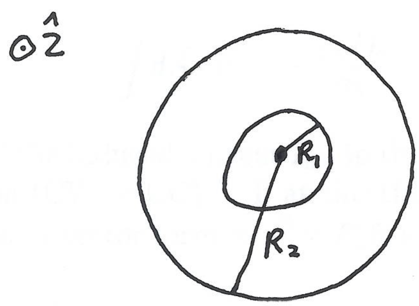
Problem 2 : The figure below shows a circular capacitor plate of radius $R_2 = 0.2\,\mathrm{m}$ which has a hole in the middle of radius $R_1 = 0.1\,\mathrm{m}$. The electric field is only non-zero between $r = R_1$ and $r = R_2$, where it is constant in space with form $\vec{E} = E(t)\,\hat{z}$ with $dE/dt = 5 \times 10^{10}\,\mathrm{V/m \cdot s}$. A wire extends through the center of the capacitor carrying an unknown current I, with positive I taken to be out of the page. The magnetic field at all points is purely tangential, and thus has the form $\vec{B} = B(r)\,\hat{\theta}$, with $\hat{\theta}$ being a unit vector in the counter-clockwise direction. If $B(r) = 0$ for $r > R_2$, find the current I running through the wire. Also find $B(r)$ for the two radii $r_1 = 0.15\,\mathrm{m}$ and $r_2 = 0.05\,\mathrm{m}$.



Use:

$$\int d\vec{l}\cdot\vec{B} = \mu_0 I + \varepsilon_0\mu_0 \frac{d}{dt}\int d\vec{A}\cdot\vec{E}$$

$r > R_2$

$$\int d\vec{A}\cdot\vec{E} = \pi(R_2^2 - R_1^2)E \qquad \int d\vec{l}\cdot\vec{B} = 0$$

$$0 = \mu_0 I + \varepsilon_0\mu_0\pi(R_2^2 - R_1^2)\frac{dE}{dt}$$

$$I = -\varepsilon_0\pi(R_2^2 - R_1^2)\frac{dE}{dt} = \underline{-4.16 \times 10^{-2}\,A}$$

$r = r_2 < R_1$

$$\int d\vec{A}\cdot\vec{E} = 0 \qquad \int d\vec{l}\cdot\vec{B} = 2\pi r_2 B = \mu_0 I$$

$$B(r_2) = \frac{\mu_0 I}{2\pi r_2} = \underline{-1.67 \times 10^{-7}\,T}$$

$r = r_1$

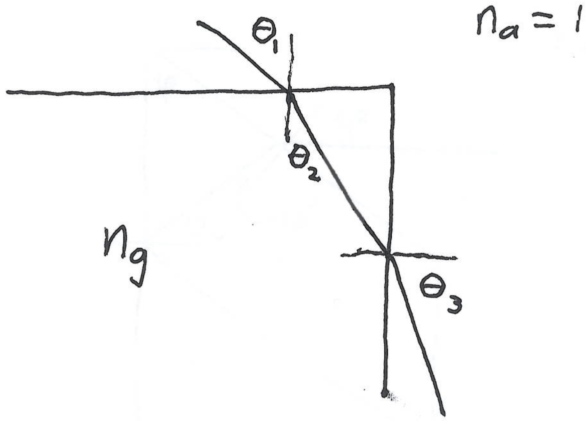
$R_2 > r_1 > R_1$

$$\int d\vec{A}\cdot\vec{E} = \pi(r_1^2 - R_1^2)E$$

$$\int d\vec{l}\cdot\vec{B} = 2\pi r_1 B = \mu_0 I + \varepsilon_0\mu_0\pi(r_1^2 - R_1^2)\frac{dE}{dt}$$

$$B(r_1) = \frac{\mu_0 I}{2\pi r_1} + \frac{\varepsilon_0\mu_0}{2}\frac{(r_1^2 - R_1^2)}{r_1}\frac{dE}{dt} = \underline{-3.24 \times 10^{-8}\,T}$$

