Higgs

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- **Higgs Phenomenon**
- **Higgs in the Standard Model**
- **Bounds on Higgs mass from theory**
  - **Unitarity**
  - **Landau pole**
  - **Perturbativity**
- **Bounds on Higgs mass from experiments**
- **Higgs Decays**
- **Higgs Production**

Abelian gauge theory

Consider the following Lagrangian

$$\mathcal{L}_{U(1)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma^\mu \mathcal{D}_\mu - m)\psi$$

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$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,, \qquad \mathcal{D}_\mu = \partial_\mu - igA_\mu \,.$$

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Two important observations:

- Gauge invariance forbids mass term for gauge fields. $M_A^2 A^\mu A_\mu$ term is not allowed
- But vector-like theories allows mass term for fermions because left handed fermions have the same charge as hight handed fermions.

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Under $U(1)$:

$$\psi_a(x) \to \psi'_a(x) = e^{i\hat{Q}\alpha(x)} \psi_a(x)$$

with $\hat{Q}$ being the charge operator such that $\hat{Q}\psi_a = Q_a \psi_a$. This means that Lagrangian $\mathcal{L}_{m_\psi}$ is invariant under $U(1)$ gauge transformation only if $Q_a - Q_b = 0$.

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- Gauge symmetry severely restricts mass terms for gauge field as well as fermionic fields.

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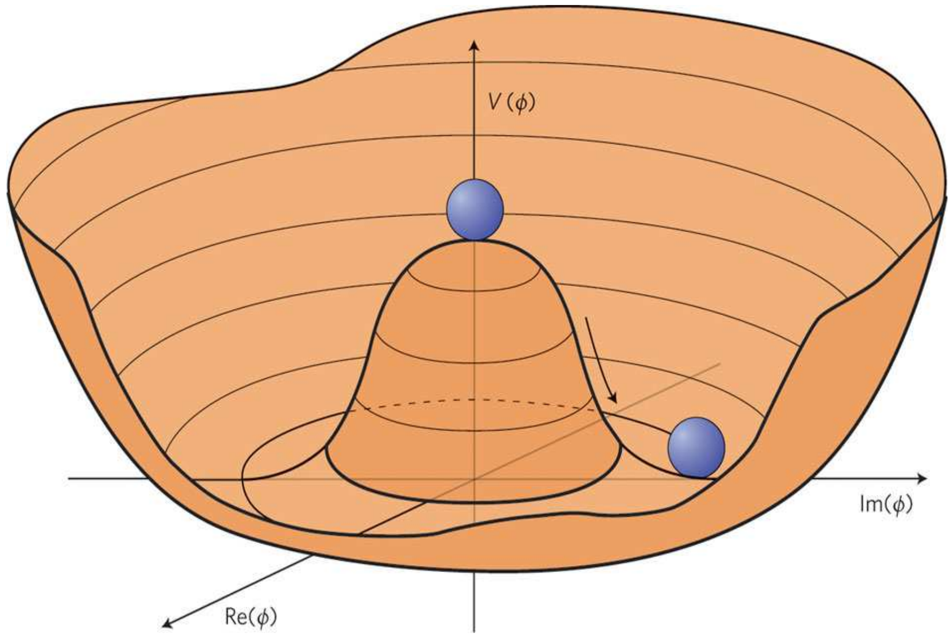
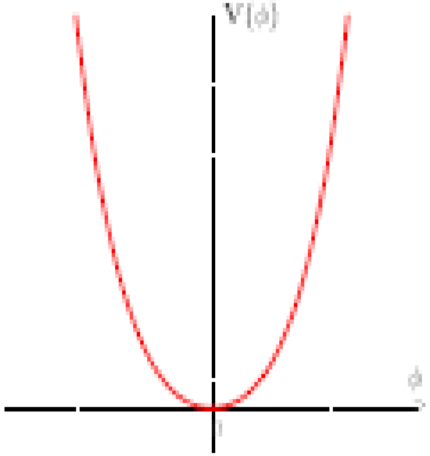
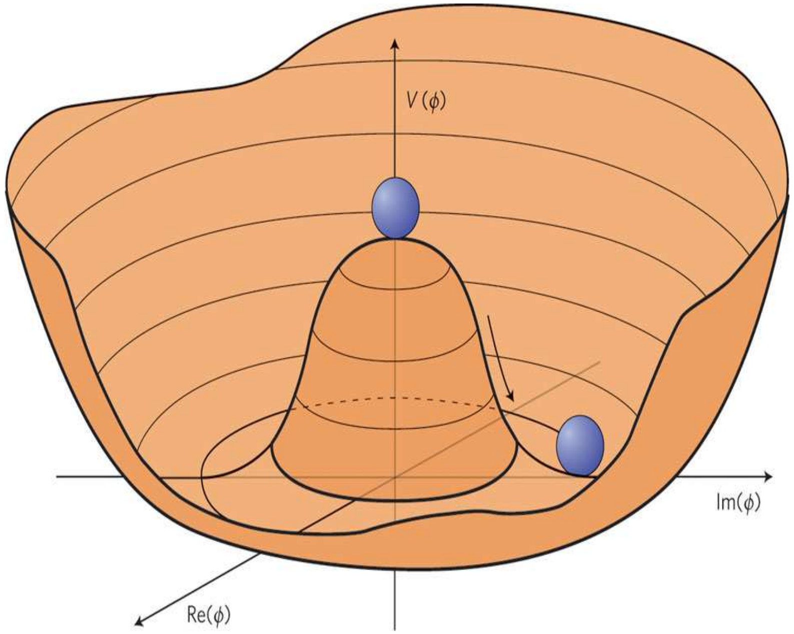


