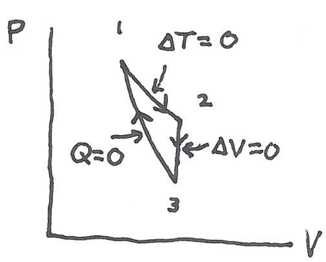
# SMU Physics 1308 : Fall 2009

## Exam 1 : Part 1

Problem 1 : Consider a closed process involving $n = 1\,\text{mol}$ of an ideal gas with $C_V = \frac{3}{2}R$. The gas is initially at $p_1 = 10^5\,\text{N/m}^2$ and $V_1 = 10^{-3}\,\text{m}^3$. It then undergoes an isothermal expansion until $V_2 = 10^{-2}\,\text{m}^3$. This is followed by a constant volume process which brings the system to a pressure $p_3$ and volume $V_3 = V_2$ which lies on the same adiabatic curve as $(p_1, V_1)$. The system then undergoes adiabatic compression along this curve to return to $(p_1, V_1)$. Draw this process in the $p - V$ plane, and find $(T_1, T_2, T_3)$ and $(p_2, p_3)$. For each individual process, find the work and heat; that is find $(W_{12}, W_{23}, W_{31})$ and $(Q_{12}, Q_{23}, Q_{31})$. Compare the efficiency of this process, defined as $e = W/Q_+$, with the Carnot efficiency $e = 1 - T_-/T_+$. Here $W$ is the total work done, $Q_+$ is the total heat added over those segments of the process in which heat is positive, and $T_+$ and $T_-$ are the highest and lowest temperatures, respectively, that the system attains.




$\gamma = C_p/C_V = 1 + R/C_V = 5/3 \qquad R = 8.31\ \text{J/K·mol}$

$T_2 = T_1 = P_1V_1/nR = \underline{12\text{ K}} \qquad P_2 = P_1V_1/V_2 = \underline{10^4\ \text{N/m}^2}$

$P_1V_1^\gamma = P_3V_3^\gamma \quad V_3 = V_2 \quad P_3 = P_1(V_1/V_2)^\gamma = \underline{2154\ \text{N/m}^2}$

$T_3 = P_3V_3/nR = \underline{2.59\text{ K}}$

$\Delta E_{12} = Q_{12} - W_{12} = nC_V\Delta T_{12} = 0 \qquad Q_{12} = W_{12} = P_1V_1\ln(V_2/V_1) = \underline{230\text{ J}}$

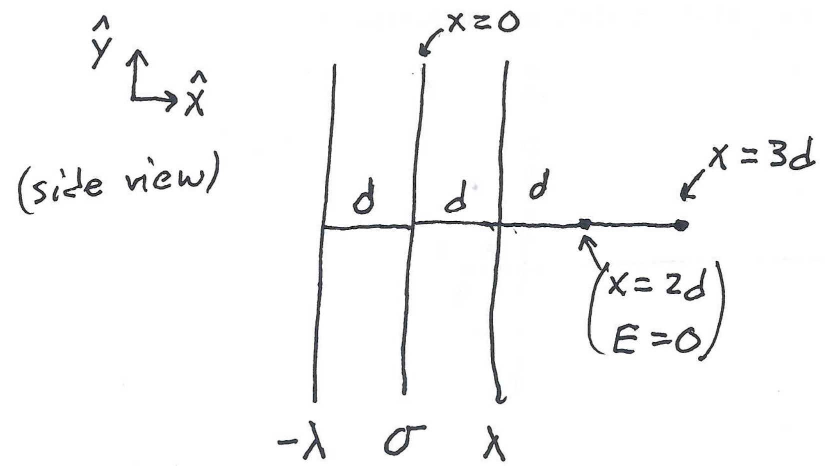
$\underline{W_{23} = 0} \qquad Q_{23} = \Delta E_{23} = nC_V\Delta T_{23} = n\tfrac{3}{2}R(T_3 - T_2) = \underline{-118\text{ J}}$

