In[3]:=
```
$Version
```
Out[3]=
```
13.3.1 for Linux x86 (64-bit) (July 24, 2023)
```
In[4]:=
```
Off[General::spell, General::spell1]
```

INDEX:

****

How small a fraction can you use to approximate Pi?

```
In[5]:= N[Pi,100]

Out[5]= 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706 8
```

In[6]:=
```
Timing[N[Pi,1000]][[1]]
```
Out[6]= 0.000079

In[7]:=
```
Do[out=Rationalize[N[Pi],10^(-m)];
   Print[m," = ",out,"    ",N[out,10]];Print["  "],{m,1,8}]
```

1 = $\frac{22}{7}$ 3.142857143

2 = $\frac{22}{7}$ 3.142857143

3 = $\frac{201}{64}$ 3.140625000

4 = $\frac{333}{106}$ 3.141509434

5 = $\frac{355}{113}$ 3.141592920

6 = $\frac{355}{113}$ 3.141592920

7 = $\frac{75948}{24175}$ 3.141592554

8 = $\frac{100798}{32085}$ 3.141592645

****

Express the Golden Ratio as a continued fraction:

In[8]:=
```
Nest[Function[x,1/(1+x)],x,3]
```
Out[8]=

$$\frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}$$

In[9]:=
```
goldenratio=1+Nest[Function[x,1/(1+x)],x,10]
```

Out[9]=

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+x}}}}}}}}}}$$

In[10]:=
```
N[goldenratio//.{x->1},20]
```

Out[10]=

1.6180555555555555556

In[11]:=
```
N[GoldenRatio,20]
```

Out[11]=

1.6180339887498948482

****

Expand a Polynomial:

In[12]:=
```
temp=Expand[(a+b)^20]
```

Out[12]=

$$a^{20}+20\,a^{19}\,b+190\,a^{18}\,b^2+1140\,a^{17}\,b^3+4845\,a^{16}\,b^4+15\,504\,a^{15}\,b^5+38\,760\,a^{14}\,b^6+77\,520\,a^{13}\,b^7+125\,970\,a^{12}\,b^8+167\,960\,a^{11}\,b^9+184\,756\,a^{10}\,b^{10}+167\,960\,a^9\,b^{11}+125\,970\,a^8\,b^{12}+77\,520\,a^7\,b^{13}+38\,760\,a^6\,b^{14}+15\,504\,a^5\,b^{15}+4845\,a^4\,b^{16}+1140\,a^3\,b^{17}+190\,a^2\,b^{18}+20\,a\,b^{19}+b^{20}$$

In[13]:=
```
Factor[temp]
```

Out[14]=

$(a+b)^{20}$

****

Solve a cubic equation:

In[15]:=
```
temp=2x^3+3x^2+5x+2
```

Out[15]=

$2+5\,x+3\,x^2+2\,x^3$

In[16]:=
```
sol=Solve[temp==0,x]
```

Out[16]=

$$\left\{\left\{x\to-\frac{1}{2}\right\},\left\{x\to\frac{1}{2}\left(-1-i\sqrt{7}\right)\right\},\left\{x\to\frac{1}{2}\left(-1+i\sqrt{7}\right)\right\}\right\}$$

In[17]:= N[sol]

Out[17]=

$\{\{x \to -0.5\}, \{x \to -0.5 - 1.32288\, i\}, \{x \to -0.5 + 1.32288\, i\}\}$

****

Solve a quartic equation:

In[18]:= temp=7x^4+2x^3+3x^2+5x+2

Out[18]=

$2 + 5\, x + 3\, x^2 + 2\, x^3 + 7\, x^4$

In[19]:= sol=Solve[temp==0,x];

In[20]:=

sol //Short[#,2]&

Out[21]//Short=

$\{\{x \to -0.523\ldots - 0.200\ldots\, i\}, \{x \to -0.523\ldots + 0.200\ldots\, i\}, \{x \to 0.380\ldots - 0.875\ldots\, i\}, \{x \to 0.380\ldots + 0.875\ldots\, i\}\}$

In[22]:= N[sol]

Out[22]=

$\{\{x \to -0.523168 - 0.200052\, i\}, \{x \to -0.523168 + 0.200052\, i\}, \{x \to 0.380311 - 0.875258\, i\}, \{x \to 0.380311 + 0.875258\, i\}\}$

In[23]:= NRoots[temp==0,x]

Out[23]=

$x == -0.523168 - 0.200052\, i \,||\, x == -0.523168 + 0.200052\, i \,||\, x == 0.380311 - 0.875258\, i \,||\, x == 0.380311 + 0.875258\, i$

****

Solve a fifth order equation:

In[24]:= temp=17x^4+7x^4+2x^3+3x^2+5x+2

Out[24]=

$2 + 5\, x + 3\, x^2 + 2\, x^3 + 24\, x^4$

In[25]:= NRoots[temp==0,x]

