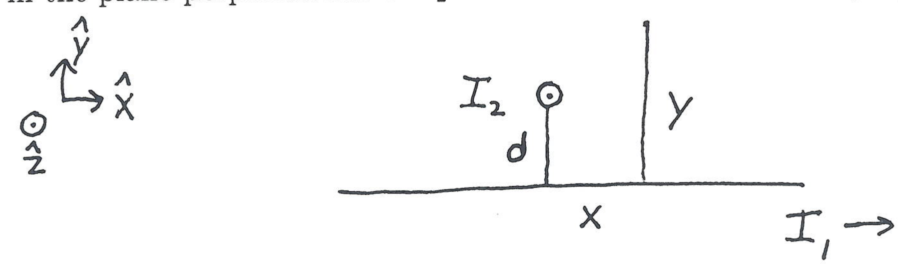
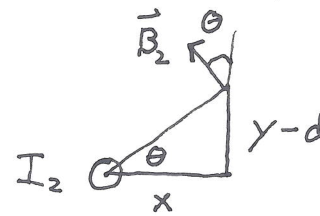
SMU Physics 1308 : Summer 2010

Exam 2

Problem 1 : The figure below shows a wire carrying a current $I_1 = 2\,\mathrm{A}$ in the $\hat{x}$ direction, and another wire carrying a current $I_2 = 3\,\mathrm{A}$ in the $\hat{z}$ direction which is positioned a distance $d = 0.05\,\mathrm{m}$ from the first wire. Find the magnetic field at a point given by $x = 0.06\,\mathrm{m}$ and $y = 0.09\,\mathrm{m}$ in the plane perpendicular to I_2 which contains the wire carrying I_1 .



$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi y}\hat{z}$$



$$\tan\theta = \frac{y-d}{x}$$

$$\cos\theta = \frac{x}{(x^2+(y-d)^2)^{1/2}}$$

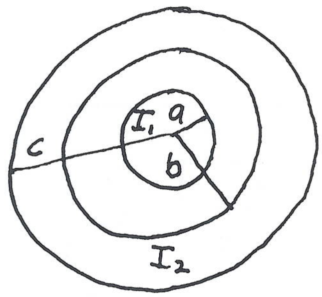
$$\sin\theta = \frac{y-d}{(x^2+(y-d)^2)^{1/2}}$$

$$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi(x^2+(y-d)^2)^{1/2}}(\cos\theta\,\hat{y} - \sin\theta\,\hat{x})$$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I_1}{2\pi y}\hat{z} + \frac{\mu_0 I_2}{2\pi(x^2+(y-d)^2)}(x\hat{y} - (y-d)\hat{x})$$

Problem 2 : The figure below shows a wire of radius $a = 0.01\,\text{m}$ carrying current $I_1 = 2\,\text{A}$ (with positive I taken to be in the $\hat{z}$ direction) inside a cylindrical wire of inner radius $b = 0.02\,\text{m}$ and outer radius $c = 0.03\,\text{m}$ which has carries $I_2 = -3\,\text{A}$. Find the respective constant current densities per unit area J_1 and J_2, and find the form of the magnetic field at all radii $r < a$, $a < r < b$, $c > r > b$, and $r > c$.

$$\vec{B} = B(r)\hat{\theta}$$



$$J_1 = \frac{I_1}{\pi a^2}$$

$$J_2 = \frac{I_2}{\pi (c^2 - b^2)}$$

$\underline{r < a}$

$$B = \frac{\mu_0 I_1 r}{2\pi a^2}$$

$\underline{b > r > a}$

$$B = \frac{\mu_0 I_1}{2\pi r}$$

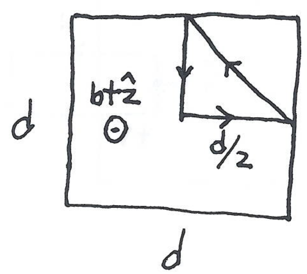
$\underline{c > r > b}$

$$B = \frac{\mu_0}{2\pi r}\left(I_1 + I_2 \frac{(r^2 - b^2)}{(c^2 - b^2)}\right)$$

