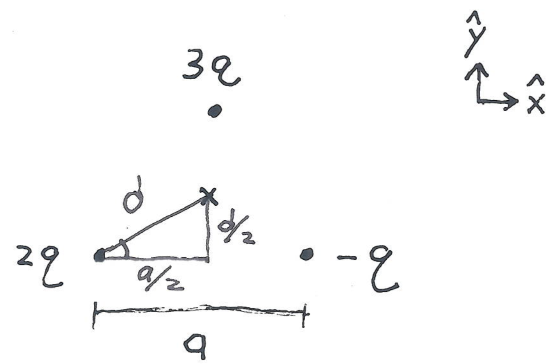
SMU Physics 1308 : Summer 2010

Exam 1

Problem 1 : The figure below shows three point charges of charge $3q$, $2q$, and $-q$, with $q = 2 \times 10^{-6}\,\mathrm{C}$, at the corners of an equilateral triangle of side $a = 0.01\,\mathrm{m}$. Find the electric field vector, with $\hat{x}$ and $\hat{y}$ axes as shown, at the center of the triangle. You will need $k = 9 \times 10^{9}\,\mathrm{N \cdot m^2/C^2}$.



$$d \sin 30^\circ = d/2$$

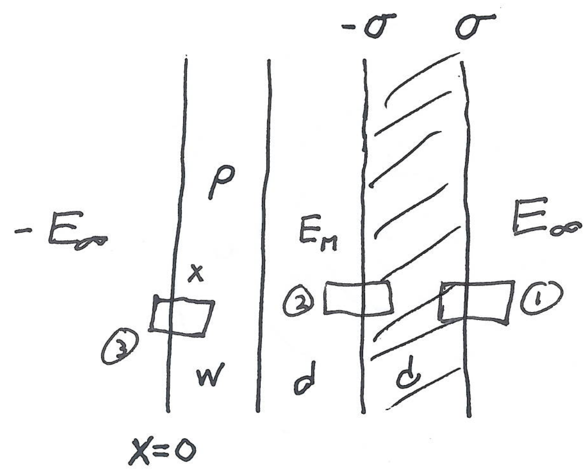
$$d \cos 30^\circ = d\sqrt{3}/2 = a/2$$

$$\underline{a = \sqrt{3}\, d}$$

$$\vec{E} = -\frac{3kq}{d^2}\hat{y} + \frac{2kq}{d^3}\left(a/2\,\hat{x} + d/2\,\hat{y}\right) - \frac{kq}{d^3}\left(-a/2\,\hat{x} + d/2\,\hat{y}\right)$$

$$= \frac{3kqa}{2d^3}\hat{x} - \frac{5kq}{2d^2}\hat{y} = \underline{\frac{9\sqrt{3}}{2}\frac{kq}{a^2}\hat{x} - \frac{15}{2}\frac{kq}{a^2}\hat{y}}$$

Problem 2 : At left in the figure below is an infinite planar slab of width $w = 0.1\,\mathrm{m}$ and constant charge per unit volume $\rho = 10^{-4}\,\mathrm{C/m^3}$. A distance $d = 0.15\,\mathrm{m}$ to the right of this slab is an infinite conducting plate of width d. The conducting plate has zero net charge, but will have constant area charge density σ on the ~~left~~ right side and $-\sigma$ on the ~~right~~ left side. Find the field at all points in the figure, indicating the value of the field where it is constant, otherwise writing it as a function of x with $x = 0$ taken to be the left side of the slab. Also find the area charge density σ.



$E = 0$ in conductor

E_∞ and E_M are constant.

$$2AE_\infty = 4\pi K \rho A W$$

$$\underline{E_\infty = 2\pi K \rho w}$$

①
$$A E_\infty = 4\pi K \sigma A$$

$$E_\infty = 4\pi K \sigma \qquad \underline{\sigma = \frac{E_\infty}{4\pi K} = \frac{\rho w}{2}}$$

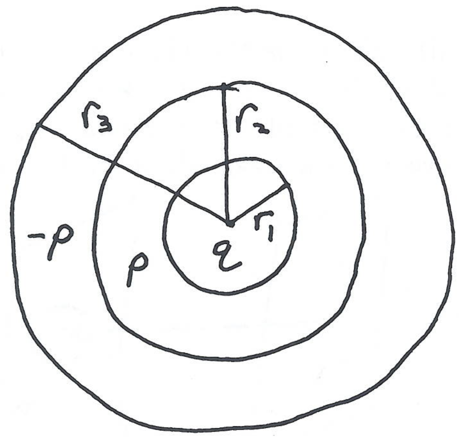
②
$$-AE_M = -4\pi K \sigma A$$

$$\underline{E_M = 4\pi K \sigma = E_\infty}$$

③
$$(E_\infty + E(x))\,A = 4\pi K \rho A x$$

$$E(x) = -E_\infty + 4\pi K \rho x = \underline{2\pi K \rho (2x - w)}$$

Problem 3 : The figure below shows a spherical cavity of radius $r_1 = 0.02\,\mathrm{m}$ with a point charge $q = 5 \times 10^{-8}\,\mathrm{C}$ at the center. Surrounding this cavity is a shell of inner radius r_1 and outer radius $r_2 = 0.05\,\mathrm{m}$ which has a constant charge per unit volume $\rho = 10^{-4}\,\mathrm{C/m^3}$. Outside of this shell is another shell of inner radius r_2 and outer radius $r_3 = 0.08\,\mathrm{m}$ which has a constant charge per unit volume $-\rho$. Find the electric field at all radii, $r < r_1$, $r_1 < r < r_2$, $r_2 < r < r_3$, $r > r_3$.



