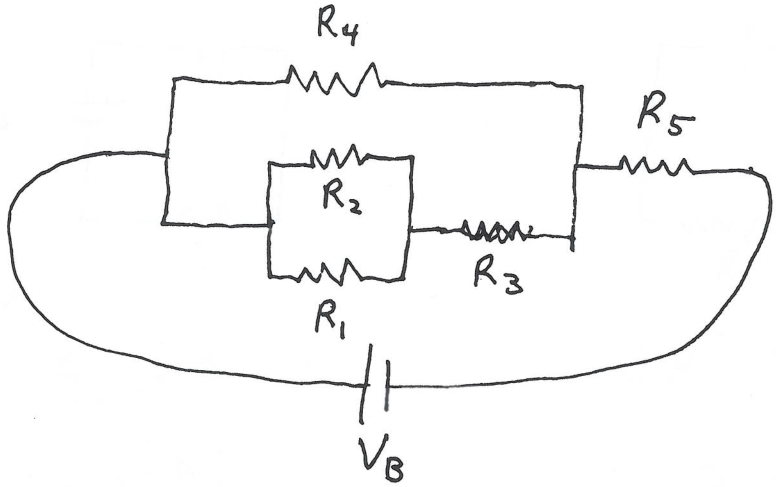
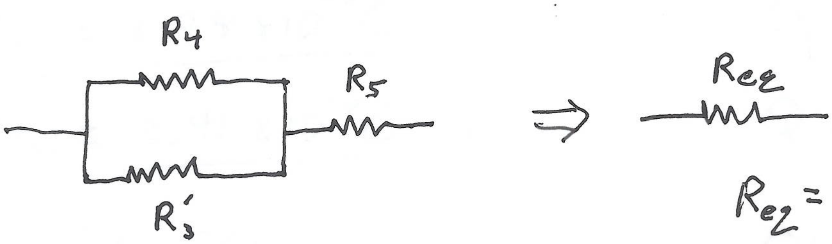
SMU Physics 1308 : Fall 2009

Final Exam

Problem 1 : The figure below shows a circuit with the corresponding resistances $R_1 = 2\Omega$, $R_2 = 7\Omega$, $R_3 = 11\Omega$, $R_4 = 13\Omega$, $R_5 = 17\Omega$. If the battery has the voltage $V_0 = 5\text{V}$, find the currents I_1, I_2, I_3, I_4, I_5 .



reduce to:



$$R_3' = R_3 + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \underline{12.56\,\Omega}$$

$$R_{eq} = R_5 + \left(\frac{1}{R_4} + \frac{1}{R_3'}\right)^{-1} = \underline{23.39\,\Omega}$$

$$I_5 = V_B / R_{eq} = \underline{\underline{0.214\,A}}$$

$$V_4 = V_B - I_5 R_5 = \underline{1.37\,V}$$

$$I_4 = V_4 / R_4 = \underline{\underline{0.105\,A}}$$

V_{12} : Voltage across R_1, R_2

$$I_3 = V_4 / R_3' = \underline{\underline{0.109\,A}}$$

$$V_{12} = V_4 - I_3 R_3 = \underline{0.169\,V}$$

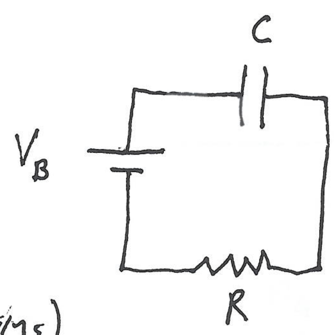
$$I_2 = V_{12} / R_2 = \underline{\underline{0.024\,A}}$$

Note: $I_1 + I_2 = I_3$

$I_3 + I_4 = I_5$

$$I_1 = V_{12} / R_1 = \underline{\underline{0.085\,A}}$$

Problem 2 : The RC circuit shown below has $R = 12\Omega$ and $C = 1.0 \times 10^{-6}\,\mathrm{F}$. Initially ($t = 0$) there is $Q_0 = 0$ on the capacitor, at which time the battery $V_B = 7\,\mathrm{V}$ is turned on. Find the charge $Q(t_1)$ at $t_1 = 4 \times 10^{-6}\,\mathrm{s}$. At $t_2 = 8 \times 10^{-6}\,\mathrm{s}$ the battery is replaced by a wire and the capacitor begins to discharge. Find the charge $Q(t_2)$ (which is the new Q_0 for the discharging capacitor) and the charge $Q(t_3)$ at $t_3 = 12 \times 10^{-6}\,\mathrm{s}$. Also find the currents I_0, $I(t_1)$, $I(t_2)$, $I(t_3)$.



