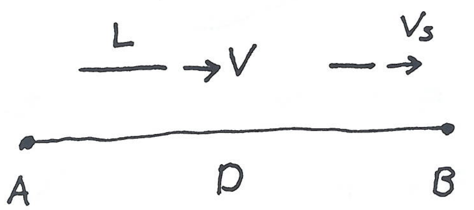
SMU Physics 1308 : Summer 2010

Final Exam

Problem 1 : The figure below shows the rest frame of two planets A and B separated by $D = 10^6\, c \cdot \mathrm{s}$. A spaceship moving from A to B with unknown velocity $v = \beta c$ has length in the planets frame given by $L = 10^{-6}\, c \cdot \mathrm{s}$. If the spaceship takes $ct' = 6 \times 10^5\, c \cdot \mathrm{s}$ in its own frame to move from A to B, find β and the length L' of the ship in its rest frame. What is the separation D' of the planets as observed by the ship? Finally, before the arrival of the ship on planet B, a smaller ship leaves the larger ship with velocity $v_s' = 0.6\, c$ as measured in the rest frame of the larger ship. What is the velocity v_s of the smaller ship as measured in the planet frame?



$$ct = \frac{cD}{V} = D/\beta \qquad ct' = ct/\gamma = \frac{D}{\beta\gamma}$$

$$(\beta\gamma)^2 = \frac{\beta^2}{1-\beta^2} = \left(\frac{D}{ct'}\right)^2 = \left(\frac{10}{6}\right)^2 \Rightarrow \beta^2 = \frac{(10/6)^2}{1+(10/6)^2}$$

$$\underline{\beta = 0.857} \qquad L' = \gamma L = \underline{1.94 \times 10^{-6}\, c \cdot s}$$

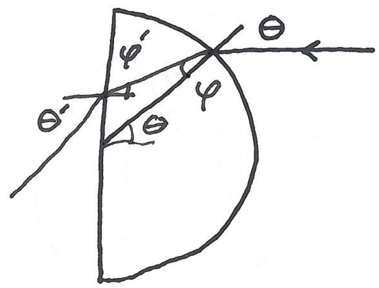
$$\underline{\gamma = 1.94} \qquad D' = D/\gamma = \underline{5.14 \times 10^{-5}\, c \cdot s}$$

$$V_s = U \qquad V_s = \frac{V_s' + V}{1 + V_s' V / c^2} = \underline{0.962\, c}$$

$$V_s' = U'$$

$$V = v = \underline{0.857c}$$

Problem 2 : The figure below shows a half-circular piece of glass of refractive index $n = 1.5$. If a horizontal ray of light is incident on the curved surface of the glass, making an angle $\theta = 50°$ with the perpendicular to the glass surface, find the angle θ' at which the ray emerges from the flat surface of the glass. Also find the angle $\theta = \theta_r$ such that total internal reflection happens on the flat surface.



$\sin\theta = n \sin\varphi$

$\sin\theta' = n \sin\varphi'$

$\sin\theta' = n \sin(\theta - \varphi)$

$\varphi + 90 - \theta + \varphi' + 90 = 180$

$\varphi' = \theta - \varphi$

$\varphi = \sin^{-1}(\sin\theta / n) = \underline{30.7°}$

$\varphi' = \theta - \varphi = \underline{19.3°}$

$\theta' = \sin^{-1}(n \sin\varphi') = \underline{29.7°}$

<u>TIR</u>

$\theta' = 90°$ $\quad \varphi' = \theta_c = \theta_r - \varphi_r$

$1 = n \sin\theta_c$ $\quad \theta_c = \sin^{-1}(1/n) = \underline{41.8°}$

$\sin\theta_r = n \sin\varphi_r = n \sin(\theta_r - \theta_c)$

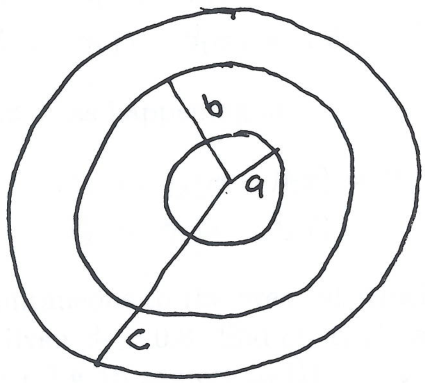
$\sin\theta_r = n \sin\theta_r \cos\theta_c - n \sin\theta_c \cos\theta_r$

$\tan\theta_r = n \cos\theta_c \tan\theta_r - n \sin\theta_c$

$$\tan\theta_r = \frac{n \sin\theta_c}{n\cos\theta_c - 1}$$

$\underline{\theta_r = 83.3°}$

Problem 3 : The figure below shows a circular capacitor plate of radius a which has a field $\vec{E} = -\alpha t\hat{z}$. Outside of this is a wire of inner radius b and outer radius c carrying a current I. Between the capacitor and wire ($b > r > a$) there is empty space. Find the function $B(r)$, where the magnetic field is given by $\vec{B} = B(r)\hat{\theta}$, for $r < a$, $b > r > a$, $c > r > b$, and $r > c$.



