1) Coulomb's Law

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

Newton's Law of Universal Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

To find the needed mass set the two forces equal and $q_1 = q_2 = q_p$, $m_1 = m_2 = m_\alpha$

$$\frac{1}{4\pi\epsilon_0}\frac{q_p q_p}{r^2} = G\frac{m_\alpha m_\alpha}{r^2}$$

$$\frac{q_p^2}{4\pi\epsilon_0 G} = m_\alpha^2$$

$$\frac{1}{4\pi\epsilon_0} = 8.9876\times10^9\ Nm^2C^{-2}$$

$$q_p = 1.6022\times10^{-19}\ C$$

$$G = 6.6742\times10^{-11}\ m^3 kg^{-1} s^{-2}$$

$$m_\alpha = 1.8593\times10^{-9}\ kg$$

Since $m_p = 1.6726\times10^{-27}$ kg

$$m_\alpha = (1.112\times10^{18})\cdot m_p$$

2) Conservation of Energy

$$U_i + K_i = U_F + K_F$$

Assuming no change in acceleration

$$U_i = K_{F_1} = mgh$$

$$g = 9.80\ m/s^2,\ h = 1\times10^6\ m$$

$$K_{F_1} = m(9.8067\times10^6\ m^2/s^2)$$

With Change in momentum

$$U_i - U_F = K_{F_2}$$

$$\frac{GmM_E}{R_E} - \frac{GmM_E}{R_E+h} = K_{F_2}$$

$$GmM_E\left(\frac{h}{R_E^2+R_E h}\right) = K_{F_2}$$

$G = 6.6742\times10^{-11} m^3 kg^{-1} s^{-2}$, $h = 1\times10^6 m$, $M_E = 5.9736\times10^{24} kg$, $R_E = 6.371\times10^6 m$

$$K_{F_2} = m(8.4898\times10^6\ m^2/s^2)$$

<u>percent error</u>

$$\frac{|\text{Approx} - \text{Exact}|}{|\text{Exact}|}\times100 = \frac{|m(9.8067\times10^6 m^2/s^2) - m(8.4898\times10^6 m^2/s^2)|}{|m(8.4898\times10^6 m^2/s^2)|}\times100$$

$$= \frac{|m(1.3168\times10^6 m^2/s^2)|}{|m(8.4898\times10^6 m^2/s^2)|}\times100 = 15.511\%$$

3) $m = 1\ Kg$

$r = 3R_E$

a) Find v & T

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$= 2\pi\sqrt{\frac{(3R_E)^3}{GM_E}}$$

$$= 2\pi\sqrt{\frac{9R_E^3}{GM_E}} = 2.6293 \times 10^4\ s$$

$$v = \frac{d}{t} = \frac{2\pi R_E}{T}$$

$$= 4.5672 \times 10^3\ m/s$$

b) Find K, U, E

$$E = K + U$$

$$K = \frac{1}{2}mv^2$$

$$U = -\frac{GMm}{r} = -\frac{GmM_E}{3R_E}$$

$$K = \frac{1}{2}(1Kg)(4.5672 \times 10^3 m/s)^2$$

$$= 1.0430 \times 10^7 \frac{Kgm^2}{s^2}$$

$$U = -(6.6742 \times 10^{-11} m^3 Kg^{-1} s^{-2})(1Kg)(5.9736 \times 10^{24} Kg) / 3(6.371 \times 10^6 m)$$

$$= -2.0860 \times 10^7\ Kg\,m^2/s^2$$

$$E = K + U = -1.0430 \times 10^7\ Kg\,m^2/s^2$$

c) ΔK required for escape velocity.

$$V_e = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2GM_E}{3R_E}}$$

But $K_e = -U$

$$\Delta K = K_e - K_{satellite} = 2.0860 \times 10^7\ Kg\,m^2/s^2 - 1.0430 \times 10^7\ Kg\,m^2/s^2 = 1.0430 \times 10^7\ Kg\,m^2/s^2$$

from rest, where $V_{satellite} = 0$ and $K_{satellite} = 0$

$$\Delta K = K_e - 0 = -U = 2.0860 \times 10^7\ Kg\,m^2/s^2$$

4) $m = m_e$

$M = 6m_p + 6m_n$

$r = 10^{-10}m$

$q_1 = q_e$

$q_2 = 6q_p$

a) v & T

For an orbit the attractive force is equal to the centripetal force

$$F_e + F_g = F_c$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} + G\frac{mM}{r^2} = m\frac{v^2}{r}$$

but the gravity term is much, much smaller than the electric term so it can be ignored as we saw in problem 1.

