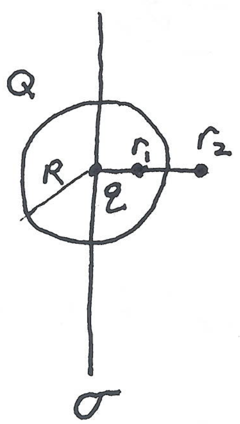
SMU Physics 1308 : Spring 2010

Exam 2

Problem 1 : The figure below shows a sheet of charge with area charge density $\sigma = 1\,\mathrm{C/m^2}$ centered on a spherical shell of radius $R = 0.2\,\mathrm{m}$ and charge $Q = 2\,\mathrm{C}$. There is also a point charge $q = 3\,\mathrm{C}$ at the center of the sphere. Find the electrical potential V_1 at $r_1 = 0.1\,\mathrm{m}$ and V_2 at $r_2 = 0.3\,\mathrm{m}$.



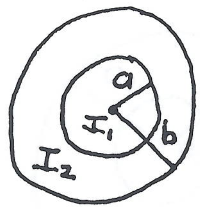
$$V_1 = \frac{Kq}{r_1} + \frac{KQ}{R} - 2\pi K\sigma r_1$$

$$= \underline{3.53 \times 10^{11}\,V}$$

$$V_2 = \frac{Kq}{r_2} + \frac{KQ}{r_2} - 2\pi K\sigma r_2$$

$$= \underline{1.33 \times 10^{11}\,V}$$

Problem 2 : The figure below shows a (very small radius) wire carrying current $I_1 = 2\,\text{A}$ (with positive current taken to be out of the page), inside a cylindrical wire of inner radius $a = 0.02\,\text{m}$ and outer radius $b = 0.03\,\text{m}$ which carries $I_2 = -3\,\text{A}$. Taking $\vec{B} = B(r)\hat{\theta}$, find $B(r)$ for $r < a$, $a < r < b$, and $r > b$.



$r<a$

$$2\pi r B = \mu_0 I_1$$

$$\vec{B} = \frac{\mu_0 I_1}{2\pi r}\hat{\theta}$$

$b>r>a$

$$2\pi r B = \mu_0\left(I_1 + I_2 (r^2-a^2)/(b^2-a^2)\right)$$

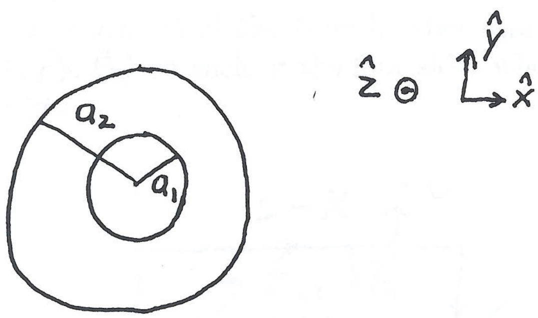
$$\vec{B} = \frac{\mu_0}{2\pi r}\left(I_1 + I_2(r^2-a^2)/(b^2-a^2)\right)\hat{\theta}$$

$r>b$

$$2\pi r B = \mu_0 (I_1 + I_2)$$

$$\vec{B} = \frac{\mu_0 (I_1+I_2)}{2\pi r}\hat{\theta}$$

Problem 3 : The figure below shows the cross section of a cylindrically symmetric magnet which has magnetic field given by $\vec{B} = b_1 t \hat{z}$ for $r < a_1$, with $b_1 = 1\,\mathrm{T/s}$ and $a_1 = 0.1\,\mathrm{m}$, and $\vec{B} = b_2 t \hat{z}$ for $a_1 < r < a_2$, with $a_2 = 0.2\,\mathrm{m}$. The magnetic field vanishes ($\vec{B} = 0$) for $r > a_2$. Taking $\vec{E} = E(r)\hat{\theta}$, find b_2 such that $E(r) = 0$ for $r > a_2$ and find $E(r)$ for $r < a_1$ and $a_1 < r < a_2$.