$r < r_1$

$$4\pi r^2 E(r) = 4\pi K q$$

$$E(r) = \frac{Kq}{r^2}$$

$r_1 < r < r_2$

$$4\pi r^2 E(r) = 4\pi K\left(q + \tfrac{4}{3}\pi\rho(r^3 - r_1^3)\right)$$

$$E(r) = \frac{K}{r^2}\left(q + \tfrac{4}{3}\pi\rho(r^3 - r_1^3)\right)$$

$r_2 < r < r_3$

$$4\pi r^2 E(r) = 4\pi K\left(q + \tfrac{4}{3}\pi\rho(r_2^3 - r_1^3) - \tfrac{4}{3}\pi\rho(r^3 - r_2^3)\right)$$

$$E(r) = \frac{K}{r^2}\left(q + \tfrac{4}{3}\pi\rho(2r_2^3 - r_1^3 - r^3)\right)$$

$r > r_3$

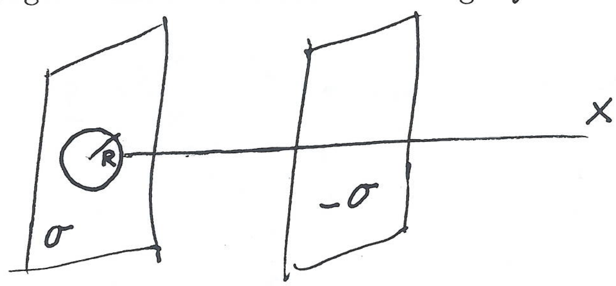
$$4\pi r^2 E(r) = 4\pi K\left(q + \tfrac{4}{3}\pi\rho(r_2^3 - r_1^3) - \tfrac{4}{3}\pi\rho(r_3^3 - r_2^3)\right)$$

$$E(r) = \frac{K}{r^2}\left(q + \tfrac{4}{3}\pi\rho(2r_2^3 - r_1^3 - r_3^3)\right)$$

Problem 4 : At left in the figure below is an infinite sheet of constant area charge density $\sigma = 10^{-5}\,\mathrm{C/m^2}$ with a hole of radius $R = 0.05\,\mathrm{m}$ cut out of it. At a distance $d = 0.2\,\mathrm{m}$ to the right is a complete infinite plane of constant area charge density $-\sigma$. A disk of radius R and constant area charge density σ has an electric field a distance x along the perpendicular through the disks center that is given by

$$\vec{E} = 2\pi k\sigma\,\hat{x}\left(\frac{x}{\sqrt{x^2}} - \frac{x}{\sqrt{x^2+R^2}}\right)$$

Taking $x = 0$ to be the location of the left sheet, find the electric field at all points ($x < 0$, $0 < x < d$, $x > d$) along the axis of the hole. Taking $R = \epsilon x$ and $d = 4R = 4\epsilon x$, expand the expression for $x > d$ in ϵ and take the first non-vanishing power of ϵ. You should find the field of a point charge. What is the associated charge Q?



(planes cancel)

$x<0$: $$\vec{E} = -2\pi K\sigma\hat{x}\left(-1 - \frac{x}{\sqrt{x^2+R^2}}\right) = 2\pi K\sigma\hat{x}\left(1 + \frac{x}{\sqrt{x^2+R^2}}\right)$$

$x>d$: $$\vec{E} = -2\pi K\sigma\hat{x}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

$d>x>0$: $$\vec{E} = -2\pi K\sigma\hat{x}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right) + 2\pi K\sigma\hat{x} - 2\pi K\sigma(-\hat{x})$$

$$\vec{E} = 2\pi K\sigma\hat{x}\left(1 + \frac{x}{\sqrt{x^2+R^2}}\right)$$

$x>d$: $$\vec{E} = -2\pi K\sigma\hat{x}\left(1 - \frac{1}{\sqrt{1+\varepsilon^2}}\right) \simeq -2\pi K\sigma\hat{x}\left(1-\left(1-\tfrac{1}{2}\varepsilon^2\right)\right)$$

~~Rot.~~

$R = \varepsilon x$

$(1+z)^p$

$z = \varepsilon^2$

$p = -\tfrac{1}{2}$

$$\vec{E} = -2\pi K\sigma\hat{x}\,\varepsilon^2/2 = -\frac{\pi R^2\sigma K}{x^2}\,\hat{x}$$

$$Q = -\pi R^2\sigma$$