Spontaneous Symmetry Breaking (SSB)





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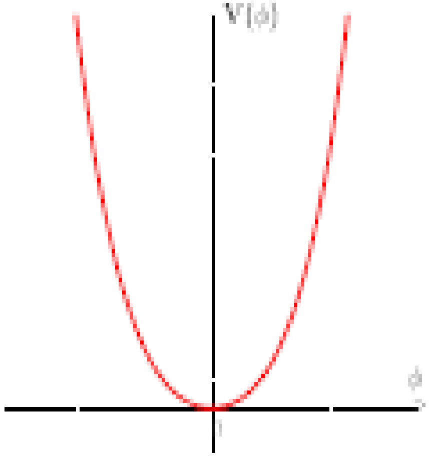

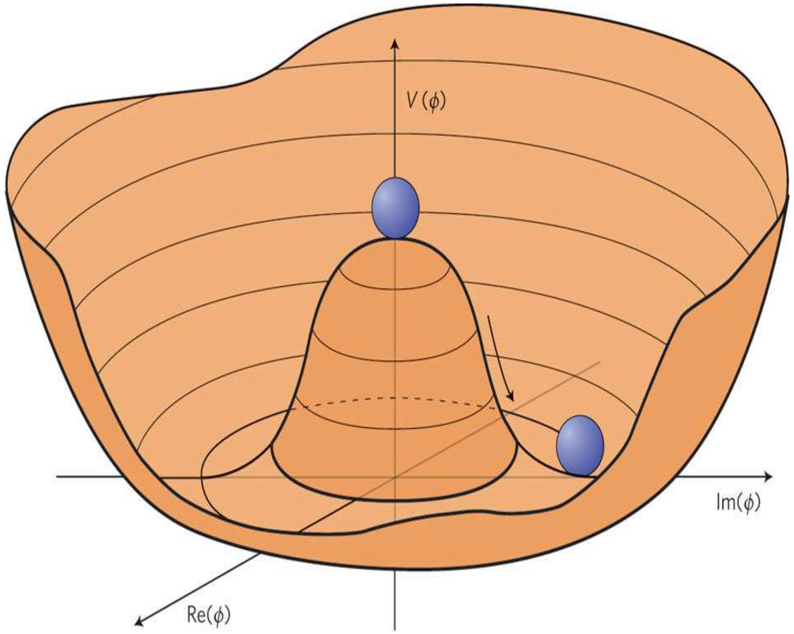

