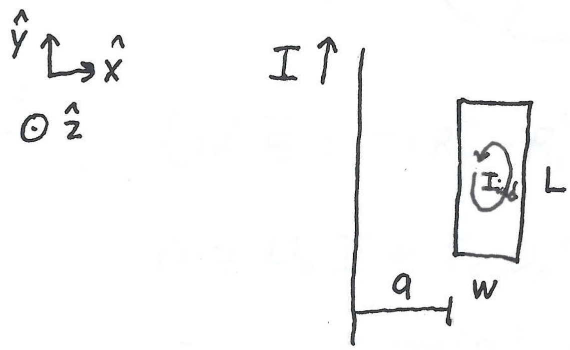
SMU Physics 1308 : Fall 2009

Exam 3

Problem 1 : The figure below shows an infinite wire carrying current I, with positive I taken to be in the $\hat{y}$ direction. At a distance $a = 0.15\,\mathrm{m}$ along the $\hat{x}$ axis is a loop of wire of length $L = 0.2\,\mathrm{m}$ and width $w = 0.1\,\mathrm{m}$ as shown. From Ampere's Law we have found that the magnetic field from the infinite wire at all points within the loop is given by $\vec{B} = -(\mu_o I/2\pi x)\hat{z}$, leading to a flux through the loop (with the choice $d\vec{A} = dA\,\hat{z}$) given by

$$\int d\vec{A}\cdot\vec{B} = -\frac{\mu_o IL}{2\pi}\ln\left(\frac{a+w}{a}\right)$$

If $dI/dt = 2\,\mathrm{A/s}$, find the induced current I_{ind} in the loop of wire if its resistance is $R = 2\,\Omega$, indicating its direction (CW or CCW). If at this time $I = 0.5\,\mathrm{A}$, also find the total force on the wire, expressing it in vector form as $\vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$.



$$\mathcal{E} = \int d\vec{L}\cdot\vec{E} = -\frac{d}{dt}\int d\vec{A}\cdot\vec{B}$$

$$= \frac{\mu_o L}{2\pi}\ln\left(\frac{a+w}{a}\right)\frac{dI}{dt}$$

$$= I_{ind}R$$

I_{ind} is CCW

$$I_{ind} = \frac{\mu_o L}{2\pi R}\ln\left(\frac{a+w}{a}\right)\frac{dI}{dt} = \underline{2.04\times10^{-8}\,A}$$

Top and Bottom forces cancel.

Right side: $\vec{F}_R = I_{ind}(L\hat{y})\times(-\mu_o I/2\pi(a+w))(\hat{z})$

$x = a+w$

$$= -\hat{x}\,I_{ind}\,\mu_o IL/(2\pi(a+w))$$

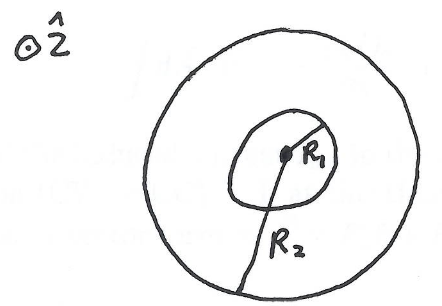
Left side: $\vec{F}_L = I_{ind}(-L\hat{y})\times(-\mu_o I/2\pi a)\,\hat{z}$

$x = a$

$$= \hat{x}\,I_{ind}\,\mu_o IL/(2\pi a)$$

$$\vec{F} = \vec{F}_R + \vec{F}_L = \hat{x}\,\frac{\mu_o IL I_{ind}}{2\pi}\left(\frac{1}{a} - \frac{1}{a+w}\right) = \underline{1.09\times10^{-14}\,N}$$

Problem 2 : The figure below shows a circular capacitor plate of radius $R_2 = 0.2\,\mathrm{m}$ which has a hole in the middle of radius $R_1 = 0.1\,\mathrm{m}$. The electric field is only non-zero between $r = R_1$ and $r = R_2$, where it is constant in space with form $\vec{E} = E(t)\,\hat{z}$ with $dE/dt = 5 \times 10^{10}\,\mathrm{V/m \cdot s}$. A wire extends through the center of the capacitor carrying an unknown current I, with positive I taken to be out of the page. The magnetic field at all points is purely tangential, and thus has the form $\vec{B} = B(r)\,\hat{\theta}$, with $\hat{\theta}$ being a unit vector in the counter-clockwise direction. If $B(r) = 0$ for $r > R_2$, find the current I running through the wire. Also find $B(r)$ for the two radii $r_1 = 0.15\,\mathrm{m}$ and $r_2 = 0.05\,\mathrm{m}$.



Use:

$$\oint d\vec{l} \cdot \vec{B} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d}{dt} \int d\vec{A} \cdot \vec{E}$$

$r > R_2$

$$\int d\vec{A} \cdot \vec{E} = \pi (R_2^2 - R_1^2) E \qquad \oint d\vec{l} \cdot \vec{B} = 0$$

$$0 = \mu_0 I + \varepsilon_0 \mu_0 \pi (R_2^2 - R_1^2) \frac{dE}{dt}$$

$$I = -\varepsilon_0 \pi (R_2^2 - R_1^2) \frac{dE}{dt} = \underline{-4.16 \times 10^{-2}\,A}$$

$r = r_2 < R_1$

$$\int d\vec{A} \cdot \vec{E} = 0 \qquad \oint d\vec{l} \cdot \vec{B} = 2\pi r_2 B = \mu_0 I$$

$$B(r_2) = \frac{\mu_0 I}{2\pi r_2} = \underline{-1.67 \times 10^{-7}\,T}$$

$r = r_1$

$R_2 > r_1 > R_1$

$$\int d\vec{A} \cdot \vec{E} = \pi (r_1^2 - R_1^2) E$$

$$\oint d\vec{l} \cdot \vec{B} = 2\pi r_1 B = \mu_0 I + \varepsilon_0 \mu_0 \pi (r_1^2 - R_1^2) \frac{dE}{dt}$$

$$B(r_1) = \frac{\mu_0 I}{2\pi r_1} + \frac{\varepsilon_0 \mu_0}{2} \frac{(r_1^2 - R_1^2)}{r_1} \frac{dE}{dt} = \underline{-3.24 \times 10^{-8}\,T}$$