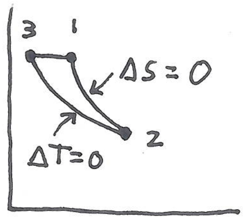
SMU Physics 1308 : Spring 2010

Exam 1

Problem 1 : Consider a closed process involving $n = 1\,\mathrm{mol}$ of an ideal gas with $C_V = \frac{3}{2}R$. The gas is initially at $p_1 = 10^5\,\mathrm{N/m^2}$ and $V_1 = 10^{-3}\,\mathrm{m^3}$. It then undergoes an adiabatic expansion until $V_2 = 5\times 10^{-3}\,\mathrm{m^3}$. This is followed by a isothermal compression ($T_3 = T_2$) which brings the system to a pressure $p_3 = p_1$. The system then undergoes a constant pressure expansion until it returns to (p_1, V_1). Draw this process in the p-V plane, and find (T_1, T_2, p_2, V_3). For each individual process, find the work and heat; that is find (W_{12}, W_{23}, W_{31}) and (Q_{12}, Q_{23}, Q_{31}). Compare the efficiency of this process, defined as $e = W/Q_+$, with the Carnot efficiency $e_C = 1 - T_-/T_+$. Here W is the total work done, Q_+ is the total heat added over those segments of the process in which heat is positive, and T_+ and T_- are the highest and lowest temperatures, respectively, that the system attains.

$P_3 = P_1$



$$\frac{P_1 V_1}{nR} = \underline{T_1}$$

$$\underline{P_2} = P_1 \left(V_1/V_2\right)^{\gamma}$$

$$\frac{P_2 V_2}{nR} = \underline{T_2}$$

$$\underline{V_3} = \frac{P_2 V_2}{P_3}$$

$$\gamma = \frac{C_P}{C_V} = \frac{5/2\,R}{3/2\,R} = 5/3$$

$$W_{12} = (P_1 V_1 - P_2 V_2)/(\gamma - 1)$$

$$W_{23} = nRT_2 \ln\left(V_3/V_2\right)$$

$$W_{31} = P_1 (V_1 - V_3)$$

$$T_+ = T_1$$

$$T_- = T_2 = T_3$$

$$Q_{12} = 0$$

$$Q_{23} = W_{23}$$

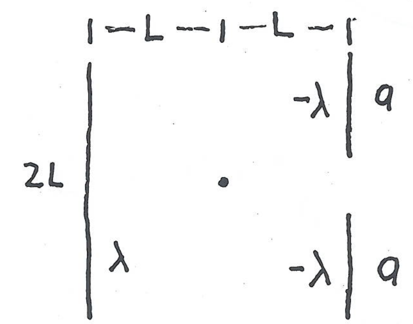
$$Q_{31} = nC_P(T_1 - T_3) = 5/2\, nR\,(T_1 - T_3) = \underline{Q_+}$$

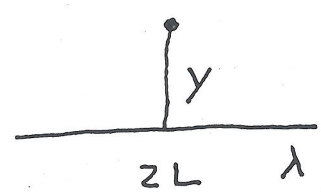
$$e = W/Q_+ \qquad e_c = 1 - T_-/T_+$$

Problem 2 : The figure at left below shows three line segments of constant linear charge density arranged on the perimeter of a square of sides $2L$, where $L = 0.1\,\mathrm{m}$. The left vertical segment has linear charge density $\lambda = 10^{-6}\,\mathrm{C/m}$ and is of length $2L$. The right two vertical segments each have linear charge density $-\lambda$ and are both of length $a = 0.05\,\mathrm{m}$. They are arranged as shown with a gap between them of width $2(L-a)$. The figure at right shows a horizontal line segment of length $2L$ and linear charge density λ. The field at a point y on the perpendicular bisector of this line charge is given by

$$\vec{E} = \frac{2k\lambda L\,\hat{y}}{y\sqrt{y^2+L^2}}$$

Use this result to find the electric field vector at the center of the square at left.





$$\vec{E} = \vec{E}_L + \vec{E}_R$$
(left) (right)

$$\vec{E}_L = \frac{2K\lambda L\,\hat{x}}{L(L^2+L^2)^{1/2}} = \frac{\sqrt{2}\,K\lambda}{L}\,\hat{x}$$