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{q_1 q_2}{4\pi\varepsilon_0 r m}} = \left[\frac{(8.9876\times10^9 Nm^2C^{-2})(1.6022\times10^{-19}C)(6\times1.6022\times10^{-19}C)}{(10^{-10}m)(9.1094\times10^{-31}kg)}\right]^{1/2}$$

$$= 3.8982\times10^6\ m/s$$

$$T = \frac{d}{v} = \frac{2\pi r}{v} = \frac{2\pi(10^{-10}m)}{(3.8982\times10^6 m/s)} = 1.6118\times10^{-16}s$$

b) Find K, U, & E

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(9.1094\times10^{-31}kg)(3.8982\times10^6 m/s)^2 = 6.9214\times10^{-18}\ kg\,m^2/s^2$$

$$U = \frac{1}{4\pi\varepsilon_0}\frac{-q_1 q_2}{r} = -(8.9876\times10^9 Nm^2C^{-2})(1.6022\times10^{-19}C)(6\times1.6022\times10^{-19}C)(10^{-10}m)^{-1}$$

$$= -1.3843\times10^{-17}\ kg\,m^2/s^2$$

$$E = K + U = -6.9214\times10^{-18}\ kg\,m^2/s^2$$

c) ΔK for escape velocity

$$K_{esc} = -U$$

$$\Delta K = K_{esc} - K_{elec} = (1.3843\times10^{-17}\ kg\,m^2/s^2) - (6.9214\times10^{-18}\ kg\,m^2/s^2) = 6.9214\times10^{-18}\ kg\,m^2/s^2$$

From rest

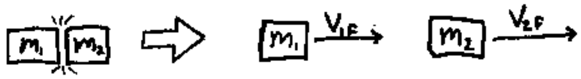
$V_{elec} = 0$ and $K_{elec} = 0$

$$\Delta K = K_{esc} - 0 = -U = 1.3843\times10^{-17}\ kg\,m^2/s^2$$

5) $m_1 = 2\ kg \quad m_2 = 1\ kg \quad m_3 = 1\ kg$

$V_{1i} = 2\ m/s \quad V_{2i} = 0\ m/s \quad V_{3i} = 0\ m/s$

Elastic Collision of m_1 and m_2



In an elastic collision, Kinetic energy and momentum are conserved

$$K_{1i} + K_{2i} = K_{1F} + K_{2F} \quad \text{and} \quad P_{1i} + P_{2i} = P_{1F} + P_{2F}$$

$$\tfrac{1}{2} m_1 V_{1i}^2 + \tfrac{1}{2} m_2 V_{2i}^2 = \tfrac{1}{2} m_1 V_{1F}^2 + \tfrac{1}{2} m_2 V_{2F}^2 \quad \text{and} \quad m_1 V_{1i} + m_2 V_{2i} = m_1 V_{1F} + m_2 V_{2F}$$

setting $V_{2i} = 0$ and multiplying Kinetic energy by 2

① $m_1 V_{1i}^2 = m_1 V_{1F}^2 + m_2 V_{2F}^2$

② $m_1 V_{1i} = m_1 V_{1F} + m_2 V_{2F}$

$m_1 V_{1i}^2 - m_1 V_{1F}^2 = m_2 V_{2F}^2 \quad$ and $\quad m_1 V_{1i} - m_1 V_{1F} = m_2 V_{2F}$

