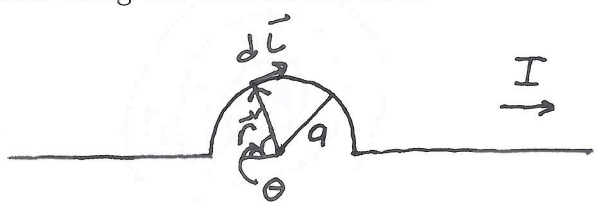
# SMU Physics 1308 : Spring 2009

## Exam 2

Problem 1 : The figure below shows an infinite wire that carries a current $I = 2\,\text{A}$. Near the origin the wire makes a semi-circle of radius $a = 0.01\,\text{m}$. Find the magnetic field at the center of the semi-circle using the Biot-Savart Law.




$\hat{y}$ $\hat{x}$ $\odot\,\hat{z}$

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int d\vec{L} \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = -\frac{\mu_0 I}{4\pi}\int d\vec{L} \times \vec{r}'/|\vec{r}'|^3$$

on straight sections:

$\underline{\underline{d\vec{L} \times \vec{r}' = 0}}$

$\underline{\vec{r} = 0}$

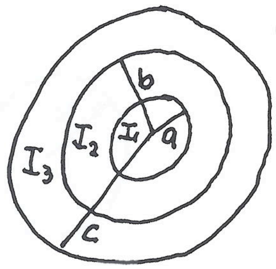
semicircle: $|\vec{r}'| = a$

$$\vec{B} = -\frac{\mu_0 I}{4\pi a^3}\int d\vec{L} \times \vec{r}'$$

$$d\vec{L} \times \vec{r}' = |d\vec{L}|\, a\,\hat{z} = a^2 d\theta\,\hat{z}$$

$$\vec{B} = -\frac{\mu_0 I}{4\pi a}\hat{z}\int_0^{\pi} d\theta = \underline{\underline{-\frac{\mu_0 I}{4a}\hat{z}}}$$

Problem 2 : The figure below shows the cross section of an infinite wire which is broken up into three conducting regions with the indicated radii $a = 0.01\,\mathrm{m}$, $b = 0.02\,\mathrm{m}$, and $c = 0.03\,\mathrm{m}$. The inner conducting region carries current $I_1 = 2\,\mathrm{A}$. The middle conducting region carries current $I_2 = -4\,\mathrm{A}$. The outer conducting region carries current $I_3 = 2\,\mathrm{A}$. Find the magnetic field in each of the regions; that is for $r < a$, $a < r < b$, $b < r < c$, and $r > c$.




$\vec{B} = B(r)\,\hat{\theta}$

Ampere: $\displaystyle\int_{L_c} d\vec{r}\cdot\vec{B} = \mu_0 I_{S_0}$

$L_c = \partial S_0$

↑ boundary

$r<a$

$2\pi r B = \mu_0 I_1\, r^2/a^2$

$B = \dfrac{\mu_0 I_1}{2\pi}\, r/a^2$

$b > r > a$

$2\pi r B = \mu_0\left(I_1 + I_2 \dfrac{(r^2-a^2)}{(b^2-a^2)}\right)$

$B = \dfrac{\mu_0}{2\pi}\left(\dfrac{I_1}{r} + \dfrac{I_2}{r}\dfrac{(r^2-a^2)}{(b^2-a^2)}\right)$

$c > r > b$

$2\pi r B = \mu_0\left(I_1 + I_2 + I_3 \dfrac{(r^2-b^2)}{(c^2-b^2)}\right)$

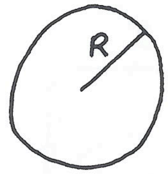
$B = \dfrac{\mu_0}{2\pi}\left(\dfrac{I_1+I_2}{r} + \dfrac{I_3}{r}\left(\dfrac{r^2-b^2}{c^2-b^2}\right)\right)$

$r>c$

$B = \dfrac{\mu_0}{2\pi r}(I_1 + I_2 + I_3)$

Problem 3 : The figure below shows the space between the plates of a circular capacitor of radius $R = 0.3\,\mathrm{m}$ which has a uniform electric field of the form $\vec{E} = at\hat{z}$, where $a = 2\,\mathrm{V/ms}$. The electric field outside the plates vanishes. Find the magnitude (and indicate its direction) of the magnetic field for $r < R$ and $r > R$. Find the magnitude (and indicate its direction) of the Poynting vector field for $r < R$ and $r > R$. Also compute the combined electromagnetic energy density $u_E + u_B$ for $r < R$ and $r > R$.