Out[25]=

$x == -0.37722 - 0.213705\, i \,||\, x == -0.37722 + 0.213705\, i \,||\, x == 0.335554 - 0.575108\, i \,||\, x == 0.335554 + 0.575108\, i$

****

Do some integrations:

In[26]:=
```
ztemp = 1/((x+I y) -1)^2 //.{x->u+v, y->u-v}
```

Out[26]=

$$\frac{1}{\left(-1+u+i\,(u-v)+v\right)^2}$$

In[27]:=
```
{x->u+v, y->u-v}
```

Out[27]=

$$\{x \to u+v,\ y \to u-v\}$$

In[28]:=
```
Integrate[ztemp,u]
```

Out[28]=

$$-\frac{1-i}{-2+(2+2\,i)\,u+(2-2\,i)\,v}$$

In[29]:=
```
Integrate[temp,x]
```

Out[29]=

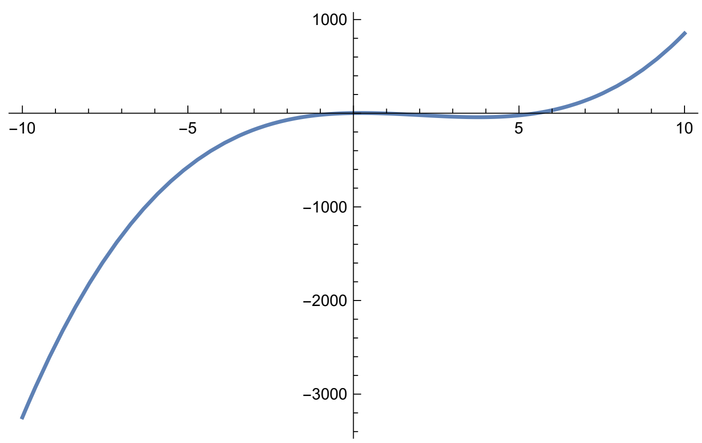
$$2\,x+\frac{5\,x^2}{2}+x^3+\frac{x^4}{2}+\frac{24\,x^5}{5}$$

****

Do some plots:

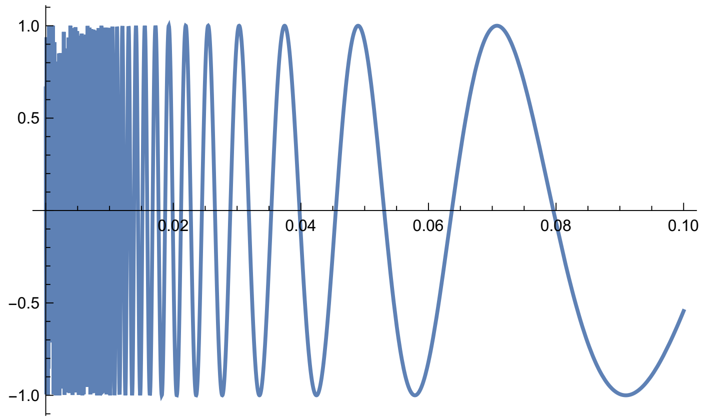
In[30]:=
```
Plot[2x^3-12x^2+5x+2,{x,-10,10}]
```

