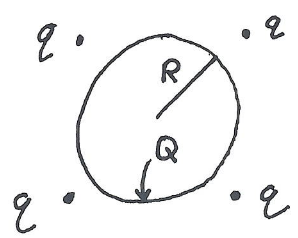
SMU Physics 1308 : Fall 2009

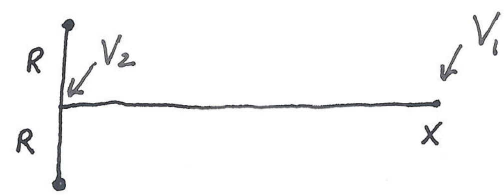
Exam 2 : Part 1

Problem 1 : The figure below shows a circular loop of charge Q and radius R with uniform linear charge density. There are four equal point charges q arranged at the corners of a square with sides of length $2R$ which is in the same plane and shares the same center as the loop. Find the potential difference ΔV between a point at the center of the loop and a point at a distance x along the axis perpendicular to the plane of the loop which extends from the center of the loop.

Front view:



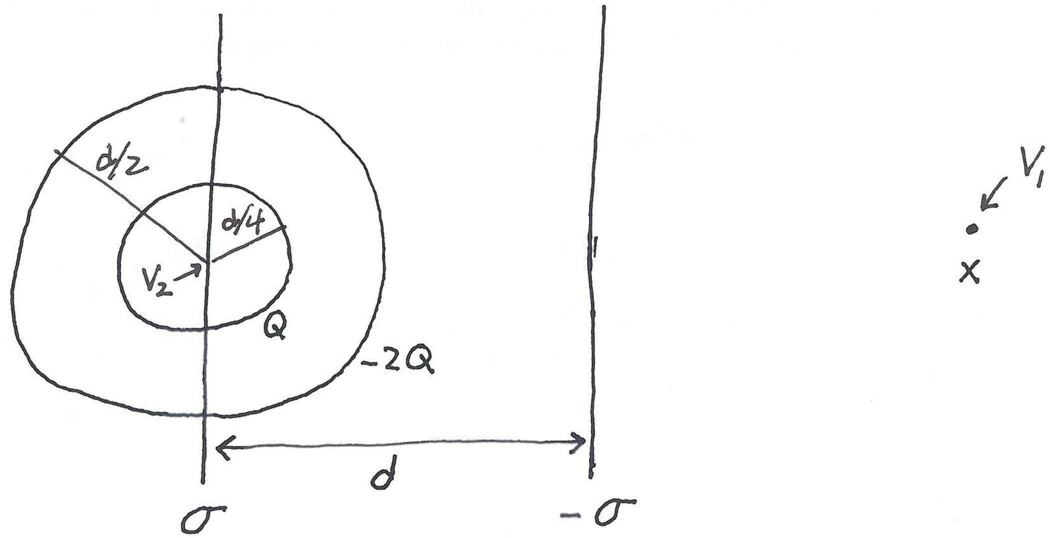
side view



$$V_2 - V_1 = 4Kq\left(\frac{1}{\sqrt{2R^2}} - \frac{1}{\sqrt{2R^2+x^2}}\right)$$

$$+ KQ\left(\frac{1}{\sqrt{R^2}} - \frac{1}{\sqrt{R^2+x^2}}\right)$$

Problem 2 : The figure below shows two parallel planes separated by a distance d. The left and right plane have uniform surface charge density σ and $-\sigma$, respectively. There are two concentric spheres with uniform surface charge density which are centered on the left plane. The inner sphere has radius $d/4$ and charge Q, and the outer sphere has radius $d/2$ and charge $-2Q$. Find the potential difference ΔV between a point at the center of the spheres and a point at a distance $x > d$ along the axis perpendicular to the planes which extends from the center of the spheres. Show that this difference is finite in the limit $x \to \infty$.



$$V_2 - V_1 = KQ\left(\frac{1}{(d/4)} - \frac{1}{X}\right) - 2KQ\left(\frac{1}{(d/2)} - \frac{1}{X}\right)$$

$$- 2\pi K\sigma(0 - X) + 2\pi K\sigma(d - X)$$

$$= \frac{KQ}{X} + 2\pi K\sigma d \xrightarrow[X \to \infty]{} 2\pi K\sigma d$$

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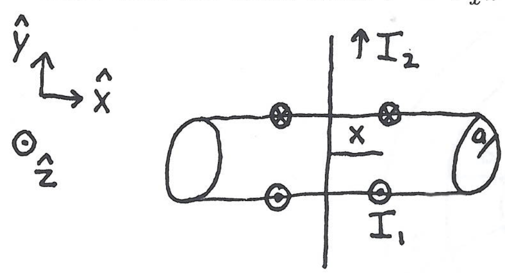
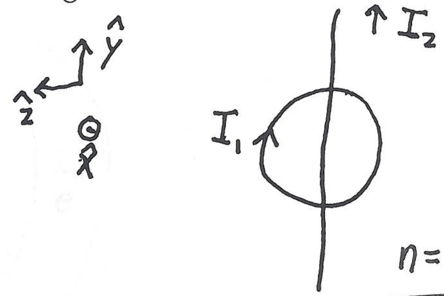
Exam 2 : Part 2