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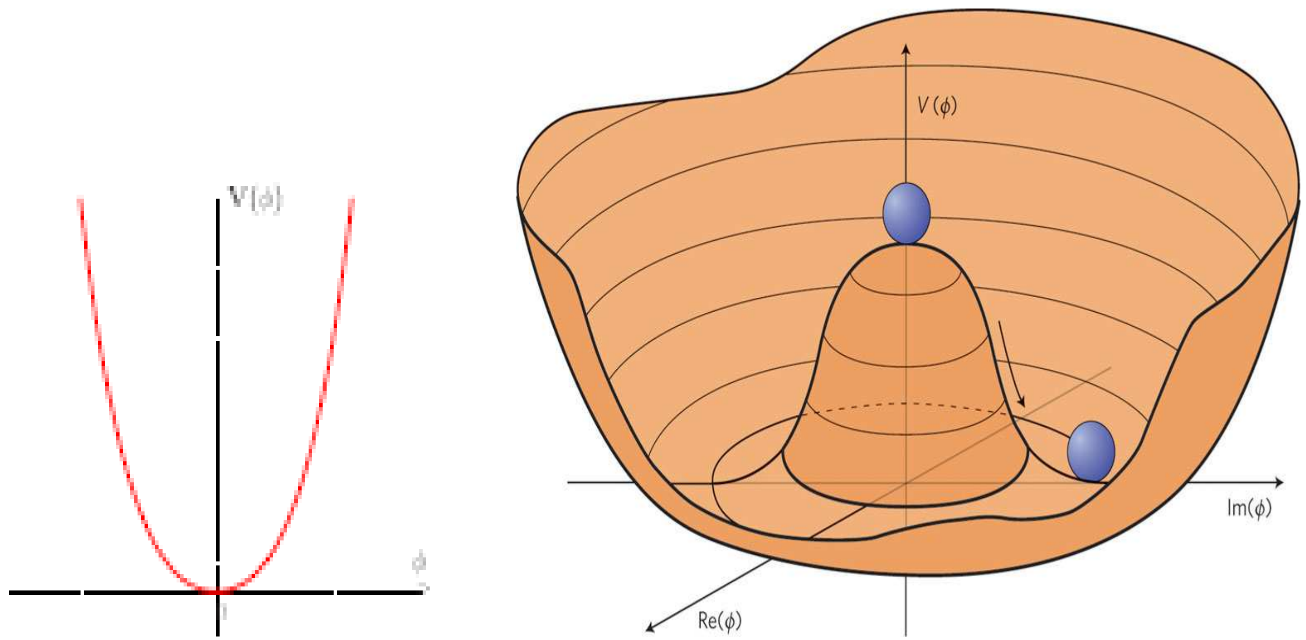

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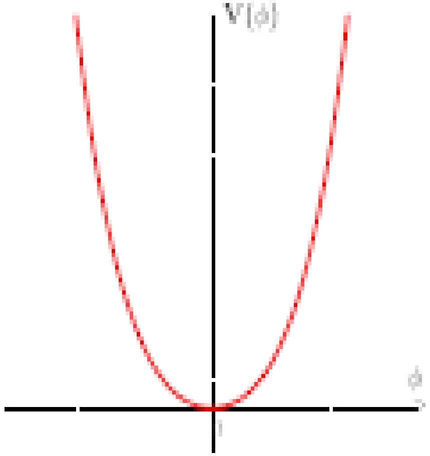