Out[30]=



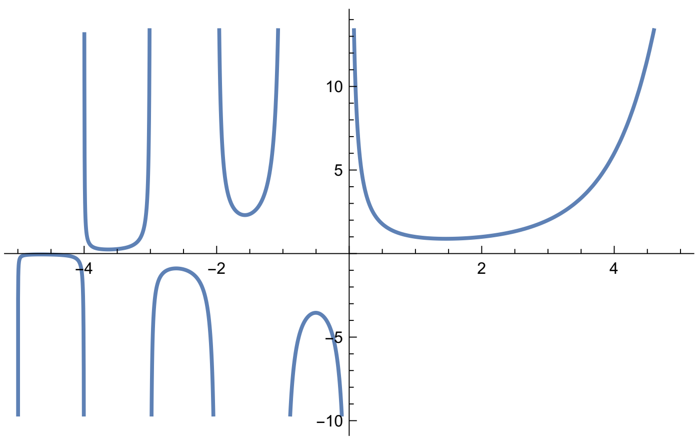
In[31]:= `Plot[Sin[1/x],{x,0,0.1}]`

Out[31]=



In[32]:= `Plot[Gamma[x],{x,-5,5}]`

Out[32]=

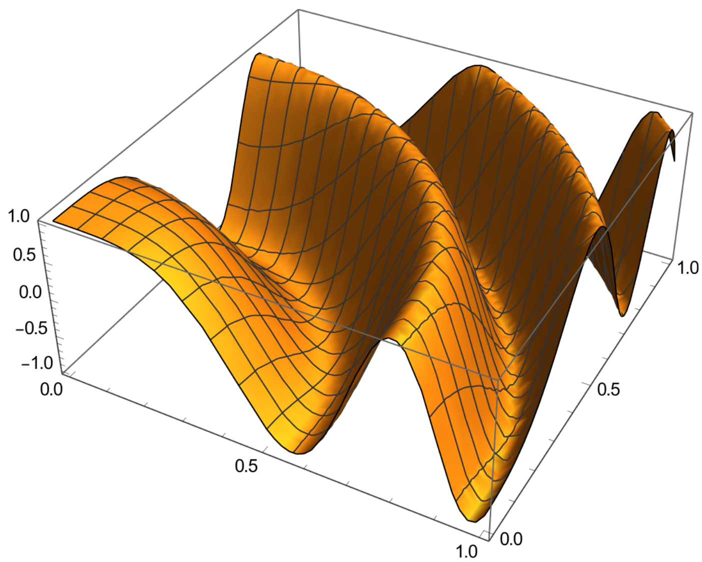


****

Do some 3D Plots:

In[33]:=
```
Plot3D[Re[Exp[10 I(x^2+y^2)]],{x,0,1},{y,0,1}]
```

Out[33]=

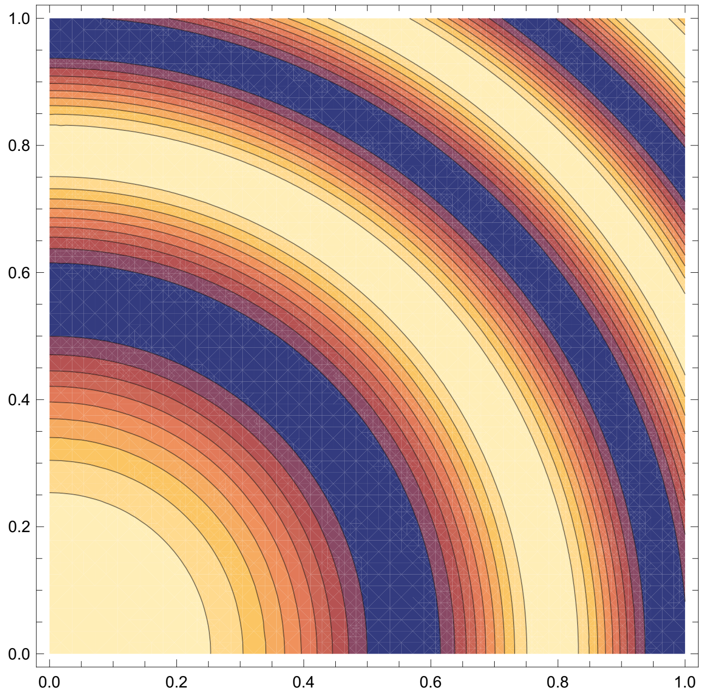


****

Do some Contour Plots:

In[34]:=
```
ContourPlot[Re[Exp[10 I(x^2+y^2)]],{x,0,1},{y,0,1}]
```

Out[34]=



****

Do some Series

In[35]:=
```
Series[Exp[x],{x,0,6}]
```

Out[35]=

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + O[x]^7$$

In[36]:=
```
Series[Exp[x],{x,a,4}]
```

Out[36]=

$$e^a + e^a (x - a) + \frac{1}{2} e^a (x - a)^2 + \frac{1}{6} e^a (x - a)^3 + \frac{1}{24} e^a (x - a)^4 + O[x - a]^5$$

****

Do some Matrices

In[37]:=
```
matrix={{a,c},{c,b}};
MatrixForm[matrix]
```

Out[38]//MatrixForm=

$$\begin{pmatrix} a & c \\ c & b \end{pmatrix}$$

In[39]:=
```
Det[matrix]
```

Out[39]=

$$a\,b - c^2$$

In[40]:=
```
Eigenvalues[matrix] //Simplify
```

Out[40]=

$$\left\{\frac{1}{2}\left(a + b - \sqrt{a^2 - 2\,a\,b + b^2 + 4\,c^2}\right),\ \frac{1}{2}\left(a + b + \sqrt{a^2 - 2\,a\,b + b^2 + 4\,c^2}\right)\right\}$$

****

Do some 3D Plots

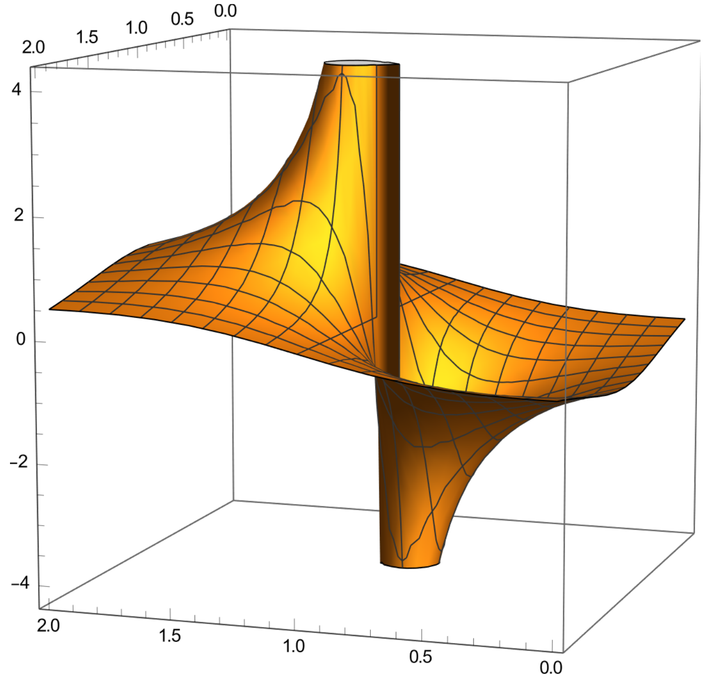
In[41]:=
```
res= 1/( (x+I y) -(1+I))
```

