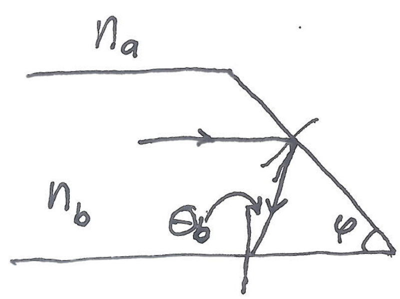
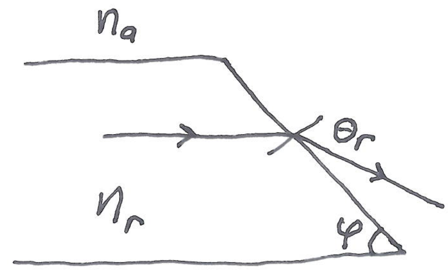
SMU Physics 1308 : Spring 2010

Final Exam

Problem 1 : The figure below shows a piece of glass which is cut at an angle φ and which is intended to filter out blue light and allow transmission of red light. Take the refractive index of the glass to be $n_b = 1.6$ for blue light and $n_r = 1.5$ for red light, and take the refractive index of the surrounding air to be $n_a = 1$. Find the angle φ at which total internal reflection occurs for a horizontal blue ray incident on the glass-air interface at right. Also find the angle θ_b at which the reflected blue ray is incident on the bottom horizontal surface. Does the blue ray then bounce back along the inside of the piece of glass? If a horizontal red ray is incident on the glass-air interface at right, at what angle θ_r does it emerge from the glass?





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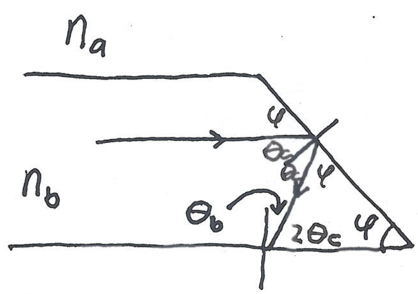
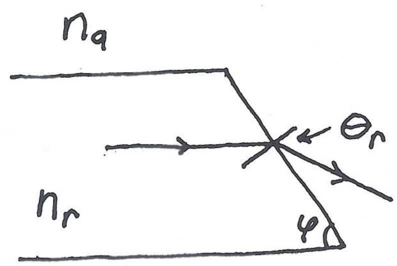




$\sin\theta_c = 1/n_b$

$\varphi = 90 - \theta_c$

$\theta_b = 90 - 2\theta_c$

$\theta_c = 38.68^\circ$

$\varphi = 51.32^\circ$

$\theta_b = 12.64^\circ$

$\theta_b < \theta_c$

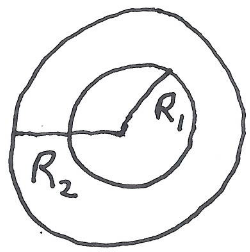
refracts through glass

$\sin\theta_c = (1/n_r)\sin\theta_r$

$\sin\theta_r = n_r/n_b$

$\theta_r = 69.64^\circ$

Problem 2 : The figure below shows a wire of radius $R_1 = 0.005\,\mathrm{m}$ carrying a current $I = 0.1\,\mathrm{A}$ which runs through a circular parallel plate capacitor of inner radius R_1 and outer radius $R_2 = 0.015\,\mathrm{m}$. The electric field between the plates of the capacitor has the form $\vec{E} = at\hat{z}$. The magnetic field at all points is purely tangential, and thus has the form $\vec{B} = B(r)\hat{\theta}$. If the magnetic field vanishes outside ($B(r) = 0$ for $r > R_2$) the capacitor, find the constant a, and find the magnetic field $B(r)$ for $r < R_1$ and for $R_1 < r < R_2$.