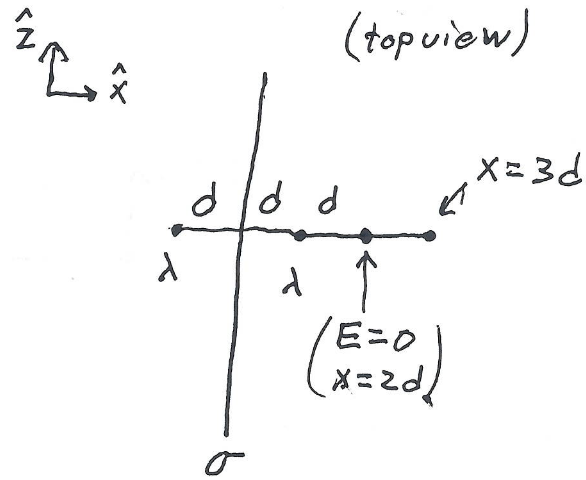
$\underline{Q_{31} = 0} \qquad W_{31} = -\Delta E_{31} = -nC_V\Delta T_{31} = n\tfrac{3}{2}R(T_1 - T_3) = \underline{-118\text{ J}}$

$W = W_{12} + W_{23} + W_{31} = \underline{112\text{ J}} \qquad Q_+ = Q_{12} = \underline{230\text{ J}}$

$e = W/Q_+ = \underline{0.49} \qquad e_c = 1 - T_-/T_+ = 1 - T_3/T_1 = \underline{0.78}$

Problem 2 : The figure below shows an infinite plane of charge at $x = 0$ with area charge density $\sigma = 10^{-6}\,\mathrm{C/m^2}$, as well as two parallel infinite line charges which extend in the $y$ direction. The first line charge has linear charge density $-\lambda$ and is located at $x = -d$ and $z = 0$. The second line charge has linear charge density $\lambda$ and is located at $x = d$ and $z = 0$. If $d = 0.1\,\mathrm{m}$ and the electric field vanishes at $(x, y, z) = (2d, 0, 0)$, find the linear charge density $\lambda$. Also find the electric field vector at $(x, y, z) = (3d, 0, 0)$.




$(y=0, z=0)$

$\vec{E} = E(x)\,\hat{x}$

$$E(x) = \frac{2K\lambda}{(x-d)} - \frac{2K\lambda}{(x+d)} + 2\pi K\sigma$$

$$= 2\pi K\sigma + \frac{4Kd\lambda}{(x^2-d^2)}$$

$x=2d \Rightarrow E=0$

$$0 = 2\pi K\sigma + \frac{4K\lambda}{3d} \Rightarrow \lambda = -\tfrac{3}{2}\pi d\sigma$$

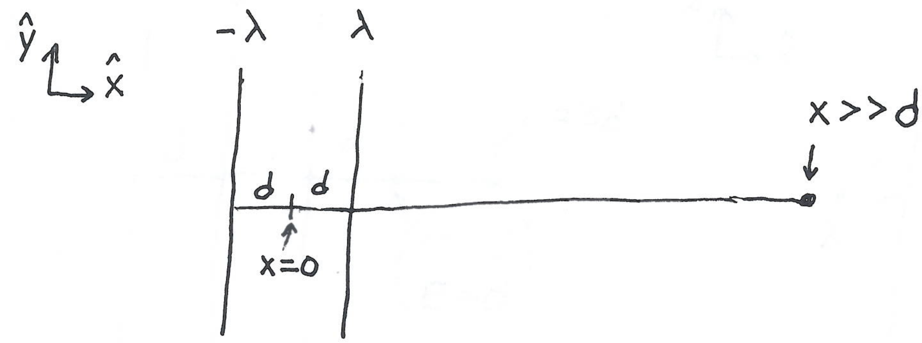
$$\lambda = -4.71\times10^{-7}\,\mathrm{C/m}$$

$x=3d$

$$E = 2\pi K\sigma + \frac{4Kd\lambda}{8d^2} = 2\pi K\sigma + \frac{K}{2}\left(-\tfrac{3}{2}\pi\sigma\right)$$

$$= \frac{5}{4}\pi K\sigma = 3.53\times10^{4}\,\mathrm{N/C}$$

Problem 3 : The figure below shows two parallel line charges with geometry identical to that in the previous problem. That is, these charges extend in the $y$ direction, and are located at $(x, z) = (-d, 0)$ and $(x, z) = (d, 0)$, respectively. Again, here $d = 0.1\,\mathrm{m}$, and the line charges have linear charge densities $-\lambda$ and $\lambda$, respectively, which are possibly different from those in the previous problem. Show that very far from the line charges along the $x$ axis, the electric field has the approximate form of a point charge. Also, find the linear charge density $\lambda$ if the equivalent point charge is $Q = 10^{-4}\,\mathrm{C}$.