$\underline{r<a}$

$$2\pi r B = \mu_0 \varepsilon_0 \frac{d}{dt}\left(-\alpha t \pi r^2\right) = -\mu_0 \varepsilon_0 \alpha \pi r^2$$

$$\vec{B} = (-\mu_0 \varepsilon_0 \alpha/2)\, r\, \hat{\theta}$$

$\underline{b>r>a}$ $\quad 2\pi r B = -\mu_0 \varepsilon_0 \alpha \pi a^2 \qquad \vec{B} = (-\mu_0 \varepsilon_0 \alpha/2)\, a^2/r\, \hat{\theta}$

$\underline{c>r>b}$ $\quad 2\pi r B = -\mu_0 \varepsilon_0 \alpha \pi a^2 + \mu_0 I (r^2 - b^2)/(c^2 - b^2)$

$\underline{r>c}$

$$2\pi r B = -\mu_0 \varepsilon_0 \pi a^2 + \mu_0 I$$

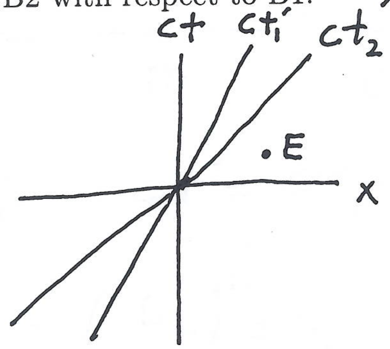
Problem 4 : The diagram below shows an event E with unknown coordinates (ct, x) in a particular inertial frame denoted by A. Also shown are two moving observers $B1$ and $B2$ with $v_{B1\to A} = \beta_1 c$ and $v_{B2\to A} = \beta_2 c$, where $\beta_2 > \beta_1 > 0$. The observer B1 sees the event as happening at

$$\begin{aligned} ct_1' &= \gamma_1 (ct - \beta_1 x) = 3.0 \times 10^5\, c\cdot s \\ x_1' &= \gamma_1 (x - \beta_1 ct) = 1.5 \times 10^6\, c\cdot s \end{aligned}$$

The observer B2 sees the event E as happening at

$$\begin{aligned} ct_2' &= \gamma_2 (ct - \beta_2 x) = 0 \\ x_2' &= \gamma_2 (x - \beta_2 ct) \end{aligned}$$

Thus, B2 sees E as being simultaneous to the event at which the three time axes cross, and x_2' is taken to be unknown. Given $\beta_2 = 0.8$, find ct, x, β_1, and x_2'. Also find $v_{B2\to B1}$, which corresponds to the velocity of B2 with respect to B1.



$ct = \beta_2 x$

$ct_1' = \gamma_1(\beta_2 - \beta_1)x$

$x_1' = \gamma_1(1 - \beta_1\beta_2)x$

$\dfrac{ct_1'}{x_1'} = \dfrac{\beta_2 - \beta_1}{1 - \beta_1\beta_2} = 0.2 \Rightarrow \beta_1 = \dfrac{\beta_2 - 0.2}{(1 - 0.2\beta_2)} = \underline{0.714}$

$x = \dfrac{x_1'}{\gamma_1(\beta_2 - \beta_1)} = \underline{2.1 \times 10^6\, c\cdot s}$ $\quad ct = \beta_2 x = \underline{1.68 \times 10^6\, c\cdot s}$

$x_2' = \gamma_2(x - \beta_2 ct) = \underline{1.26 \times 10^6\, c\cdot s}$

$V = V_{B2\to B1} = \beta c$ $\quad V = \dfrac{U' - U}{1 - UU'/c^2} = \underline{0.2c}$

$U = V_{A\to B1} = -\beta_1 c$

$U' = V_{A\to B2} = -\beta_2 c$

$\beta = V/c = \dfrac{\beta_1 - \beta_2}{1 - \beta_1\beta_2} = \underline{0.2}$