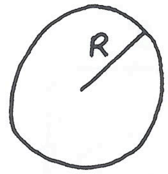
Problem 3 : The figure below shows the space between the plates of a circular capacitor of radius $R = 0.3\,\mathrm{m}$ which has a uniform electric field of the form $\vec{E} = at\hat{z}$, where $a = 2\,\mathrm{V/ms}$. The electric field outside the plates vanishes. Find the magnitude (and indicate its direction) of the magnetic field for $r < R$ and $r > R$. Find the magnitude (and indicate its direction) of the Poynting vector field for $r < R$ and $r > R$. Also compute the combined electromagnetic energy density $u_E + u_B$ for $r < R$ and $r > R$.

$\vec{B} = B(r)\,\hat{\theta}$



$\odot\,\hat{z}$

$\underline{\vec{E} = at\,\hat{z}}$

<u>Ampere/Maxwell</u>

$$\oint_{L_c} d\vec{r}\cdot\vec{B} = \mu_0\varepsilon_0 \frac{d}{dt}\int d\vec{A}\cdot\vec{E}$$

<u>$r<R$</u>

$$2\pi r B = \mu_0\varepsilon_0 a \pi r^2$$

$$\vec{B} = \tfrac{1}{2}\mu_0\varepsilon_0 a r\,\hat{\theta}$$

<u>$r>R$</u>

$$2\pi r B = \mu_0\varepsilon_0 a \pi R^2$$

$$\vec{B} = \tfrac{1}{2}\mu_0\varepsilon_0 a R^2/r\,\hat{\theta}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$\vec{S} = 0$ outside since $\vec{E} = 0$

inside: $\vec{S} = t/2\,\varepsilon_0 a^2 r \underbrace{\hat{z}\times\hat{\theta}}_{-\hat{r}} = -t/2\,\varepsilon_0 a^2 r\,\hat{r}$

(points inward)

$u = u_B + u_E$

$u_E = \tfrac{1}{2}\varepsilon_0 \vec{E}^2$

$u_B = \tfrac{1}{2}\mu_0^{-1}\vec{B}^2$

$u_E = 0$ <u>$r>R$</u>

$$u = u_B = \tfrac{1}{2}\mu_0^{-1}\left(\tfrac{1}{2}\mu_0\varepsilon_0 a R^2/r\right)^2$$

<u>$r<R$</u>

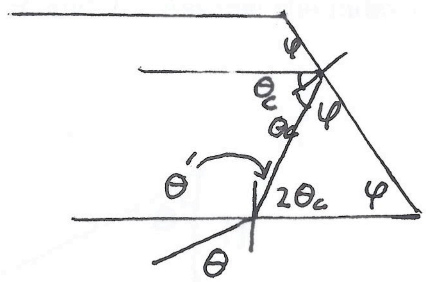
$u = u_B + u_E$

$$u_E = \tfrac{1}{2}\varepsilon_0 (at)^2$$

$$u_B = \tfrac{1}{2}\mu_0^{-1}\left(\tfrac{1}{2}\mu_0\varepsilon_0 a r\right)^2$$

Problem 3 : The figure below shows a piece of glass of refractive index $n = 1.8$ which is cut at an angle φ such that the ray shown strikes the glass-air interface an infinitesimal amount beyond the critical angle θ_c. That is assume the angle shown below is the critical angle, but a reflection takes place as shown. Find the angle φ, and the angle θ that the ray emerges from the glass.

$n = 1.8$



$\sin\theta_c = \frac{1}{n}$

$\theta_c = 33.75^\circ$

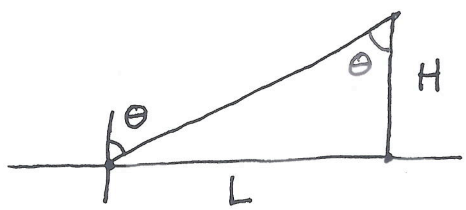
$\varphi = 90^\circ - \theta_c = 56.25^\circ$

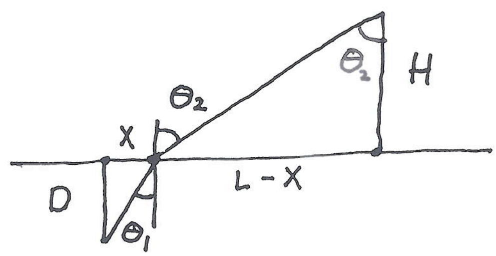
$\theta' = 90^\circ - 2\theta_c = 22.50^\circ$

$n \sin\theta' = \sin\theta$

$\theta = 43.54^\circ$

Problem 4 : The first figure below depicts a small submarine at the surface of the water. The submarine is looking at a lighthouse of height H which is a horizontal distance L away. The angle from the vertical that the submarine sees the lighthouse is $\theta = 75°$. The second figure shows the submarine submerged at a depth $D = 20\,\mathrm{m}$ at the same horizontal distance L from the lighthouse. The angle from the vertical that the submarine sees the lighthouse when submerged is $\theta_1 = 46°$. Find the height of the lighthouse H and the horizontal distance L. The distance x and the angle θ_2 in the figure below will have to be eliminated (or solved for) in order to find H and L. Assume the index of refraction of air is $n_2 = 1$ and that of water is $n_1 = 1.33$.





$$\tan\theta = \frac{L}{H} = 3.73$$

$$\tan\theta_1 = \frac{X}{D} = 1.04$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\tan\theta_2 = \frac{L-X}{H} = 3.29$$

$$\sin\theta_2 = 1.33 \sin\theta_1$$

$$X = D\tan\theta_1 = 20.71$$

$$\theta_2 = 73.1°$$

$$\tan\theta_2 = \frac{\tan\theta\, H - D\tan\theta_1}{H}$$

$$H = \frac{D\tan\theta_1}{\tan\theta - \tan\theta_2} = 46.60\,\mathrm{m}$$

$$L = H\tan\theta = 173.93\,\mathrm{m}$$