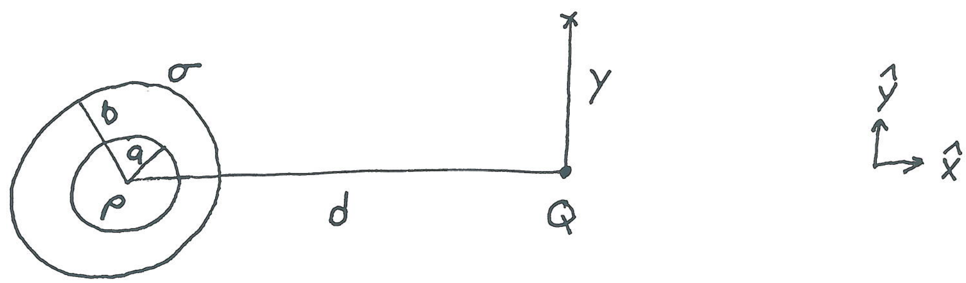
# SMU Physics 1304 : Spring 2011

## Exam 1

Problem 1 : The figure below shows a spherical region of constant charge per unit volume $\rho = 10^{-4}\,\mathrm{C/m^3}$ and radius $a = 10^{-2}\,\mathrm{m}$ centered at $x = 0$ and $y = 0$. This is surrounded by a spherical shell of surface charge density $\sigma = 3 \times 10^{-6}\,\mathrm{C/m^2}$ and radius $b = 2 \times 10^{-2}\,\mathrm{m}$. To the right is a point charge of $Q = -4 \times 10^{-10}\,\mathrm{C}$ at $x = d = 6 \times 10^{-2}\,\mathrm{m}$ and $y = 0$. Find the electric field vector as a function of $x$ for $0 < x < a$, $a < x < b$, and $b < x < d$ with $y = 0$. Also find the electric field vector as a function of $y$ at $x = d$.




$y=0$

$0<x<a$

$$\vec{E} = \frac{K\,\frac{4}{3}\pi x^3 \rho\,\hat{x}}{x^2} - \frac{KQ\hat{x}}{(d-x)^2} = K\left(\tfrac{4}{3}\pi x\rho - \frac{Q}{(d-x)^2}\right)\hat{x}$$

$a<x\,b$

$$\vec{E} = \frac{K\,\frac{4}{3}\pi a^3 \rho}{x^2}\hat{x} - \frac{KQ\hat{x}}{(d-x)^2} = K\left(\frac{\frac{4}{3}\pi a^3\rho}{x^2} - \frac{Q}{(d-x)^2}\right)\hat{x}$$

$b<x<d$

$$\vec{E} = K\left(\frac{\left(\frac{4}{3}\pi a^3\rho + 4\pi b^2\sigma\right)}{x^2} - \frac{Q}{(d-x)^2}\right)\hat{x}$$

$x=d$ (all $y$)

$$\vec{E} = \frac{KQ}{y^2}\hat{y} + K\left(\tfrac{4}{3}\pi a^3\rho + 4\pi b^2\sigma\right)\vec{r}/r^3$$

$$\vec{r} = d\hat{x} + y\hat{y}$$

Problem 2 : The figure below shows an empty cylindrical cavity of radius $a = 10^{-3}$ m surrounded by a cylindrical region of constant charge per unit volume $\rho = 10^{-5}\,\mathrm{C/m^3}$ with inner radius $a$ and outer radius $b = 2 \times 10^{-3}$ m. Outside this is a cylindrical conductor with zero net charge, with respective inner radius and surface charge density $c = 3 \times 10^{-3}$ m and $\sigma_c$, and respective outer radius and surface charge density $d = 4 \times 10^{-3}$ m and $\sigma_d$. Find $\sigma_c$ and $\sigma_d$, and find the electric field as a function of $r$ for $r < a$, $a < r < b$, $b < r < c$, $c < r < d$, and $r > d$.