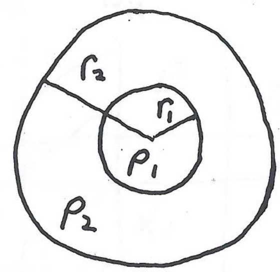
$$\vec{E}_R = \frac{\sqrt{2}\,K\lambda}{L}\,\hat{x} - \frac{2K\lambda(L-a)\,\hat{x}}{L\left(L^2+(L-a)^2\right)^{1/2}}$$

Problem 3 : The figure below shows a sphere of radius $r_1 = 0.02\,\mathrm{m}$ with a uniform volume charge density $\rho_1 = 2 \times 10^{-8}\,\mathrm{C/m^3}$ surrounded by a shell of outer radius $r_2 = 0.04\,\mathrm{m}$ with a uniform volume charge density $\rho_2 = -1 \times 10^{-8}\,\mathrm{C/m^3}$. Find the electric field at all points $r < r_1$, $r_1 < r < r_2$, and $r > r_2$.

$$Q_1 = \tfrac{4}{3}\pi \rho_1 r_1^3$$

$$Q_2 = \tfrac{4}{3}\pi \rho_2 (r_2^3 - r_1^3)$$

$$\vec{E} = E(r)\,\hat{r}$$



$\underline{r < r_1}$

$$E = KQ_1 r / r_1^3$$

$\underline{r > r_2}$

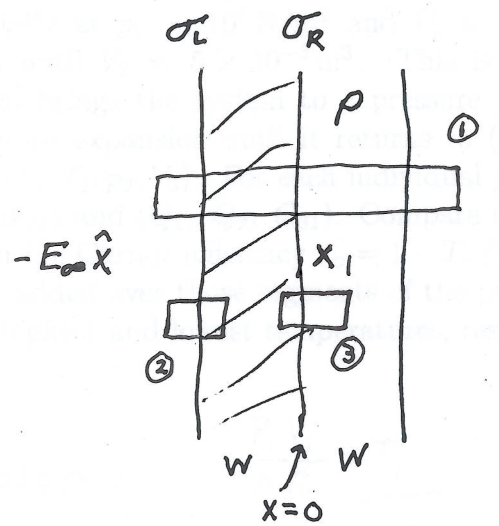
$$E = K(Q_1 + Q_2)/r^2$$

$\underline{r_2 > r > r_1}$

$$E = \frac{KQ_1}{r^2} + \frac{K}{r^2}\,\tfrac{4}{3}\pi \rho_2 (r^3 - r_1^3)$$

$$= \frac{KQ_1}{r^2} + \frac{K}{r^2}\, Q_2 \frac{(r^3 - r_1^3)}{(r_2^3 - r_1^3)}$$

Problem 4 : The figure below shows an infinite conducting slab of width $w = 0.01\,\mathrm{m}$ next to a slab of material of constant volume charge density $\rho = 3 \times 10^{-8}\,\mathrm{C/m^3}$ which also has width w. If the conducting slab has zero net charge, find the area charge densities σ_L and σ_R on its left and right side, respectively. Taking $x = 0$ to be the plane separating the two slabs, also find the electric field at all points $x < -w$, $-w < x < 0$, $0 < x < w$, and $x > w$.



$\sigma_L + \sigma_R = 0$

$-w < x < 0$

$\vec{E} = 0$ (conductor)

Box 1: $2AE_\infty = 4\pi K(\sigma_L + \sigma_R + \rho w)A$

$E_\infty = 2\pi K \rho w$

$x > w$ $\vec{E} = 2\pi K \rho w \hat{x}$

$x < -w$ $\vec{E} = -2\pi K \rho w \hat{x}$

Box 2: $AE_\infty = 4\pi K \sigma_L A$

$2\pi K \rho w = 4\pi K \sigma_L$

$\sigma_L = -\sigma_R = \rho w/2$

Box 3: $0 < x < w$

$AE(x) = 4\pi K(\sigma_R + \rho x)A$

$E(x) = 4\pi K \rho (x - w/2)$