Problem 1 : The figure below shows a circular loop of charge Q and radius R with uniform linear charge density. There are four equal point charges q arranged at the corners of a square with sides of length $2R$ which is in the same plane and shares the same center as the loop. Find the potential difference ΔV between a point at the center of the loop and a point at a distance x along the axis perpendicular to the plane of the loop which extends from the center of the loop.

Front view:
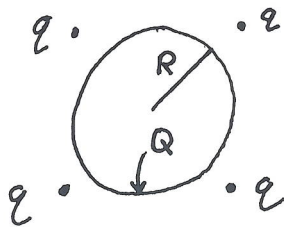
side view
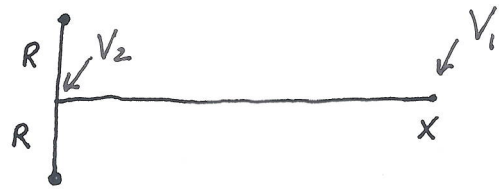
$$V_2 - V_1 = 4kq\left(\frac{1}{\sqrt{2R^2}} - \frac{1}{\sqrt{2R^2 + x^2}}\right)$$

$$+ kQ\left(\frac{1}{\sqrt{R^2}} - \frac{1}{\sqrt{R^2 + x^2}}\right)$$

Problem 2 : The figure below shows two parallel planes separated by a distance d. The left and right plane have uniform surface charge density σ and $-\sigma$, respectively. There are two concentric spheres with uniform surface charge density which are centered on the left plane. The inner sphere has radius $d/4$ and charge Q, and the outer sphere has radius $d/2$ and charge $-2Q$. Find the potential difference ΔV between a point at the center of the spheres and a point at a distance $x > d$ along the axis perpendicular to the planes which extends from the center of the spheres. Show that this difference is finite in the limit $x \to \infty$.
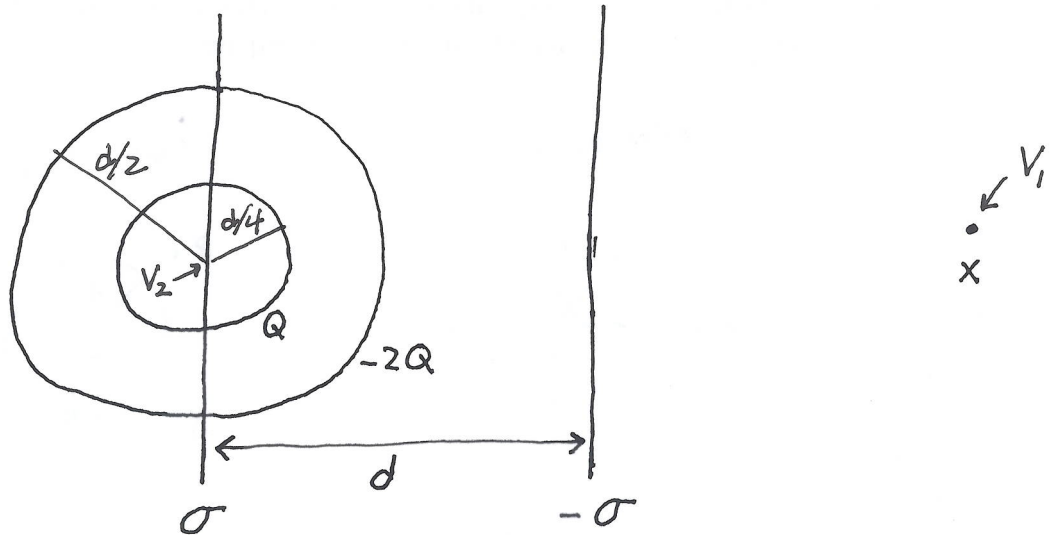
$$V_2 - V_1 = KQ\left(\frac{1}{(d/4)} - \frac{1}{X}\right) - 2KQ\left(\frac{1}{(d/2)} - \frac{1}{X}\right)$$