④ $m_1 (V_{1i} - V_{1F}) = m_2 V_{2F}$

$m_1 (V_{1i}^2 - V_{1F}^2) = m_2 V_{2F}^2$

③ $m_1 (V_{1i} - V_{1F})(V_{1i} + V_{1F}) = m_2 V_{2F}^2$

Dividing ③ by ④

$$\frac{m_1 (V_{1i} - V_{1F})(V_{1i} + V_{1F})}{m_1 (V_{1i} - V_{1F})} = \frac{m_2 V_{2F}^2}{m_2 V_{2F}}$$

⑤ $V_{1i} + V_{1F} = V_{2F}$

Substituting ⑤ into ④

$$m_1 V_{1i} - m_1 V_{1F} = m_2 V_{1i} + m_2 V_{1F}$$

$$m_1 V_{1i} - m_2 V_{1i} = m_1 V_{1F} + m_2 V_{1F}$$

$$(m_1 - m_2) V_{1i} = (m_1 + m_2) V_{1F}$$

$$V_{1F} = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1i}$$

$$V_{1F} = \tfrac{1}{3} V_{1i} = \tfrac{2}{3}\ m/s$$

$$V_{2F} = \frac{m_1 (V_{1i} - V_{1F})}{m_2} = \frac{8}{3}\ m/s$$

5 con't) Inelastic Collision between m_2 and m_3

$m_2 = 1\,Kg$ $\quad m_3 = 5\,Kg$

$V_i = \frac{8}{3}\ m/s$ $\quad V_3 = 0\ m/s$

Momentum is conserved but Kinetic Energy is not

$$m_2 V_i + m_3 V_3 = (m_2 + m_3) V_f$$

$$m_2 V_i = (m_2 + m_3) V_f \qquad \text{since } V_3 = 0\ m/s$$

$$V_f = \frac{m_2}{(m_2 + m_3)} V_i$$

$$V_f = \frac{1}{6} V_i = \frac{1}{6}\left(\frac{8}{3}\ m/s\right) = \boxed{\frac{4}{9}\ m/s = V_f}$$

Yes, since the combined m_2/m_3 is moving slower than m_1 now, m_1 will catch up and collide a third time.

6) Car 1 $\qquad$ Car 2

$m_1 = 1000\ Kg$ $\quad m_2 = 1500\ Kg$

$V_1 = 20\ m/s$ $\quad V_2 = 0\ m/s$

a) elastic collision

Since the second car is at rest, we can use our solutions from the previous problem.

$$V_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} V_{1i} \quad \text{and} \quad V_{2f} = \frac{m_1}{m_2}(V_{1i} - V_{1f}) = \frac{-m_1(V_{1f} - V_{1i})}{m_2}$$

$$\Delta V_1 = V_{1f} - V_{1i} \qquad \Delta V_2 = V_{2f} - V_{2i} = V_{2f} = -\frac{m_1}{m_2}\Delta V_1$$

$$= \left(\frac{(m_1 - m_2)}{(m_1 + m_2)} - 1\right) V_{1i}$$

$$\boxed{\Delta V_1 = -24\ m/s} \qquad \boxed{\Delta V_2 = 16\ m/s}$$

b) inelastic collisions

Once again we can use our work from the previous problem.

$$V_f = \frac{m_1}{(m_1 + m_2)} V_{1i} = 8\ m/s \qquad \Delta V_1 = V_f - V_{1i} = 8\ m/s - 20\ m/s = \boxed{-12\ m/s = \Delta V_1}$$

$$\Delta V_2 = V_f - V_{2i} = V_f = \boxed{8\ m/s = \Delta V_2}$$

Kinetic Energy

$$K_i = \frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2$$

$$= 2.0 \times 10^5\ Kg\,m^2/s^2 \qquad \Delta K = K_f - K_i = -1.2 \times 10^5\ Kg\,m^2/s^2$$

$$K_f = \frac{1}{2}(m_1 + m_2) V_f^2$$

$$= 8.0 \times 10^4\ Kg\,m^2/s^2$$