Out[41]=

$$\frac{1}{(-1 - i) + x + i\,y}$$

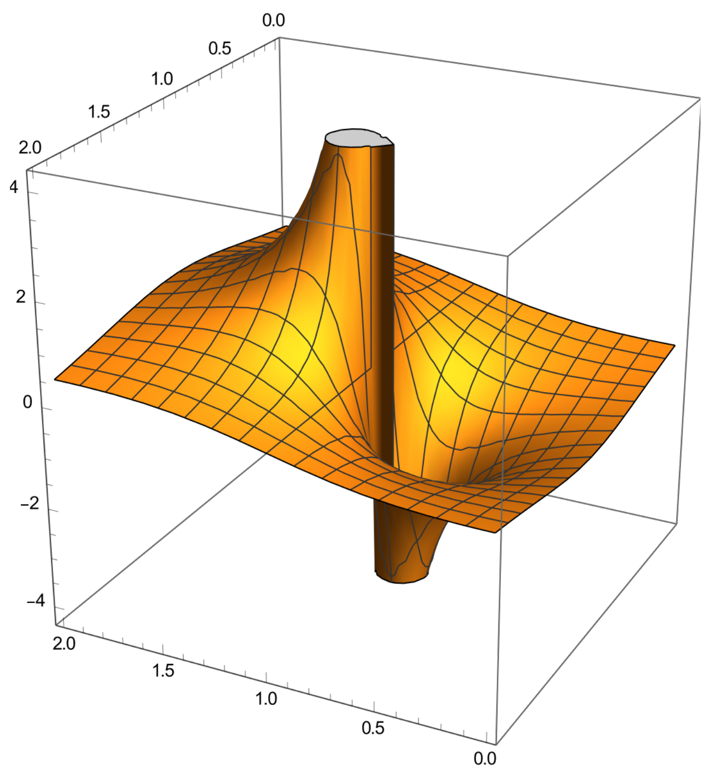
In[43]:=
```
pic=Plot3D[Re[res],{x,0,2},{y,0,2},
                    BoxRatios->{1,1,1},
                    ViewPoint->{-2,6,1}]
```

Out[43]=



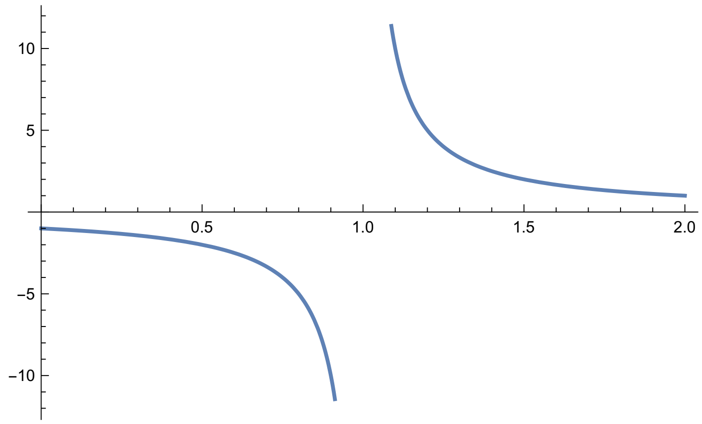
In[44]:=
```
Show[pic, ViewPoint->{-2,4,2}, BoxRatios->{1,1,1}]
```

Out[44]=



In[45]:=
```
Plot[Re[res] /.{y->1} //Evaluate ,{x,0,2}]
```

Out[45]=



****

Do some Bessel Functions

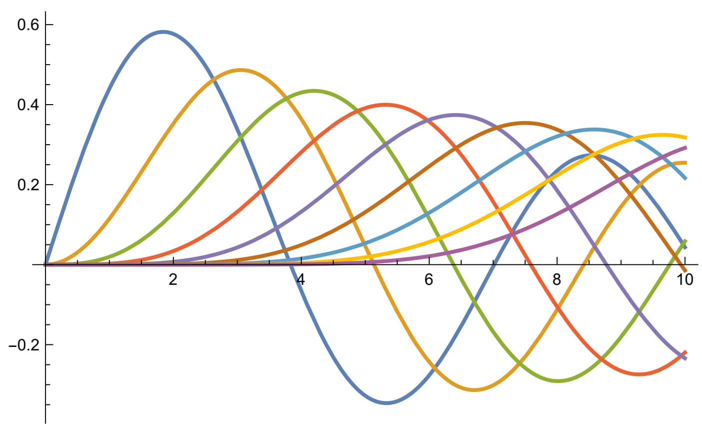
In[46]:=
```
list={BesselJ[1,z],BesselJ[2,z]}
```