$$- 2\pi K\sigma (0 - X) + 2\pi K\sigma (d - X)$$

$$= \frac{KQ}{X} + 2\pi K\sigma d \xrightarrow[X \to \infty]{} 2\pi K\sigma d$$

Problem 1 : The figure below shows an infinite solenoid of radius $a = 0.05\,\text{m}$ which extends along the x axis, and which carries a current $I_1 = 2\,\text{A}$ in the indicated direction. Also shown is an infinite wire which carries a current $I_2 = 3\,\text{A}$ with indicated direction along the y axis, thus coinciding with a diameter of the solenoid. Find the magnetic field vector $\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$ at a point $x = 0.1\,\text{m}$ along the x axis (thus at $z = 0$ and $y = 0$). Also find the total force $\vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$ acting on the wire.
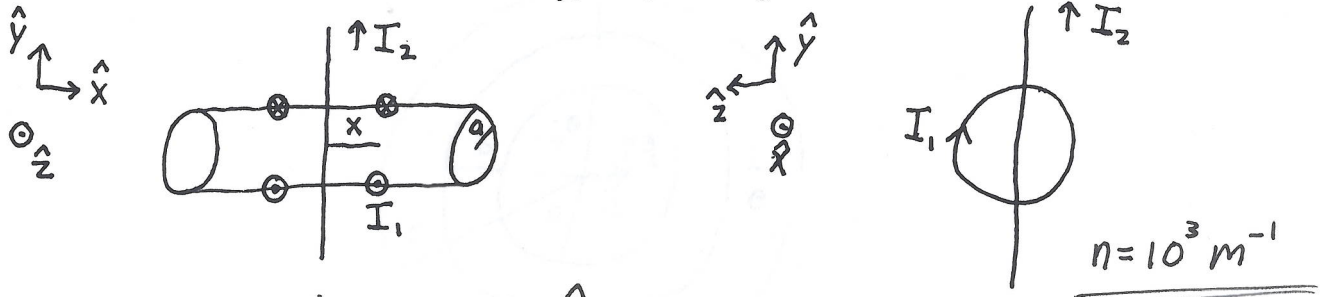
inside solenoid: $\vec{B}_s = -\mu_0 n I_1 \hat{x}$

wire at $x\hat{x}$: $\vec{B}_w = -\dfrac{\mu_0 I_2}{2\pi x}\hat{z}$

Thus at $x\hat{x}$: $\vec{B} = -\mu_0 n I_1 \hat{x} - \dfrac{\mu_0 I_2}{2\pi x}\hat{z} = -2.52 \times 10^{-3}\hat{x}\,T$
$$-6.02 \times 10^{-6}\hat{z}\,T$$

Force on wire:

field $\vec{B}_s$ only non-zero inside solenoid.

$$\vec{F}_w = I_2 \vec{L}_2 \times \vec{B}_s \qquad \vec{L}_2 = 2a\hat{y}$$

$$= I_2 (2a\hat{y}) \times (-\mu_0 n I_1 \hat{x})$$

$$= \mu_0 I_1 I_2 (2an)\hat{z}$$

$$= 7.56 \times 10^{-4}\hat{z}\,N$$

$n = 10^3\,m^{-1}$

Problem 2 : The figure below shows a coaxial cable extending along the z axis with an inner wire of radius $a = 0.005\,\text{m}$ carrying current I_1, and an outer wire with inner radius $b = 0.01\,\text{m}$ and outer radius $c = 0.02\,\text{m}$ carrying current I_2. The magnetic field at $\vec{r}_1 = r_1 \hat{x}$ with $r_1 = 0.002\,\text{m}$ is given by $\vec{B}_1 = B_1 \hat{y}$ with $B_1 = 0.01\,\text{T}$. The magnetic field at $\vec{r}_2 = r_2 \hat{y}$ with $r_2 = 0.03\,\text{m}$ is given by $\vec{B}_2 = B_2 \hat{x}$ with $B_2 = 0.04\,\text{T}$. Find the currents I_1 and I_2 with positive values defined as being along the z axis.
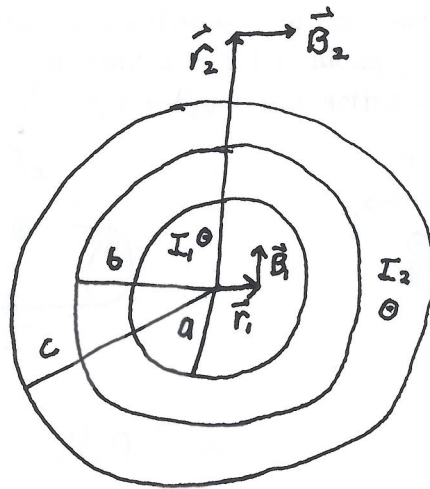
$\underline{r < a}$

$$2\pi r_1 B_1 = \mu_0 I_1 \frac{r_1^2}{a^2} \qquad I_1 = B_1 \frac{2\pi}{\mu_0} \frac{a^2}{r_1} = \underline{623\,A}$$