$\underline{r > c}$

$$B = \frac{\mu_0}{2\pi r}(I_1 + I_2)$$

Problem 3 : The figure below shows the cross section of a square magnet of sides $d = 0.02\,\mathrm{m}$ which has magnetic field given by $\vec{B} = bt\hat{z}$, with $b = 1\,\mathrm{T/s}$. The magnetic field vanishes ($\vec{B} = 0$) outside of the square. Also shown is a wire in the shape of a right triangle with two equal sides of length $d/2$. If the resistance in the wire is $R = 1\,\Omega$, find the induced current I_{ind} in the wire (with positive current defined to be counter-clockwise). Also find the respective forces on each segment (left, bottom, hypotenuse) of the wire. Label these $\vec{F}_L$, $\vec{F}_B$, and $\vec{F}_H$ and write them in terms of the $\hat{x}$, $\hat{y}$, $\hat{z}$ unit vectors. Also find the net force $\vec{F}_{\text{net}}$ on the wire.



$$\int d\vec{A}\cdot\vec{B} = bt\,(d/2)^2/2 = btd^2/8$$

$$I_{ind}R = \oint d\vec{r}\cdot\vec{E} = -\frac{d}{dt}\int d\vec{A}\cdot\vec{B} = -bd^2/8$$

$$\underline{I_{ind} = -\frac{bd^2}{8R}}$$

$$\vec{F}_L = I_{ind}(-d/2\,\hat{y})\times(bt\hat{z}) = -I_{ind}\frac{dbt}{2}\hat{x} = \frac{b^2d^3t}{16R}\hat{x}$$

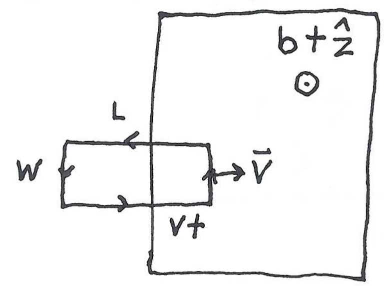
$$\vec{F}_B = I_{ind}(d/2\,\hat{x})\times(bt\hat{z}) = -I_{ind}\frac{dbt}{2}\hat{y} = \frac{b^2d^3t}{16R}\hat{y}$$

$$\vec{F}_H = I_{ind}\left(-\hat{x}/\sqrt{2} + \hat{y}/\sqrt{2}\right)\times(bt\hat{z})\,(d/\sqrt{2})$$

$$= +I_{ind}\left(\frac{dbt}{2}\right)(\hat{y}+\hat{x}) = -\frac{b^2d^3t}{16R}(\hat{y}+\hat{x})$$

$$\underline{\vec{F}_{net} = 0}$$

Problem 4 : The figure below shows a loop of wire of resistance R, length L, and width w which is entering a region of non-zero magnetic field given by $\vec{B} = bt\hat{z}$. If the wire moves at constant speed v so that the right side of the loop is at a distance vt into the magnetic field, find the induced current I_{ind} as a function of time t. You will need $\frac{d}{dt}t^2 = 2t$. Also find the force $\vec{F}_R$ on the right side of the loop.



$$\int d\vec{A}\cdot\vec{B} = W(Vt)\,bt = WVbt^2$$

$$I_{ind}\,R = -\frac{d}{dt}(WVbt^2) = -2WVbt$$

$$\underline{I_{ind} = -2WVbt/R}$$

$$\vec{F}_R = I_{ind}(W\hat{y})\times(bt\hat{z}) = I_{ind}(Wbt)\hat{x}$$

$$\underline{\vec{F}_R = -\frac{2V}{R}(Wbt)^2\hat{x}}$$