$r < a$

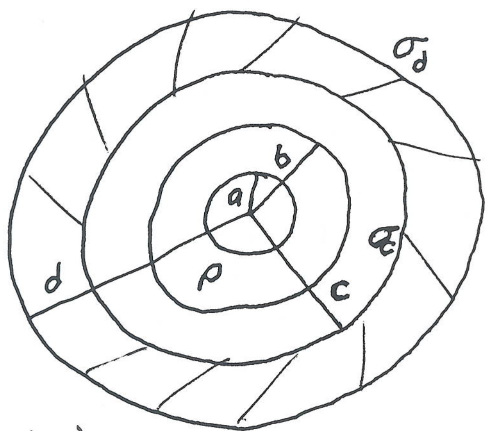
$E = 0$

$a < r < b$

$\cancel{2\pi r L E = 4\pi K}$

$2\pi r L E = 4\pi K(\pi(r^2 - a^2)\rho) L$

$E = \frac{2K}{r}\pi(r^2 - a^2)\rho$




$d > r > c$

$E = 0$

$b < r < c$

$2\pi r L E = 4\pi K(\pi(b^2 - a^2)\rho) L$

$E = \frac{2K}{r}\pi(b^2 - a^2)\rho$

$r > d$

$2\pi r L E = 4\pi K(\pi(b^2 - a^2)\rho) L$

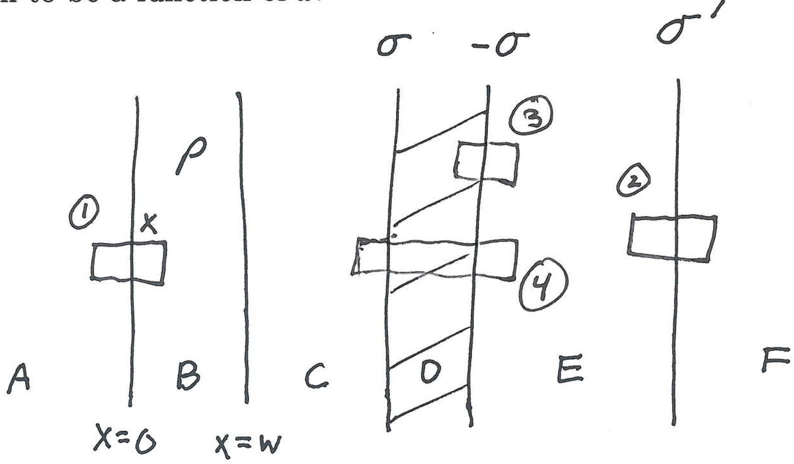
$E = \frac{2K}{r}\pi(b^2 - a^2)\rho$

define: $Q = L\pi(b^2 - a^2)\rho$

then $\sigma_d 2\pi d L = -\sigma_c 2\pi c L = Q$

$\sigma_d = \frac{\pi(b^2 - a^2)\rho}{2\pi d}$ $\sigma_c = \frac{-\pi(b^2 - a^2)\rho}{2\pi c}$

Problem 3 : The figure below shows, from left to right, an infinite planar slab of constant charge per unit volume $\rho = 10^{-4}\,\mathrm{C/m^3}$ extending from $x = 0$ to $x = w = 10^{-2}\,\mathrm{m}$, a conducting plane with zero net charge with respective left and right surface charge densities $\sigma$ and $-\sigma$, and an infinite plane with surface charge density $\sigma' = -2\times10^{-6}\,\mathrm{C/m^2}$. In terms of $\rho$ and $\sigma'$, find $\sigma$ and the electric field in the six regions A through F, with the field in region B taken to be a function of $x$.




$E_A = -E_\infty$

$E_F = E_\infty$

$E_D = 0$

box in A through F: $2E_\infty A = 4\pi K(\rho w + \sigma')A$

$E_\infty = 2\pi K(\rho w + \sigma')$

① $E_\infty A + E(x)A = 4\pi K \rho x A$ $\qquad E(x) = 4\pi K\rho x - E_\infty$ (in B)

② $(E_\infty - E_E)A = 4\pi K\sigma' A$ $\qquad E_E = E_\infty - 4\pi K\sigma'$

③ $E_E A = -4\pi K\sigma A$

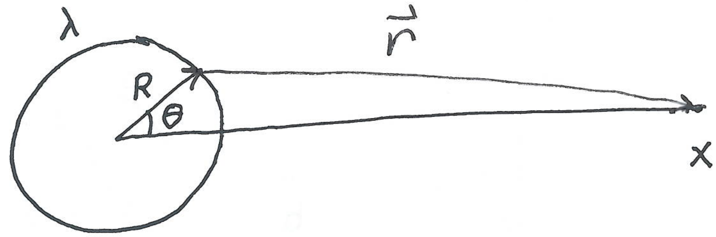
$\sigma = -E_E/4\pi K$

④ $A(E_E - E_c) = 0$

$E_C = E_E$

Problem 4 : The figure below shows a circular loop of radius $R = 10^{-2}\,\mathrm{m}$ with constant charge per unit length $\lambda = 10^{-6}\,\mathrm{C/m}$. Find the electrical potential at a point in the plane of the loop which is outside the circle at a distance $x$ from the center of the loop. Show that if the resulting integral is simplified by making the replacement $x = zR$ then the electrical potential may be written as

$$V(x) = k\lambda \int_0^{2\pi} d\theta\,(1 + z^2 - 2z\cos\theta)^{-1/2}$$




$dq = \lambda R\, d\theta$

$\vec{r} = x\hat{x} - (\hat{x}\cos\theta + \hat{y}\sin\theta)R$

$= (x - R\cos\theta)\hat{x} - R\sin\theta\,\hat{y}$

$V(x) = K\int dq/|\vec{r}| = K\lambda R \int_0^{2\pi} d\theta\,\left((x - R\cos\theta)^2 + (R\sin\theta)^2\right)^{-1/2}$

$= K\lambda R \int_0^{2\pi} d\theta\,(x^2 + R^2 - 2Rx\cos\theta)^{-1/2}$

choose $x = zR$

$V(x) = K\lambda \int_0^{2\pi} d\theta\,(1 + z^2 - 2z\cos\theta)^{-1/2}$