Out[46]=

```
{BesselJ[1, z], BesselJ[2, z]}
```

In[47]:=
```
pic2=Plot[{BesselJ[1,z],BesselJ[2,z],BesselJ[3,z],
              BesselJ[4,z],BesselJ[5,z],BesselJ[6,z],
              BesselJ[7,z],BesselJ[8,z],BesselJ[9,z]
               }  ,{z,0,10}]
```

Out[47]=



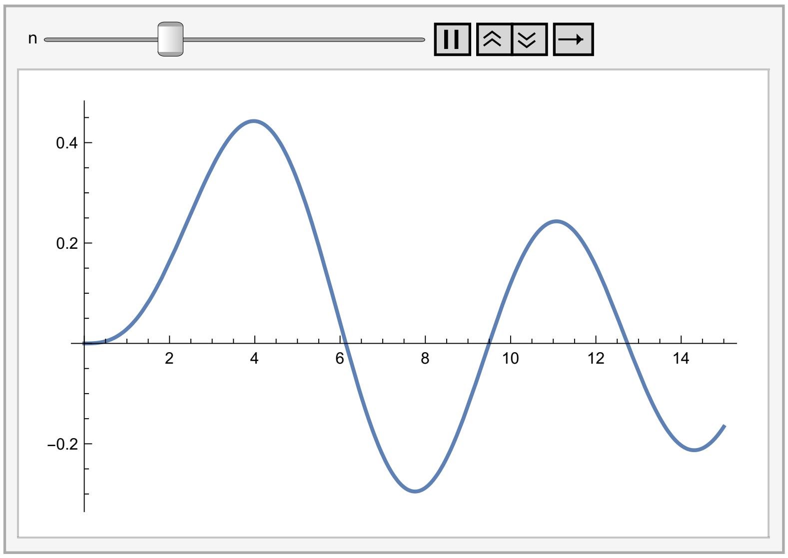
****

How to View Animations in *Mathematica*

In[48]:=

```
Animate[Plot[BesselJ[n, x], {x, 0, 15}], {n, 1, 7, 0.1}]
```

Out[49]=

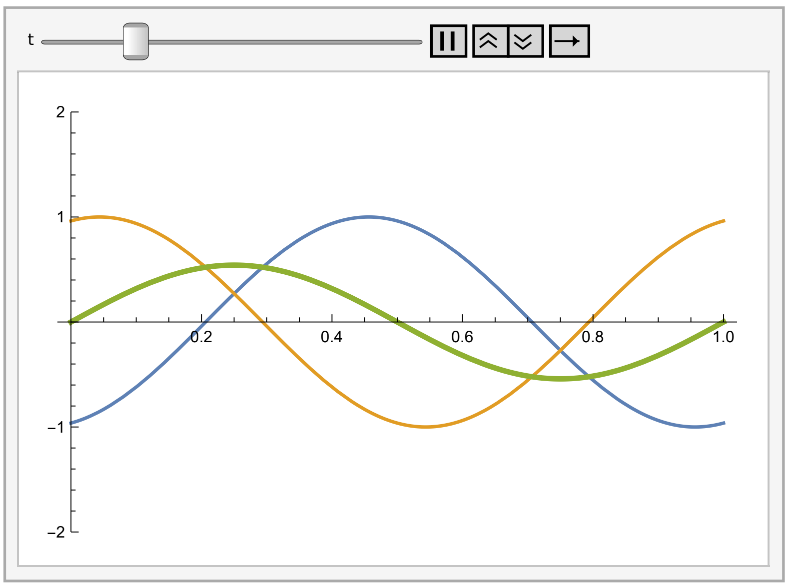


****

Standing Wave Animation

In[50]:=
```
Animate[
Plot[
   {Sin[2 Pi ( x - t) ],
    Sin[2 Pi ( x + t) ],
    Sin[2 Pi ( x - t) ]+
    Sin[2 Pi ( x + t) ]},{x,0,1},
PlotStyle->{Thickness[0.005],Thickness[0.005],Thickness[0.008]},
PlotRange->{-2,2} ]
,{t,0,1}]
```

Out[50]=



****

Play with Loops and Conditionals:

In[52]:=
```
For[i=1, i<4,  Print[i++]]
```

1

2

3

In[53]:=
```
For[i=1, i<4,  Print[++i]]
```

```
2

3

4
```

In[54]:=
```
For[i=2, i<8,  Print[i+=2]]
```

```
4

6

8
```

In[55]:=
```
For[i=2, i<64,  Print[i*=2]]
```

```
4

8

16

32

64
```

In[56]:=
```
Clear[x,t];
t=x;
Do[t=1/(1+t), {3}];t
```

Out[58]=

$$\frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}$$

In[60]:=
```
Clear[x,t];
t=x;
table=Table[t=1/(1+t), {4}]
```

Out[62]=

$$\left\{\frac{1}{1+x}, \frac{1}{1+\frac{1}{1+x}}, \frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}, \frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}}\right\}$$