looks like :

for $x >> d$


$$E = \frac{2K\lambda}{x-d} - \frac{2K\lambda}{x+d} = \frac{4Kd\lambda}{x^2-d^2}$$

look at $x >> d$

$$E = \frac{4Kd\lambda}{x^2(1-d^2/x^2)} \rightarrow \frac{4Kd\lambda}{x^2}$$

looks like $E = \dfrac{KQ}{x^2}$ with $Q = 4d\lambda$

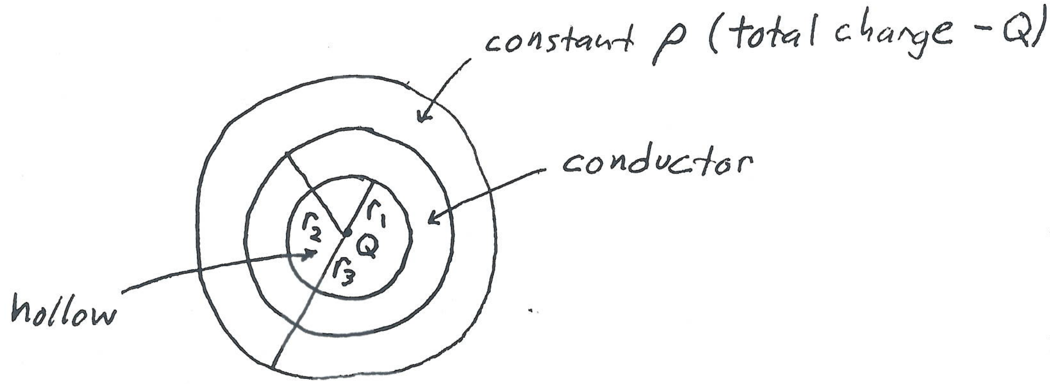
for $Q = 10^{-4}\,C$

$$\lambda = \frac{Q}{4d} = \underline{\underline{2.5 \times 10^{-4}\,C/m}}$$

# SMU Physics 1308 : Fall 2009

## Exam 1 : Part 2

Problem 4 : The figure below shows a point charge $Q = 3\,\mathrm{C}$ at the center of a hollowed out conducting sphere of inner radius $r_1 = 0.10\,\mathrm{m}$ and outer radius $r_2 = 0.12\,\mathrm{m}$. Just outside the conducting sphere is a spherical shell of constant volume charge density with inner radius $r_2$ and outer radius $r_3 = 0.15\,\mathrm{m}$. The total charge in this shell is $-Q$, and the conducting shell has zero net charge. Find the form of the electric field at points $< r_1$, $r_1 < r < r_2$, $r_2 < r < r_3$, $r > r_3$, in terms of $(k, Q, r_1, r_2, r_3)$. Evaluate these expressions at $r = 0.05\,\mathrm{m}$, $r = 0.11\,\mathrm{m}$, $r = 0.14\,\mathrm{m}$ and $r = 0.16\,\mathrm{m}$ to find numerical values for the field.




$\underline{r < r_1}$ $\quad 4\pi r^2 E = 4\pi K Q_{enc}$

$Q_{enc} = 0$ $\quad E = \dfrac{QK}{r^2}$

$\underline{E(.05m) = 1.08 \times 10^{13} N}$

$\underline{r_1 < r < r_2}$ $\quad Q_{enc} = 0$

$E = 0$ so $E(.11m) = 0$

$\underline{r > r_3}$ $\quad Q_{enc} = 0$

$E = 0$ so $E(.16m) = 0$

$\underline{r_2 < r < r_3}$

$$4\pi r^2 E(r) = 4\pi K Q(r)$$

$Q(r) = \rho V_s(r) + Q$

$V_s(r)$: Volume within shell at $r > r_2$

$\rho$: charge density within shell

total charge on shell:

$-Q = \rho V_s(r_3)$

$$V_s(r) = \tfrac{4}{3}\pi (r^3 - r_2^3)$$

$$\rho V_s(r) = -Q\, V_s(r)/V_s(r_3) = -Q \frac{(r^3 - r_2^3)}{(r_3^3 - r_2^3)}$$

$$E(r) = \frac{KQ(r)}{r^2} = \frac{KQ}{r^2}\left(1 - (r^3 - r_2^3)/(r_3^3 - r_2^3)\right)$$

$$= \underline{\frac{KQ}{r^2} \frac{(r_3^3 - r^3)}{(r_3^3 - r_2^3)}}$$

so, $\underline{E(.14m) = 4.93 \times 10^{11} N}$