$\vec{B} = B(r)\,\hat{\theta}$




$\odot\,\hat{z}$

$\vec{E} = at\,\hat{z}$

Ampere/Maxwell

$$\oint_{L_c} d\vec{r}\cdot\vec{B} = \mu_0\varepsilon_0 \frac{d}{dt}\int d\vec{A}\cdot\vec{E}$$

$r<R$

$$2\pi r B = \mu_0\varepsilon_0 a\pi r^2$$
$$\vec{B} = \tfrac{1}{2}\mu_0\varepsilon_0 a r\,\hat{\theta}$$

$r>R$

$$2\pi r B = \mu_0\varepsilon_0 a\pi R^2$$
$$\vec{B} = \tfrac{1}{2}\mu_0\varepsilon_0 a R^2/r\,\hat{\theta}$$

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$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$\vec{S} = 0$ outside since $\vec{E} = 0$

inside: $\vec{S} = t/2\,\varepsilon_0 a^2 r\,\underbrace{\hat{z}\times\hat{\theta}}_{-\hat{r}} = -t/2\,\varepsilon_0 a^2 r\,\hat{r}$

(points inward)

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$U = U_B + U_E$

$U_E = 1/2\,\varepsilon_0\vec{E}^2$

$U_B = 1/2\,\mu_0^{-1}\vec{B}^2$

$U_E = 0 \quad r>R$

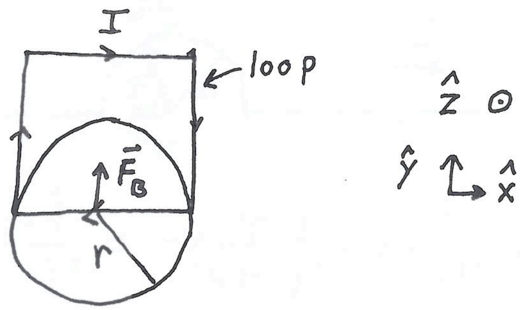
$$U = U_B = 1/2\,\mu_0^{-1}\left(1/2\,\mu_0\varepsilon_0 a R^2/r\right)^2$$

$r<R$

$$U = U_B + U_E \qquad U_E = 1/2\,\varepsilon_0 (at)^2$$

$$U_B = 1/2\,\mu_0^{-1}\left(1/2\,\mu_0\varepsilon_0 a r\right)^2$$

Problem 4 : The circular electromagnet shown in the figure below has a radius of $r = 0.2\,\mathrm{m}$ and produces a uniform magnetic field of the form $\vec{B} = bt\hat{z}$, where $b = 2\,\mathrm{T/s}$. A square loop of wire of side length $2r$ is suspended in the electromagnet so that one side extends along the horizontal diameter. The other three sides do not lie between the poles. If the loop has resistance $R = 1\,\Omega$, find the direction and magnitude of the current $I$ in the loop. Because of the current $I$, the loop feels a force $\vec{F_B}$ in the magnetic field which changes with time. Taking $\hat{z}$ out of the page, $\hat{x}$ to the right, and $\hat{y}$ upward, express $\vec{F_B}$ as a vector. If the wire weighs $m = 10^{-3}\,\mathrm{kg}$, find the time at which the wire would support its own weight.




Faraday:

$$IR = \mathcal{E} = -\frac{d}{dt}\int d\vec{A}\cdot\vec{B} = -\frac{d}{dt}\left(\frac{\pi}{2} r^2 b t\right) = -\frac{\pi}{2} r^2 b$$

$$\vec{L} = 2r\,\hat{x}$$
$$\vec{B} = bt\,\hat{z}$$

$$I = -\frac{\pi r^2 b}{2R} \quad \text{(clockwise)}$$

$$\vec{F_B} = I\vec{L}\times\vec{B}$$

$$\vec{F_B} = \left(-\frac{\pi r^2 b}{2R}\right)(2rbt)\underbrace{\hat{x}\times\hat{z}}_{=-\hat{y}}$$

$$\boxed{\vec{F_B} = \frac{\pi r^3 b^2}{R}t\,\hat{y}}$$

at $t_c$:

$$\vec{F_B} - mg\hat{y} = 0$$

$$mg = \frac{\pi r^3 b^2}{R}t_c \qquad t_c = \frac{mgR}{\pi r^3 b^2}$$

$$\boxed{I = -0.126\,A}$$

$$\boxed{\vec{F_B} = 0.1t\,\hat{y}}$$

$$\boxed{t_c = 0.097\,s}$$