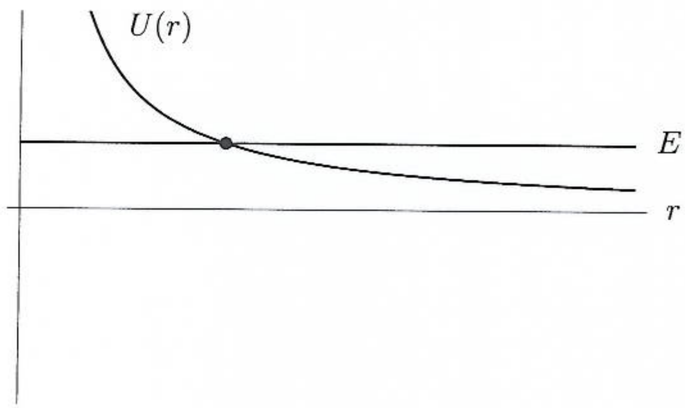
SMU Physics 1313 : Fall 2008

Exam 2

Questions 1-3 refer to the energy diagram shown below. The curve denotes the potential energy $U(r)$ and the straight line denotes the total mechanical energy E. The axes cross at $r = 0$ and $U = 0$. The initial radius of the moving object is denoted by the point on the E line. The moving object has non-zero velocity only along the line between it and the fixed object at the origin; that is, we are not considering circular motion. Assume that if the object is not initially at rest then its initial velocity is outward.



1. Which statement best describes the system as a whole :

[a] The system involves a repulsive force.

(b) The system involves an attractive force with a bound object.

(c) The system involves an attractive force with an unbound object.

(d) The system can only be gravitational.

2. Which statement best describes the initial state of the object :

(a) The object has more total energy than potential energy.

[b] The object is at rest.

(c) The object has more kinetic energy than total energy.

(d) The object is being attracted to the origin.

3. Which statement best describes the subsequent motion of the object :

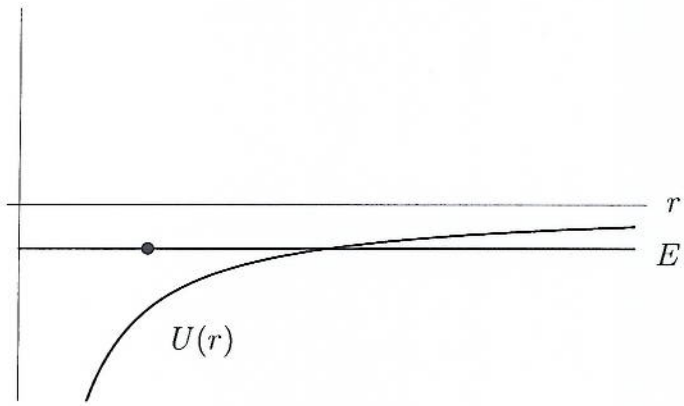
(a) The object will drop toward the origin.

(b) The object will stay at rest.

[c] The object will gain kinetic energy as it escapes to infinity.

(d) The object will reach a maximum radius and fall back toward the origin.

Questions 4-6 refer to the energy diagram shown below. The curve denotes the potential energy $U(r)$ and the straight line denotes the total mechanical energy E. The axes cross at $r = 0$ and $U = 0$. The initial radius of the moving object is denoted by the point on the E line. The moving object has non-zero velocity only along the line between it and the fixed object at the origin; that is, we are not considering circular motion. Assume that if the object is not initially at rest then its initial velocity is outward.



4. Which statement best describes the system as a whole :

(a) The system involves a repulsive force.

[b] The system involves an attractive force with a bound object.

(c) The system involves an attractive force with an unbound object.

(d) The system can only be gravitational.

5. Which statement best describes the initial state of the object :

[a] The object has more total energy than potential energy.

(b) The object is at rest.

(c) The object has more total energy than kinetic energy.

(d) The object is being repelled from the origin.