In[64]:=
```
table //.{x->1} //N
```

Out[64]=

```
{0.5, 0.666667, 0.6, 0.625}
```

In[65]:=
```
FixedPoint[1/(1+#)&,1.0,20]   //N[#,20]&
```

Out[65]=

```
0.618034
```

In[66]:=
```
FixedPoint[1/(1+#)&,1.0]    //N[#,20]&
```

Out[66]=

```
0.618034
```

In[67]:= `Nest[ 1/(1+#)&,x,4]`

Out[67]=

$$\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{1+x}}}}$$

In[68]:= `t1=Nest[ 1/(1+#)&,1,100] //N[#,20]&`

Out[68]=

0.61803398874989484820

In[69]:= `t2=1/Nest[ 1/(1+#)&,1,100] //N[#,20]&`

Out[69]=

1.6180339887498948482

In[70]:= `t1-t2 //N[#,20]&`

Out[70]=

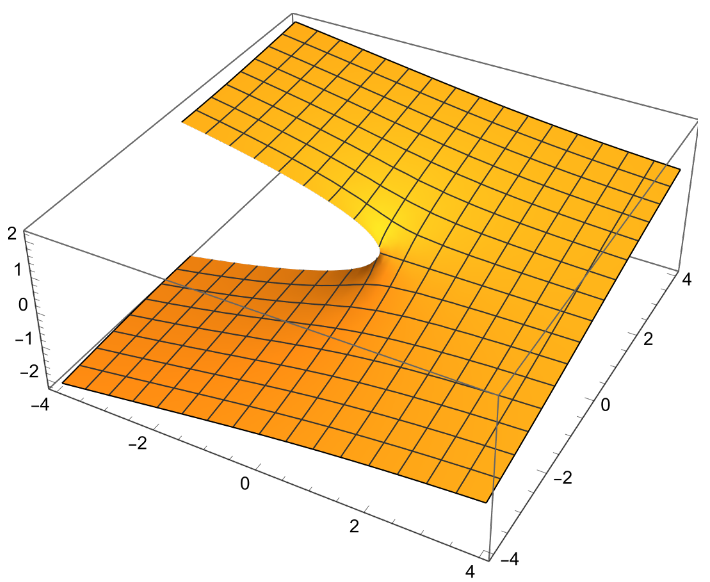
-1.0000000000000000000

****

3D Plots of Complex Functions

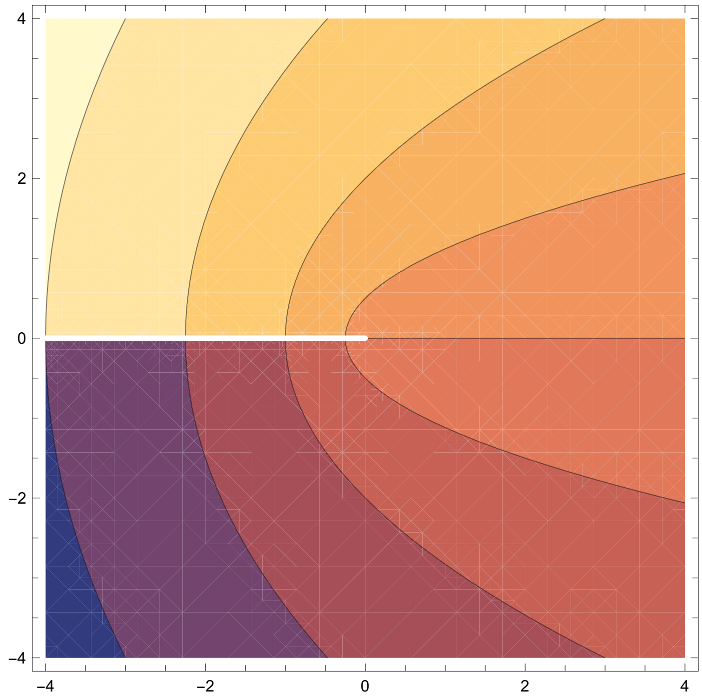
In[71]:= `Plot3D[Im[Sqrt[x+ I y]],{x,-4,4},{y,-4,4}, Lighting->Automatic]`