Problem 3 : As shown in the figure below, a ray of light is incident from air ($n_a = 1$) at an angle $\theta_1 = 50°$ from the vertical on a piece of glass which makes a right angle. The light ray bends to an angle θ_2 from the vertical upon passing into the glass, after which it exits the glass at an angle $\theta_3 = 67°$ from the horizontal. Find the index of refraction n_g of the glass, and find the angle θ_2. You will need $\sin^2(\theta) + \cos^2(\theta) = 1$.



$$n_a \sin\theta_1 = n_g \sin\theta_2$$

$$n_g \sin(90-\theta_2) = n_a \sin\theta_3$$

$$\Rightarrow \quad \sin\theta_1 = n_g \sin\theta_2$$

$$\sin\theta_3 = n_g \cos\theta_2$$

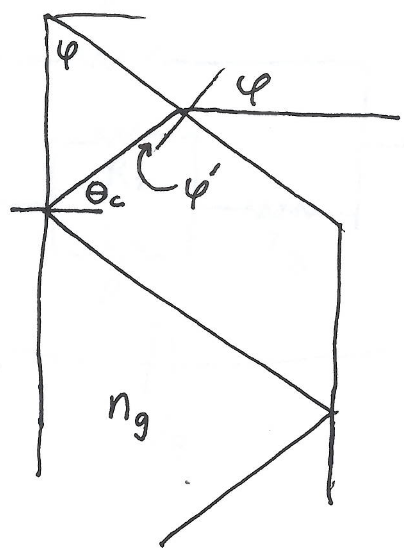
$$\sin^2\theta_1 + \sin^2\theta_3 = n_g^2$$

$$n_g = (\sin^2\theta_1 + \sin^2\theta_2)^{1/2} = \underline{\underline{1.198}}$$

$$\sin\theta_2 = \sin\theta_1 / n_g$$

$$\theta_2 = \underline{\underline{39.77°}}$$

Problem 4 : The figure below shows a piece of glass of refractive index $n_g = 1.6$ which is cut at an angle φ such that a ray which enters the glass from the surrounding air $n_a = 1.0$ along the horizontal line shown will reflect off the interior surfaces of the glass. Find the smallest angle φ above which this total internal reflection is possible, and find the corresponding reflection angle θ_c. Find the smallest index of refraction n_c such that this scenario is possible (this corresponds to requiring that $\tan(\varphi) = \sin(\varphi)/\cos(\varphi) > 0$). You will need $\sin(\varphi - \theta_c) = \sin(\varphi)\cos(\theta_c) - \cos(\varphi)\sin(\theta_c)$.



$n_a = 1$

$\sin\varphi = n_g \sin\varphi'$

$\varphi + 90 - \theta_c + 90 - \varphi' = 180$

$\underline{\varphi' = \varphi - \theta_c}$

TIR: $\underline{n_g \sin\theta_c = 1}$

$$\sin\varphi = n_g \sin(\varphi - \theta_c) = n_g(\sin\varphi\cos\theta_c - \cos\varphi\sin\theta_c)$$

$$\tan\varphi = n_g \cos\theta_c \tan\varphi - \underbrace{n_g \sin\theta_c}_{=1}$$

$$\tan\varphi = (n_g \cos\theta_c - 1)^{-1}$$

$$n_g \cos\theta_c = n_g(1 - \sin^2\theta_c)^{1/2} = n_g(1 - n_g^{-2})^{1/2} = (n_g^2 - 1)^{1/2}$$

$$\tan\varphi = \left((n_g^2 - 1)^{1/2} - 1\right)^{-1}$$

$\tan\varphi > 0$ requires:

$n_g = 1.6$: $\varphi = \underline{\underline{76^\circ}}$

$n_g > \underline{\underline{n_c = \sqrt{2}}}$