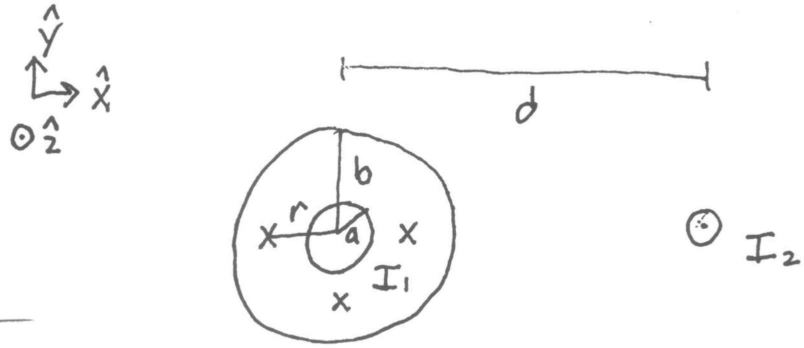
for $x >> d$: $\vec{B}_1 \simeq \frac{\mu_0 I_1 d}{2\pi x^2}(-\hat{z})$

$$\vec{F} = (I_2 L \hat{y}) \times \vec{B}_1$$

$$= (I_2 L)\left(\frac{\mu_0 I_1}{2\pi}\right)\frac{d}{x(x+d)} \overbrace{\hat{y} \times (-\hat{z})}^{-\hat{x}}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi}\frac{dL}{x(x+d)}(-\hat{x})$$

Problem 3 : The figure below shows a wire of outer radius b which contains a cavity of radius a and carries a total current I_1 in the $\hat{z}$ direction. Another wire, carrying a current I_2 in the $\hat{z}$ direction is a distance d from the first wire as shown. Find the total magnetic field vector at the three indicated points at radius r for $b > r > a$.



3 points: $r\hat{x}, -r\hat{x}, -r\hat{y}$

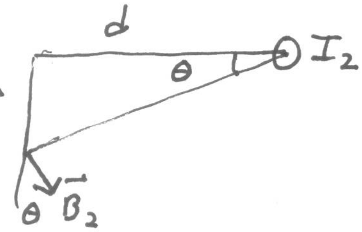
$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$\vec{B}_2$:

$r\hat{x}$: $\vec{B}_2 = \dfrac{\mu_0 I_2}{2\pi(d-r)}(-\hat{y})$ $\qquad$ $-r\hat{x}$: $\vec{B}_2 = \dfrac{\mu_0 I_2}{2\pi(d+r)}(-\hat{y})$

$-r\hat{y}$:

$\tan\theta = r/d$



$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi} \frac{(\sin\theta\,\hat{x} - \cos\theta\,\hat{y})}{(r^2+d^2)^{1/2}}$$

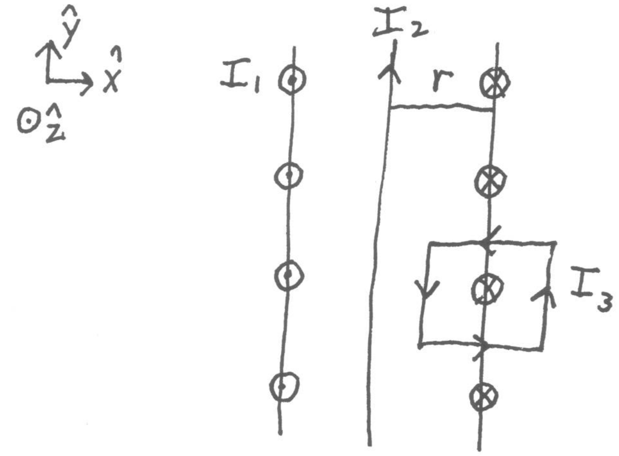
$\vec{B}_1$: $\quad 2\pi r\, B(r) = \mu_0 I_1 \dfrac{(r^2-a^2)}{(b^2-a^2)}$ $\qquad$ $\vec{B}_1 = B(r)\,\hat{\theta}$

$r\hat{x}$: $\hat{\theta} = \hat{y}$

$-r\hat{x}$: $\hat{\theta} = -\hat{y}$

$-r\hat{y}$: $\hat{\theta} = \hat{x}$

Problem 4 : The figure below shows a solenoid of radius r with axis pointing in the $\hat{y}$ direction carrying a current I_1 around n turns per unit length in the direction indicated. Another wire is placed along the central axis of the solenoid and carries a current I_2 in the $\hat{y}$ direction. A square loop of wire of side r carrying current I_3 in the counter-clockwise direction is placed partly inside and partly outside the solenoid. The left segment is at a distance $r/2$ from I_2 and the right segment is at a distance $3r/2$ from I_2. Considering infinitesimal elements of the top and bottom of the loop, argue that the total force on these segments is zero. Find the forces $\vec{F}_L$ and $\vec{F}_R$ on the left and right segments.



Top and Bottom segments of loop see identical fields but carry current in opposite directions thus forces are opposite.

$$\vec{F}_R : \quad \vec{B}_R = \frac{\mu_0 I_2}{2\pi(3/2\, r)}(-\hat{z}) \qquad \vec{F}_R = (I_3 r \hat{y}) \times \vec{B}_R = \frac{\mu_0 I_2 I_3}{3\pi}(-\hat{x})$$

$$\vec{F}_L : \quad \vec{B}_L = \frac{\mu_0 I_2}{2\pi(1/2\, r)}(-\hat{z}) + \mu_0 I_1 n \hat{y}$$

$$\vec{F}_L = (-I_3 r \hat{y}) \times \vec{B}_L = \frac{\mu_0 I_2 I_3}{\pi}\hat{x}$$

$$\vec{F} = \vec{F}_L + \vec{F}_R = \frac{2}{3}\frac{\mu_0 I_2 I_3}{\pi}\hat{x}$$