$\underline{Q_0 = 0}$

$\underline{t < t_2}$ (charging)

$$V_B - IR - Q/C = 0$$

$$Q = CV_B\left(1 - e^{-t/RC}\right)$$

$$Q(t_1) = \underline{1.98 \times 10^{-6}\,C}$$

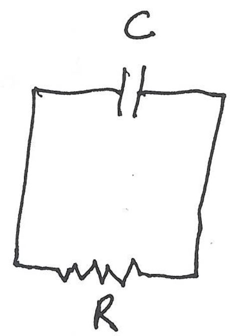
$$Q(t_2) = \underline{3.41 \times 10^{-6}\,C}$$

$$I = (V_B - Q/C)/R$$

$$I_0 = V_B/R = \underline{0.583\,A}$$

$$I(t_1) = \underline{0.418\,A}$$

$$I(t_2) = \underline{0.299\,A}$$



$\underline{t > t_2}$

$t' = t - t_2$

(uncharging)

$$-IR - Q/C = 0$$

$$Q = Q_0' e^{-t'/RC}$$

$$Q_0' = Q(t_2)$$

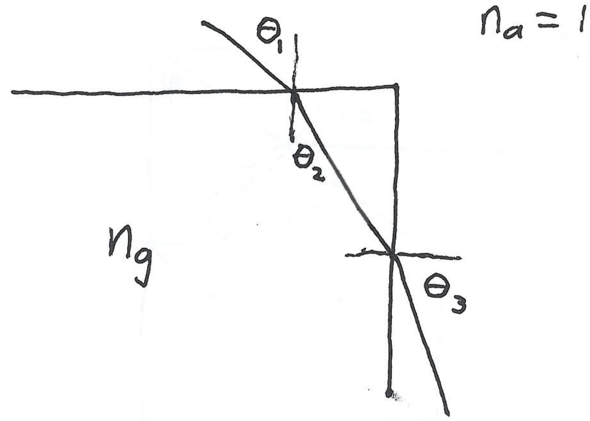
$$Q = Q(t_2)\, e^{-(t - t_2)/RC}$$

$$Q(t_3) = \underline{2.44 \times 10^{-6}\,C}$$

$$I = -Q/RC$$

$$I(t_3) = \underline{-0.203\,A}$$

Problem 3 : As shown in the figure below, a ray of light is incident from air ($n_a = 1$) at an angle $\theta_1 = 50°$ from the vertical on a piece of glass which makes a right angle. The light ray bends to an angle θ_2 from the vertical upon passing into the glass, after which it exits the glass at an angle $\theta_3 = 67°$ from the horizontal. Find the index of refraction n_g of the glass, and find the angle θ_2. You will need $\sin^2(\theta) + \cos^2(\theta) = 1$.



$$n_a \sin\theta_1 = n_g \sin\theta_2$$

$$n_g \sin(90-\theta_2) = n_a \sin\theta_3$$

$\Rightarrow$

$$\sin\theta_1 = n_g \sin\theta_2$$

$$\sin\theta_3 = n_g \cos\theta_2$$

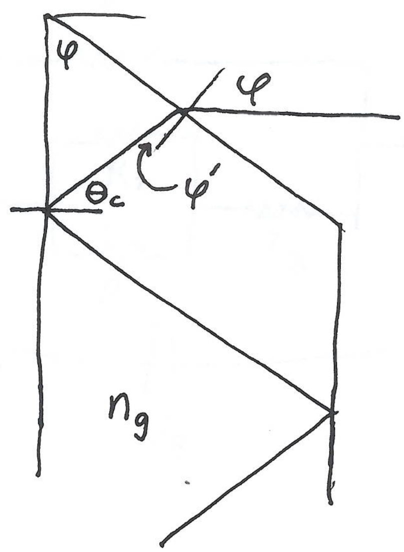
$$\sin^2\theta_1 + \sin^2\theta_3 = n_g^2$$

$$n_g = (\sin^2\theta_1 + \sin^2\theta_2)^{1/2} = \underline{\underline{1.198}}$$

$$\sin\theta_2 = \sin\theta_1 / n_g$$

$$\theta_2 = \underline{\underline{39.77°}}$$

Problem 4 : The figure below shows a piece of glass of refractive index $n_g = 1.6$ which is cut at an angle φ such that a ray which enters the glass from the surrounding air $n_a = 1.0$ along the horizontal line shown will reflect off the interior surfaces of the glass. Find the smallest angle φ above which this total internal reflection is possible, and find the corresponding reflection angle θ_c. Find the smallest index of refraction n_c such that this scenario is possible (this corresponds to requiring that $\tan(\varphi) = \sin(\varphi)/\cos(\varphi) > 0$). You will need $\sin(\varphi-\theta_c) = \sin(\varphi)\cos(\theta_c) - \cos(\varphi)\sin(\theta_c)$.



$n_a = 1$

$\sin\varphi = n_g \sin\varphi'$

$\varphi + 90 - \theta_c + 90 - \varphi' = 180$

$\underline{\varphi' = \varphi - \theta_c}$

TIR: $\underline{n_g \sin\theta_c = 1}$

$$\sin\varphi = n_g \sin(\varphi - \theta_c) = n_g(\sin\varphi\cos\theta_c - \cos\varphi\sin\theta_c)$$

$$\tan\varphi = n_g\cos\theta_c \tan\varphi - \underbrace{n_g \sin\theta_c}_{=1}$$

$$\tan\varphi = (n_g\cos\theta_c - 1)^{-1}$$

$$n_g\cos\theta_c = n_g(1-\sin^2\theta_c)^{1/2} = n_g(1-n_g^{-2})^{1/2} = (n_g^2-1)^{1/2}$$

$$\tan\varphi = ((n_g^2-1)^{1/2}-1)^{-1}$$

$\tan\varphi > 0$ requires:

$n_g = 1.6$: $\varphi = \underline{\underline{76^\circ}}$

$n_g > \underline{\underline{n_c = \sqrt{2}}}$