Problem 1 : The figure below shows an infinite solenoid of radius $a = 0.05\,\mathrm{m}$ which extends along the x axis, and which carries a current $I_1 = 2\,\mathrm{A}$ in the indicated direction. Also shown is an infinite wire which carries a current $I_2 = 3\,\mathrm{A}$ with indicated direction along the y axis, thus coinciding with a diameter of the solenoid. Find the magnetic field vector $\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$ at a point $x = 0.1\,\mathrm{m}$ along the x axis (thus at $z = 0$ and $y = 0$). Also find the total force $\vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$ acting on the wire.





$n = 10^3\,\mathrm{m}^{-1}$

inside solenoid : $\vec{B}_s = -\mu_0 n I_1 \hat{x}$

wire at $x\hat{x}$: $\vec{B}_w = -\dfrac{\mu_0 I_2}{2\pi x}\hat{z}$

Thus at $x\hat{x}$: $\vec{B} = -\mu_0 n I_1 \hat{x} - \dfrac{\mu_0 I_2}{2\pi x}\hat{z} = -2.52\times10^{-3}\hat{x}\,T - 6.02\times10^{-6}\hat{z}\,T$

Force on wire :

field $\vec{B}_s$ only non-zero inside solenoid.

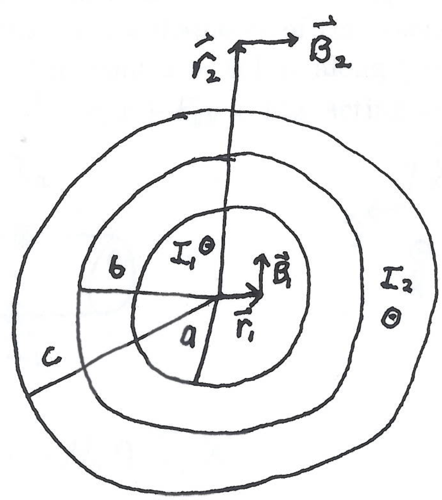
$$\vec{F}_w = I_2 \vec{L}_2 \times \vec{B}_s \qquad \vec{L}_2 = 2a\hat{y}$$

$$= I_2(2a\hat{y})\times(-\mu_0 n I_1 \hat{x})$$

$$= \mu_0 I_1 I_2 (2an)\hat{z}$$

$$= 7.56\times10^{-4}\hat{z}\,N$$

Problem 2 : The figure below shows a coaxial cable extending along the z axis with an inner wire of radius $a = 0.005\,\text{m}$ carrying current I_1, and an outer wire with inner radius $b = 0.01\,\text{m}$ and outer radius $c = 0.02\,\text{m}$ carrying current I_2. The magnetic field at $\vec{r}_1 = r_1\hat{x}$ with $r_1 = 0.002\,\text{m}$ is given by $\vec{B}_1 = B_1\hat{y}$ with $B_1 = 0.01\,\text{T}$. The magnetic field at $\vec{r}_2 = r_2\hat{y}$ with $r_2 = 0.03\,\text{m}$ is given by $\vec{B}_2 = B_2\hat{x}$ with $B_2 = 0.04\,\text{T}$. Find the currents I_1 and I_2 with positive values defined as being along the z axis.



$\underline{r<a}$

$$2\pi r_1 B_1 = \mu_0 I_1 \frac{r_1^2}{a^2} \qquad I_1 = B_1 \frac{2\pi}{\mu_0}\frac{a^2}{r_1} = \underline{623\,A}$$

$\vec{B}_1$: CCW $d\vec{L}$: CCW

$d\vec{L}\cdot\vec{B}_1 > 0$

$d\vec{L} = dL\hat{y}$ at $\vec{r}_1$

$\underline{r>c}$

$$-2\pi r_2 B_2 = \mu_0(I_1 + I_2)$$

$\vec{B}_2$: CW $d\vec{L}$: CCW

$$I_2 = -\frac{2\pi}{\mu_0} r_2 B_2 - I_1 = \underline{-6607\,A}$$

$d\vec{L}\cdot\vec{B}_2 < 0$

$d\vec{L} = -dL\hat{x}$ at $\vec{r}_2$