6. Which statement best describes the subsequent motion of the object :

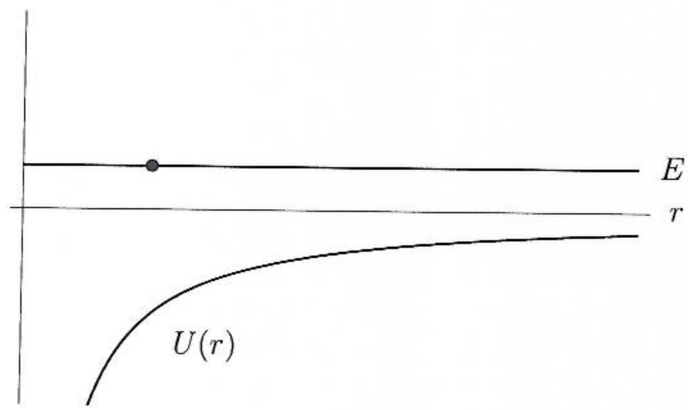
(a) The object will drop toward the origin.

(b) The object will stay at rest.

(c) The object will gain kinetic energy as it escapes to infinity.

[d] The object will reach a maximum radius and fall back toward the origin.

Questions 7-9 refer to the energy diagram shown below. The curve denotes the potential energy $U(r)$ and the straight line denotes the total mechanical energy E. The axes cross at $r = 0$ and $U = 0$. The initial radius of the moving object is denoted by the point on the E line. The moving object has non-zero velocity only along the line between it and the fixed object at the origin; that is, we are not considering circular motion. Assume that if the object is not initially at rest then its initial velocity is outward.



7. Which statement best describes the system as a whole :

(a) The system involves a repulsive force.

(b) The system involves an attractive force with a bound object.

[c] The system involves an attractive force with an unbound object.

(d) The system can only be gravitational.

8. Which statement best describes the initial state of the object :

(a) The object has more energy than kinetic energy.

(b) The object is at rest.

(c) The object has more potential energy than total energy.

[d] The object is being attracted to the origin.

9. Which statement best describes the subsequent motion of the object :

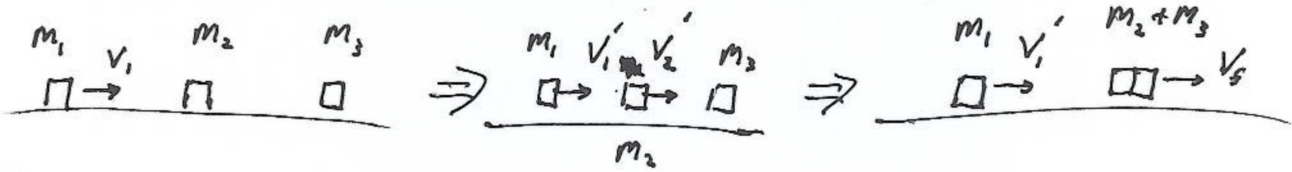
(a) The object will drop toward the origin.

(b) The object will stay at rest.

[c] The object will lose kinetic energy as it escapes to infinity.

(d) The object will reach a maximum radius and fall back toward the origin.

Problem 1 : A mass $m_1 = 3\,\mathrm{kg}$ is given an initial velocity $v_1 = 2\,\mathrm{m/s}$ to the right. It then collides elastically with a mass $m_2 = 1\,\mathrm{kg}$ which is initially at rest. Find the velocities v_1' and v_2' after this collision. The mass m_2 then collides completely inelastically with a mass m_3 which is initially at rest. Find the mass m_3 such that the final velocity v_f of the resulting combined mass is equal to v_1'. How much total kinetic energy is lost in this entire process?



$$V_1' = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_1 = \frac{2}{4} V_1 = 1\,\mathrm{m/s}$$

$$V_2' = \frac{2m_1}{(m_1 + m_2)} V_1 = \frac{6}{4} V_1 = 3\,\mathrm{m/s}$$

$$\underline{V_f = V_1'}$$

$$m_2 V_2' = (m_2 + m_3) V_f = (m_2 + m_3) V_1'$$

$$m_3 = m_2 \left(V_2'/V_1' - 1 \right) = \underline{\underline{2\,\mathrm{Kg}}}$$

