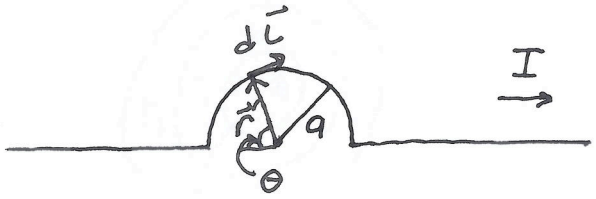
SMU Physics 1308 : Spring 2009

Exam 2

Problem 1 : The figure below shows an infinite wire that carries a current $I = 2\,\text{A}$. Near the origin the wire makes a semi-circle of radius $a = 0.01\,\text{m}$. Find the magnetic field at the center of the semi-circle using the Biot-Savart Law.



$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int d\vec{L}\times\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = -\frac{\mu_0 I}{4\pi}\int d\vec{L}\times\frac{\vec{r}'}{|\vec{r}'|^3}$$

on straight sections: $d\vec{L}\times\vec{r}' = 0$

$\vec{r}=0$

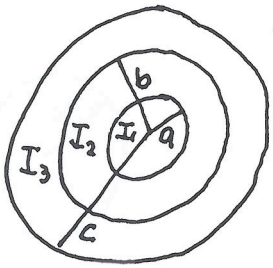
semicircle: $|\vec{r}'| = a$

$$\vec{B} = \frac{-\mu_0 I}{4\pi a^3}\int d\vec{L}\times\vec{r}'$$

$$d\vec{L}\times\vec{r}' = |d\vec{L}|\,a\,\hat{z} = a^2 d\theta\,\hat{z}$$

$$\vec{B} = \frac{-\mu_0 I}{4\pi a}\hat{z}\int_0^{\pi} d\theta = -\frac{\mu_0 I}{4a}\hat{z}$$

Problem 2 : The figure below shows the cross section of an infinite wire which is broken up into three conducting regions with the indicated radii $a = 0.01\,\mathrm{m}$, $b = 0.02\,\mathrm{m}$, and $c = 0.03\,\mathrm{m}$. The inner conducting region carries current $I_1 = 2\,\mathrm{A}$. The middle conducting region carries current $I_2 = -4\,\mathrm{A}$. The outer conducting region carries current $I_3 = 2\,\mathrm{A}$. Find the magnetic field in each of the regions; that is for $r < a$, $a < r < b$, $b < r < c$, and $r > c$.



$\vec{B} = B(r)\,\hat{\theta}$

Ampere: $\oint_{L_c} d\vec{r}\cdot\vec{B} = \mu_0 I_{S_0}$

$L_c = \partial S_0$ ↑ boundary

$r<a$

$2\pi r B = \mu_0 I_1 r^2/a^2$

$B = \frac{\mu_0 I_1}{2\pi} r/a^2$

$b > r > a$

$2\pi r B = \mu_0\left(I_1 + I_2\frac{(r^2-a^2)}{(b^2-a^2)}\right)$

$B = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r} + \frac{I_2}{r}\frac{(r^2-a^2)}{(b^2-a^2)}\right)$

$c > r > b$

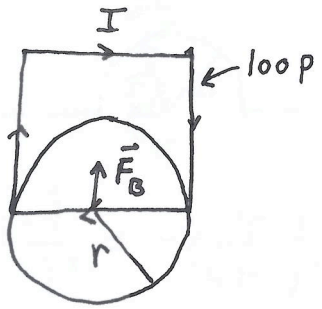
$2\pi r B = \mu_0\left(I_1+I_2+I_3\frac{(r^2-b^2)}{(c^2-b^2)}\right)$

$B = \frac{\mu_0}{2\pi}\left(\frac{I_1+I_2}{r} + \frac{I_3}{r}\left(\frac{r^2-b^2}{c^2-b^2}\right)\right)$

$r > c$

$B = \frac{\mu_0}{2\pi r}(I_1+I_2+I_3)$

Problem 4 : The circular electromagnet shown in the figure below has a radius of $r = 0.2\,\mathrm{m}$ and produces a uniform magnetic field of the form $\vec{B} = bt\hat{z}$, where $b = 2\,\mathrm{T/s}$. A square loop of wire of side length $2r$ is suspended in the electromagnet so that one side extends along the horizontal diameter. The other three sides do not lie between the poles. If the loop has resistance $R = 1\,\Omega$, find the direction and magnitude of the current I in the loop. Because of the current I, the loop feels a force $\vec{F}_B$ in the magnetic field which changes with time. Taking $\hat{z}$ out of the page, $\hat{x}$ to the right, and $\hat{y}$ upward, express $\vec{F}_B$ as a vector. If the wire weighs $m = 10^{-3}\,\mathrm{kg}$, find the time at which the wire would support its own weight.



$\hat{z}$ ⊙

$\hat{y}$ ↑ → $\hat{x}$

Faraday:

$$IR = \mathcal{E} = -\frac{d}{dt}\int d\vec{A}\cdot\vec{B} = -\frac{d}{dt}\left(\frac{\pi}{2}r^2bt\right) = -\frac{\pi}{2}r^2b$$

$$I = -\frac{\pi r^2 b}{2R} \quad \text{(clockwise)}$$

$\vec{L} = 2r\,\hat{x}$

$\vec{B} = bt\,\hat{z}$

$$\vec{F}_B = I\vec{L}\times\vec{B}$$

$$\vec{F}_B = \left(-\frac{\pi r^2 b}{2R}\right)(2rbt)\,\hat{x}\times\hat{z} = -\hat{y}$$

at t_c:

$$\vec{F}_B - mg\hat{y} = 0$$

$$\boxed{\vec{F}_B = \frac{\pi r^3 b^2}{R}t\,\hat{y}}$$

$$mg = \frac{\pi r^3 b^2}{R}t_c \qquad t_c = \frac{mgR}{\pi r^3 b^2}$$

$$\boxed{I = -0.126\,\mathrm{A}}$$

$$\boxed{\vec{F}_B = 0.1t\,\hat{y}}$$

$$\boxed{t_c = 0.097\,\mathrm{s}}$$

Problem 1 : The figure below shows the circular cross section of radius $R = 0.003\,\mathrm{m}$ of an infinitely long wire with uniform current density carrying a time dependent current $I(t) = 2\,t\,\mathrm{A/s}$ which is in the $\hat{z}$ direction. Embedded within the wire in the x-z plane is a square loop of insulated wire which extends a distance $L = 0.01\,\mathrm{m}$ in the z direction and from $x_1 = 0.001\,\mathrm{m}$ to $x_2 = 0.002\,\mathrm{m}$ in the x direction. If the resistance in the loop is $R_{res} = 1\,\Omega$, find the current $I_\ell(t)$ in the loop, and indicate its direction in a diagram where the y axis points out of the page.

Problem 2 : The figure below shows the circular cross section of radius $R = 0.03\,\mathrm{m}$ between two poles of an electromagnet. The magnetic field is uniform between the poles and vanishes outside them. It points in the z direction and increases as $B = 0.1\,t\,\mathrm{T/s}$. Find the electric field $\vec{E}(r,t)$, indicating its direction, for radii $r < R$ and $r > R$.