$\vec{B}_1 : CCW \quad d\vec{L} : CCW$

$d\vec{L} \cdot \vec{B}_1 > 0$

$d\vec{L} = dL \hat{y} \text{ at } \vec{r}_1$

$\underline{r > c}$

$$-2\pi r_2 B_2 = \mu_0 (I_1 + I_2)$$

$\vec{B}_2 : CW \quad d\vec{L} : CCW \qquad I_2 = -\frac{2\pi}{\mu_0} r_2 B_2 - I_1 = \underline{-6607\,A}$

$d\vec{L} \cdot \vec{B}_2 < 0$

$d\vec{L} = -dL \hat{x} \text{ at } \vec{r}_2$

**Problem 1 :** The figure below shows an infinite wire carrying current I, with positive I taken to be in the $\hat{y}$ direction. At a distance $a = 0.15\,\mathrm{m}$ along the $\hat{x}$ axis is a loop of wire of length $L = 0.2\,\mathrm{m}$ and width $w = 0.1\,\mathrm{m}$ as shown. From Ampere's Law we have found that the magnetic field from the infinite wire at all points within the loop is given by $\vec{B} = -(\mu_0 I/2\pi x)\hat{z}$, leading to a flux through the loop (with the choice $d\vec{A} = dA\,\hat{z}$) given by

$$\int d\vec{A} \cdot \vec{B} = -\frac{\mu_0 I L}{2\pi} \ln\left(\frac{a+w}{a}\right)$$

If $dI/dt = 2\,\mathrm{A/s}$, find the induced current I_{ind} in the loop of wire if its resistance is $R = 2\,\Omega$, indicating its direction (CW or CCW). If at this time $I = 0.5\,\mathrm{A}$, also find the total force on the wire, expressing it in vector form as $\vec{F} = F_x\hat{x} + F_y\hat{y} + F_z\hat{z}$.

$\hat{y}\,\llcorner\!\!\rightarrow\hat{x}$

$\odot\,\hat{z}$

$I\uparrow$

L

$a \quad w$
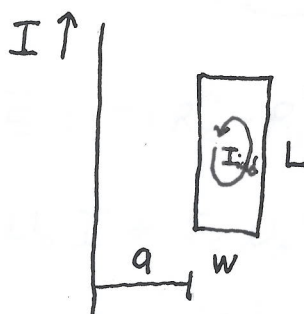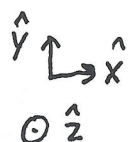
$\mathcal{E} = \oint d\vec{L}\cdot\vec{E} = -\frac{d}{dt}\int d\vec{A}\cdot\vec{B}$

$= \frac{\mu_0 L}{2\pi}\ln\left(\frac{a+w}{a}\right)\frac{dI}{dt}$

$= I_{ind}\,R$

I_{ind} is CCW

$$I_{ind} = \frac{\mu_0 L}{2\pi R}\ln\left(\frac{a+w}{a}\right)\frac{dI}{dt} = \underline{2.04\times10^{-8}\,A}$$

Top and Bottom forces cancel.

Right side : $\vec{F}_R = I_{ind}(L\hat{y})\times\left(-\mu_0 I/2\pi(a+w)\right)(\hat{z})$

$x = a+w$

$\qquad = -\hat{x}\,I_{ind}\,\mu_0 I L/(2\pi(a+w))$

Left side : $\vec{F}_L = I_{ind}(-L\hat{y})\times\left(-\mu_0 I/2\pi a\right)\hat{z}$

$x = a$

$\qquad = \hat{x}\,I_{ind}\,\mu_0 I L/(2\pi a)$

1

$$\vec{F} = \vec{F}_R + \vec{F}_L = \hat{x}\,\frac{\mu_0 I L I_{ind}}{2\pi}\left(\frac{1}{a} - \frac{1}{a+w}\right) = \underline{1.09\times10^{-14}\,N}$$