$r < R_1$

$\int d\vec{A} \cdot \vec{E} = 0$

$$2\pi r B = \mu_0 I r^2 / R_1^2$$

$$\vec{B} = \frac{\mu_0 I r}{2\pi R_1^2} \hat{\theta}$$

$r > R_2$

$B(r) = 0$

$\frac{dE}{dt} = a$

$\vec{B} = 0$

$$0 = 2\pi r B = \mu_0 I + \mu_0 \varepsilon_0 a \pi (R_2^2 - R_1^2)$$

$$\varepsilon_0 a = -I / \pi (R_2^2 - R_1^2)$$

$R_2 > r > R_1$

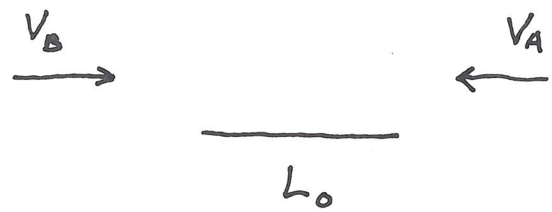
$$2\pi r B = \mu_0 I + \mu_0 \varepsilon_0 a \pi (r^2 - R_1^2)$$

$$= \mu_0 I - \mu_0 I (r^2 - R_1^2) / (R_2^2 - R_1^2)$$

$$= \mu_0 I \frac{(R_2^2 - r^2)}{(R_2^2 - R_1^2)}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \frac{(R_2^2 - r^2)}{(R_2^2 - R_1^2)} \hat{\theta}$$

Problem 3 : You are in a spaceship which has a rest (proper) length of $L_0 = 100\,\text{m}$. An observer A with negative velocity V_A with respect to your ship claims it is $L_A = 85\,\text{m}$ long. An observer B with positive velocity V_B with respect to your ship claims it is $L_B = 75\,\text{m}$ long. Find V_A and V_B, and find the relative velocity V_R of B with respect to A.



$$L_A/L_o = \gamma_A^{-1} = \sqrt{1 - V_A^2/c^2} \qquad L_B/L_o = \gamma_B^{-1} = \sqrt{1 - V_B^2/c^2}$$

$V_A < 0$

$$V_A = -c\left(1 - L_A^2/L_o^2\right)^{1/2}$$

$$V_A = -0.526c$$

$V_B > 0$

$$V_B = c\left(1 - L_B^2/L_o^2\right)^{1/2}$$

$$V_B = 0.661c$$

define:

$$V = V_{B \to A}$$

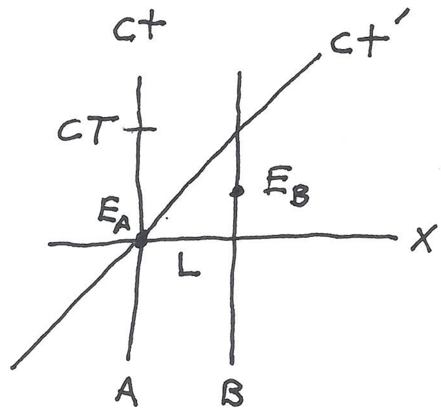
$$U = V_{S \to A} = -V_{A \to S} = -V_A = 0.526c$$

$$U' = V_{S \to B} = -V_{B \to S} = -V_B = -0.661c$$

$$U' = \frac{U - V}{1 - UV/c^2} \Rightarrow V = \frac{U - U'}{1 - UU'/c^2}$$

$$V = 0.881c$$

Problem 4 : Two stars A and B which are not moving with respect to one another are separated by $L = 10^6\, c \cdot \mathrm{s}$ as measured in the frame that they share. An event E_A happens on A which is given coordinates $c t_A = 0$ and $x_A = 0$. Another event E_B which occurs on B takes place at a time which is simultaneous to E_A as seen by an observer on a ship which moves from A to B with velocity $v = 0.8\, c$. Find $c t_B$ of this event for the stars frame, and find the distance x'_B of this event in the ships frame. How long cT does the ship take to get from star A to star B in the stars frame. How long cT' does the ship take to get from star A to star B in the ships frame. How far apart L' are the stars in the ships frame.



$$\beta = v/c$$

$$\gamma = (1 - v^2/c^2)^{-1/2}$$

$$L' = L/\gamma$$

$$vT = L \Rightarrow cT = L/\beta$$

$$cT' = cT/\gamma$$

$$ct'_B = 0 \qquad ct'_B = \gamma(ct_B - \beta x_B)$$

$$x_B = L \qquad x'_B = \gamma(x_B - \beta c t_B)$$

Thus

$$ct_B = \beta L$$

$$x'_B = \gamma(L - \beta^2 L) = L/\gamma = L'$$