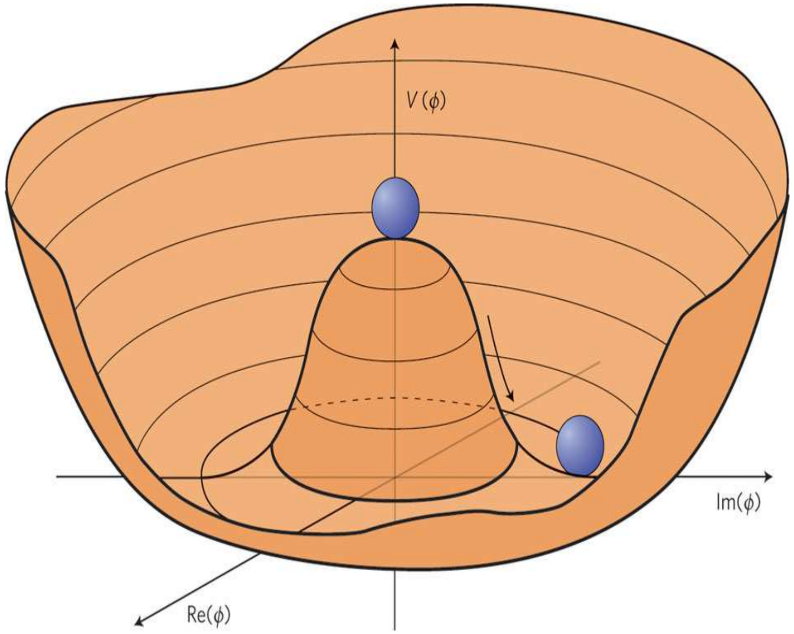

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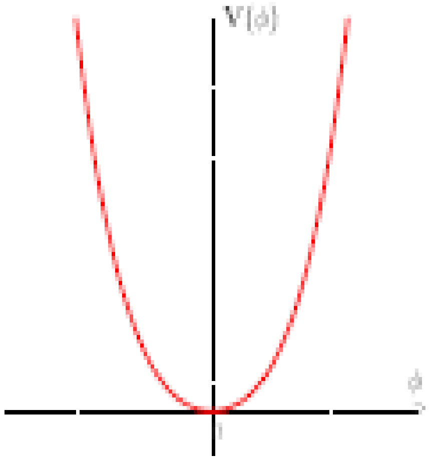

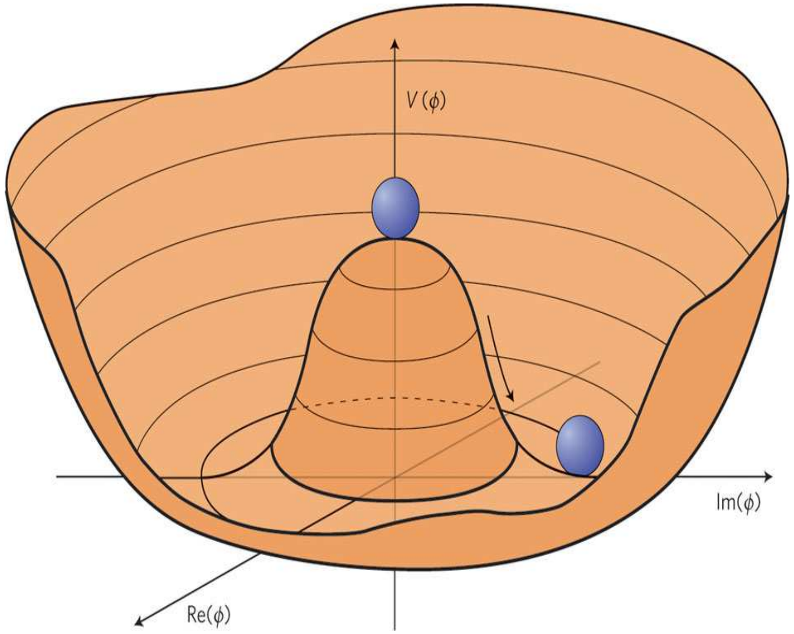

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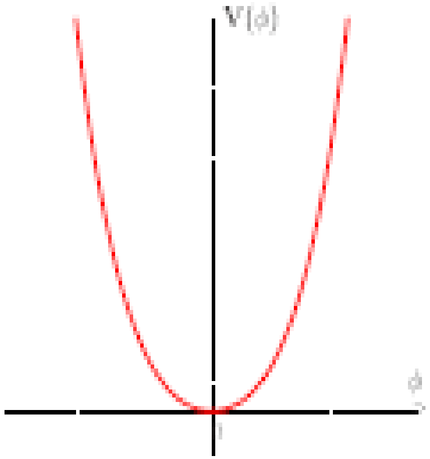