Out[71]=



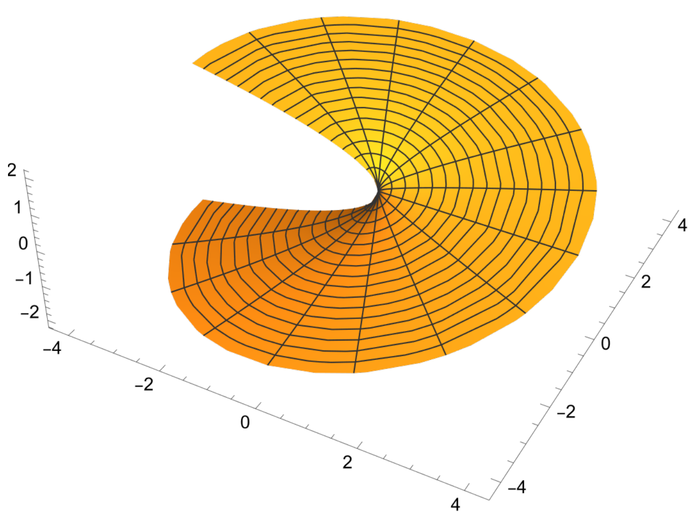
In[72]:=
```
ContourPlot[Im[Sqrt[x+ I y]],{x,-4,4},{y,-4,4}]
```

Out[72]=



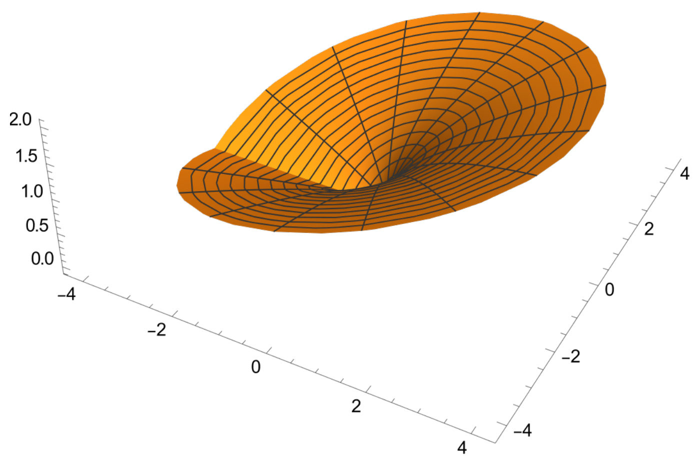
In[74]:=
```
ParametricPlot3D[
    {r Cos[theta]
    ,r Sin[theta]
    ,Im[Sqrt[r Cos[theta] + I r Sin[theta]]]}
,{r,0,4}
,{theta,-Pi+0.0001, Pi-0.0001}
,Boxed->False
,BoxRatios->{1,1,0.4}]
```

Out[74]=



In[75]:=
```
ParametricPlot3D[
    {r Cos[theta]
    ,r Sin[theta]
    ,Re[Sqrt[r Cos[theta] + I r Sin[theta]]]}
,{r,0,4}
,{theta,-Pi+0.0001, Pi-0.0001}
,Boxed->False
,BoxRatios->{1,1,0.4}]
```

Out[75]=



****

Prime Numbers

In[76]:=
```
Clear[i];
Do[Print[i,"  ",Prime[i]],{i,1000,2000,100}]
```

1000	7919
1100	8831
1200	9733
1300	10 657
1400	11 657
1500	12 553
1600	13 499
1700	14 519
1800	15 401
1900	16 381
2000	17 389

In[78]:=
```
Plot[Prime[Floor[i]],{i,1,20}];
```

In[79]:=
```
Plot[Prime[Floor[i]],{i,1,100}];
```

****

The Factorial Function: Old Fashioned Way

In[80]:=
```
Clear[fac2]
fac2[n_Integer?(Function[n,n>0])]:=
 Module[{total=1},
    Do[ total=total * i,{i,1,n}];
    Return[total];
 ]

fac2[0]=1
```