$r < a_1$

$$2\pi r E = -\frac{d}{dt}\left(b_1 t \pi r^2\right) = -b_1 \pi r^2$$

$$\vec{E} = -\frac{b_1 r}{2}\hat{\theta}$$

$r > a_2$

$\vec{E} = 0$

$$2\pi r E = -\frac{d}{dt}\left(b_1 t \pi a_1^2 + b_2 t \pi (a_2^2 - a_1^2)\right)$$

$$= -\left(b_1 \pi a_1^2 + b_2 \pi (a_2^2 - a_1^2)\right) = 0$$

$$b_2 = -b_1 a_1^2 / (a_2^2 - a_1^2)$$

$a_2 > r > a_1$

$$2\pi r E = -\frac{d}{dt}\left(b_1 t \pi a_1^2 + b_2 t \pi (r^2 - a_1^2)\right)$$

$$= -\left(b_1 \pi a_1^2 + b_2 \pi (r^2 - a_1^2)\right)$$

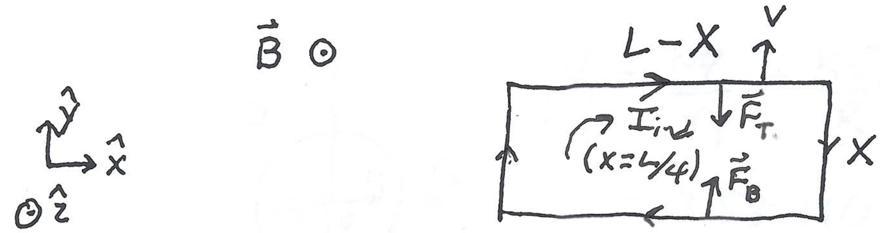
$$= -\pi b_1 \left(a_1^2 - a_1^2 (r^2 - a_1^2)/(a_2^2 - a_1^2)\right)$$

$$= -\pi b_1 a_1^2 (a_2^2 - r^2)/(a_2^2 - a_1^2)$$

$$\vec{E} = \frac{-\pi b_1 a_1^2 (a_2^2 - r^2)}{2\pi r (a_2^2 - a_1^2)}\hat{\theta}$$

Problem 4 : In the figure below there is a uniform magnetic field $\vec{B} = B\hat{z}$ coming out of the page with $B = 1\,\mathrm{T}$. A current loop is shown in the x-y plane which is has width $L - x$ and height x. It is being deformed at a constant rate so that $L = 0.1\,\mathrm{m}$ and $x = vt$ with $v = 0.2\,\mathrm{m/s}$. Find an expression for the induced current I_{ind} in the loop (with positive I_{ind} taken to be counter-clockwise) as a function of time (this will be valid between $t = 0$ when $x = 0$ and $t = L/v$ when $x = L$). Find the time t_c when $I_{\mathrm{ind}} = 0$, and find $x_c = vt_c$. Also find the forces ($\vec{F}_{\mathrm{R}}, \vec{F}_{\mathrm{L}}, \vec{F}_{\mathrm{T}}, \vec{F}_{\mathrm{B}}$) on each of the four sides when $x = L/4$. This problem requires $\frac{d}{dt}t = 1$ and $\frac{d}{dt}t^2 = 2t$.

$\tilde{R} = 3\,\Omega$



$$I_{ind}\tilde{R} = -\frac{d}{dt}\left(Bx(L-x)\right) = -\frac{d}{dt}\left(BvtL - Bv^2t^2\right)$$

$$= -BLv + 2Bv^2t$$

$$I_{ind} = -\frac{vB}{\tilde{R}}(L-2vt) = -\frac{vB}{\tilde{R}}(L-2x)$$

so, $I_{ind} = 0$ for $L = 2vt_c$ $\quad t_c = \frac{L}{2v}$ $\quad x_c = L/2$

for $x = L/4$ $\quad I_{ind} = -\frac{vB}{\tilde{R}}(L - L/2) = -\frac{vBL}{2\tilde{R}}$ (CW)

$\vec{L}_T = -3L/4\,\hat{x}$ $\quad \vec{F}_T = I_{ind}\vec{L}_T \times (B\hat{z}) = -\frac{vB^2L^2}{2\tilde{R}}(3/4)\,\hat{y}$

$\vec{L}_B = 3L/4\,\hat{x}$ $\quad \vec{F}_B = I_{ind}\vec{L}_B \times (B\hat{z}) = -\vec{F}_T$

$\vec{L}_R = L/4\,\hat{y}$ $\quad \vec{F}_R = I_{ind}\vec{L}_R \times (B\hat{z}) = -\frac{vB^2L^2}{2\tilde{R}}(1/4)\,\hat{x}$

$\vec{L}_L = -L/4\,\hat{y}$ $\quad \vec{F}_L = I_{ind}\vec{L}_L \times (B\hat{z}) = -\vec{F}_R$