$$K_o = \tfrac{1}{2} m_1 V_1^2$$

$$K_o = \tfrac{1}{2}(3\,\mathrm{Kg})(4\,\mathrm{m^2/s^2})$$

$$\underline{K_o = 6\,\mathrm{J}}$$

$$K_f = \tfrac{1}{2} m_1 V_1'^2 + \tfrac{1}{2}(m_2 + m_3) V_f^2$$

$$K_f = \tfrac{1}{2}(m_1 + m_2 + m_3) V_1'^2$$

$$K_f = \tfrac{1}{2}(6\,\mathrm{Kg})(1\,\mathrm{m^2/s^2})$$

$$\underline{K_f = 3\,\mathrm{J}}$$

$$\underline{K_f - K_o = -3\,\mathrm{J}}$$

Problem 2 : Suppose you are brought back to earth millions of years from now, and the moon looks twice as big as it does today; that is, its distance from the earth is half of what it is today, but the mass of the earth is unchanged. What is the ratio $T_{\text{then}}/T_{\text{now}}$ of the respective periods? What is the ratio $K_{\text{then}}/K_{\text{now}}$ of the kinetic energies of the orbits? Find out if total energy has been gained or lost by computing the ratio $(E_{\text{then}} - E_{\text{now}})/E_{\text{now}}$. Note that you will not need any numerical values to compute these ratios.

$$T = 2\pi \left(\frac{r^3}{G M_e}\right)^{1/2}$$

$$\frac{T_{then}}{T_{now}} = \left(r_{then}/r_{now}\right)^{3/2} = \left(1/2\right)^{3/2} = 1/\sqrt{8}$$

$$\frac{V_{then}}{V_{now}} = \left(\frac{2\pi r_{then}}{T_{then}}\right) \Big/ \left(\frac{2\pi r_{now}}{T_{now}}\right) = \left(\frac{r_{then}}{r_{now}}\right)\left(\frac{T_{now}}{T_{then}}\right)$$

$$\frac{V_{then}}{V_{now}} = \left(\frac{1}{2}\right)\sqrt{8} = \sqrt{2}$$

$$\frac{K_{then}}{K_{now}} = \frac{\frac{1}{2} M V_{then}^2}{\frac{1}{2} M V_{now}^2} = \left(\frac{V_{then}}{V_{now}}\right)^2 = 2$$

$$E = K + U \qquad U = -\frac{G M_e M}{r} \qquad K = \tfrac{1}{2} m v^2$$

$$\boxed{\frac{m v^2}{r} = \frac{G M_e M}{r^2} \qquad v^2 = G M_e / r}$$

$$K = \frac{G M_e M}{2r}$$

$$E = -\frac{G M_e M}{2r}$$

$$\frac{E_{then} - E_{now}}{E_{now}} = \frac{E_{then}}{E_{now}} - 1 = \frac{r_{now}}{r_{then}} - 1 = \tfrac{1}{2} - 1 = -\tfrac{1}{2}$$

Problem 3 : An alpha particle ($q_2 = 2e$, $m_2 = 4m_p$) escapes from a uranium nucleus, and is initially at rest at $r = 10^{-15}\,\mathrm{m}$ outside the resulting Thorium nucleus ($q_1 = 90e$). Find the final velocity v_∞ of the alpha particle after it escapes to infinity. Find out how this velocity compares to the speed of light $c = 3.0 \times 10^8\,\mathrm{m/s}$ by computing v_∞/c. You will need $k = 8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2}$, where the definition of the Newton is $\mathrm{N} = \mathrm{kg \cdot m/s^2}$, as well as $m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$ and $e = 1.60 \times 10^{-19}\,\mathrm{C}$.

$$U = \frac{K q_1 q_2}{r} = \frac{180 K e^2}{r}$$

$$K_0 + U_0 = K_f + U_f$$

$$K_0 = 0 \quad U_0 = \frac{180 K e^2}{10^{-15}\,m} = \underline{4.15 \times 10^{-11}\,J}$$

$$U_f = 0 \quad K_f = \tfrac{1}{2}(4M_p)V_\infty^2$$

$$V_\infty = \left(U_0/2M_p\right)^{1/2}$$

$$= \underline{1.11 \times 10^8\,m/s}$$

$$= \underline{0.371\,c}$$