Out[83]=
```
1
```

In[84]:=
```
{fac2[4],fac2[0],fac2[4.0],fac2[4.5],fac2[-4]}
```

Out[84]=
```
{24, 1, fac2[4.], fac2[4.5], fac2[-4]}
```

****

The Factorial Function: New Way

In[85]:=
```
Clear[fac]
fac[n_Integer]:= n fac[n-1] /; n>=1
fac[0]=1;
```

In[88]:= `?fac`

Out[88]=

Symbol

Global`fac

Definitions

```
fac[0] = 1

fac[n_Integer] := n fac[n - 1] /; n ≥ 1
```

Full Name Global`fac

In[89]:= `{fac[4],fac[0],fac[4.0],fac[4.5],fac[-4]}`

Out[89]=

`{24, 1, fac[4.], fac[4.5], fac[-4]}`

In[90]:= `Log[10,1000.!]`

Out[90]=

`2567.604644222133`

** **

** ** ** ***

Different Bases

In[91]:= `BaseForm[#, 2] & /@ {$MaxMachineNumber, $MaxPrecision}`

Out[91]=

$\{1._2 \times 2^{1024}, \infty\}$

In[92]:= `BaseForm[Pi // N, 2]`

Out[92]//BaseForm=

11.00100100001111111_2

In[93]:= `BaseForm[Pi // N, 16]`

Out[93]//BaseForm=

$3.243f_{16}$

** **

** ** ** ***

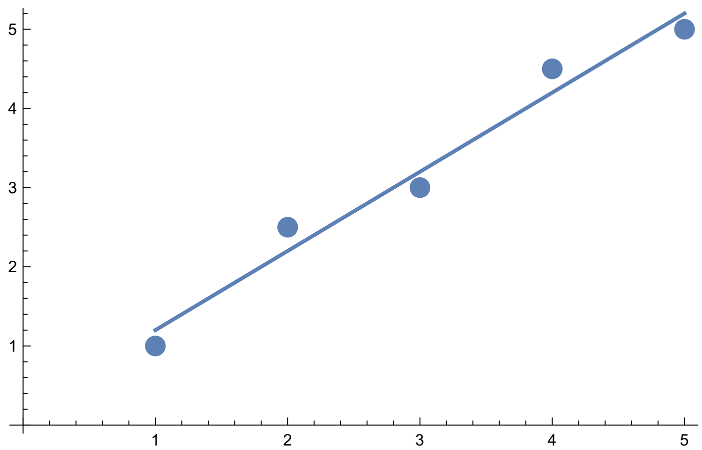
Least Squares Fitting : b

In[94]:=
```
list = {1, 2.5, 3.0, 4.5, 5.0};
fit[x_] = Fit[list, {1, x}, x]
```
Out[95]=
```
0.2 + 1. x
```

In[96]:=
```
p1 = ListPlot[list, PlotStyle → {PointSize[0.03]}];
```

In[97]:=
```
p2 = Plot[fit[x], {x, 1, 5}]; 1
```
Out[97]=
```
1
```

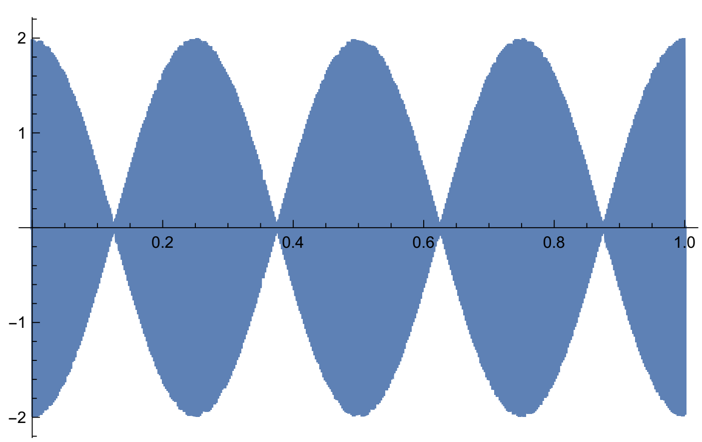
In[98]:=
```
Show[p1, p2]
```
Out[98]=



** ***

Musical Beats

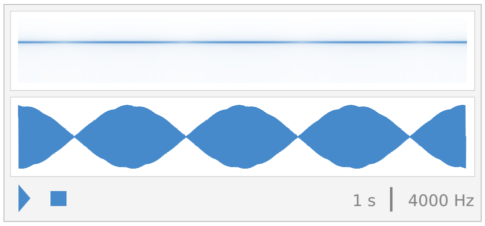
In[99]:=
```
Plot[Sin[2 Pi 440 t] + Sin[2 Pi 444 t], {t, 0, 1}, PlotPoints → 100]
```
Out[99]=



In[100]:=

```
f = 440;
Play[Sin[2 Pi 440 t] + Sin[2 Pi 444 t], {t, 0, 1}, SampleRate → 4000]
```

Out[101]=



** **

** ** ** ***

Solve and Reduce1

In[102]:=

```
Solve[x ^ 3 == 1, x]
```

Out[102]=

$\{\{x \to 1\}, \{x \to -(-1)^{1/3}\}, \{x \to (-1)^{2/3}\}\}$

In[103]:=

```
Solve[x a == a, x]
```

Out[103]=

$\{\{x \to 1\}\}$

In[104]:=

```
Reduce[x a == a, x]
```

Out[104]=

a == 0 || x == 1

** **

** ** ** ***

FractalsP

In[105]:=

```
Clear[reduce, transx, transy];
reduce[point_] := point / 3;
transx[point_] := point + (2 / 3 {1, 0});
transy[point_] := point + (2 / 3 {0, 1});
```

In[109]:=

```
p0 = {Random[], Random[]}
list = {};
print = False;

Do[p1 = reduce[p0];
 p1 = If[Random[] > 0.5, transx[p1], p1];
 p1 = If[Random[] > 0.5, transy[p1], p1];
 list = Append[list, p1];
 If[print, Print[p1]];
 p0 = p1;, {i, 1, 3000}]
```

Out[109]=

```
{0.53539, 0.0691571}
```

In[113]:=

```
ListPlot[list, PlotStyle → {PointSize[0.005]},
 PlotRange → {{0, 1}, {0, 1}}, Axes → False, AspectRatio → 1]
```

Out[113]=