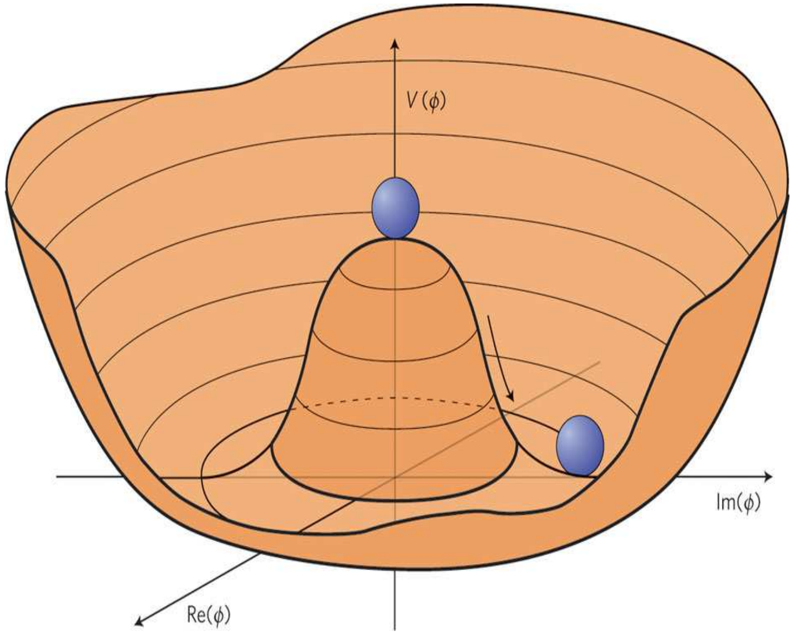

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♣ Massless modes are called Goldston bosons and the massive modes are called Higgs bosons.

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Pleasant surprise: Mass term for gauge fields

$$\boxed{\mathcal{L}_{M_A} = \frac{g^2v^2}{2}A_\mu A^\mu}$$

The gauge boson now becomes massive with the mass $M_A = gv$ (g: gauge coupling, v: vev)

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Peculiar term: ϕ_2 interacts with A_μ in a peculiar way

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$$\Phi(x) = \frac{1}{\sqrt{2}}\Big(v + h(x)\Big) \exp\Big(i\xi(x)/v\Big)$$

such that

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For small fluctuations,

$$\Phi(x) = \frac{1}{\sqrt{2}} (v + h(x) + i\xi(x)) + \mathcal{O}(h^2, \xi^2)$$

$h(x)$ and $\xi(x)$ will coincide with $\phi_1'(x)$ and $\phi_2'(x)$ respectively.

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where $F^U_{\mu\nu} = \partial_\mu A^U_\nu - \partial_\nu A^U_\mu$

$\xi(x)$ has disappeared but will reappear soon!.

Unitary gauge · · ·

$$\mathcal{L}_{A,h} = \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{4}F^U_{\mu\nu}F^{U\,\mu\nu} \quad \Longleftarrow \textbf{K.E} \quad \textbf{terms}$$

$$-\mu^2 h^2(x) + \frac{1}{2}g^2 v^2 A^U_\mu A^{U\,\mu} \quad \Longleftarrow \textbf{mass} \quad \textbf{terms}$$

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Unitary gauge ...

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$$\boxed{m_A^2 = g^2 v^2}, \qquad \boxed{m_h^2 = \mu^2 = 2\lambda v}$$

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- Massless guage fields have two transverse degrees of freedom while massive ones have two transverse and one longitudinal.
- The disappeared $\xi(x)$ field reappears as longitudinal degrees of freedom of massive gauge fields.

Fermion mass

Consider the following Yukawa term:

$$\mathcal{L}_Y = Y_{ab}\overline{\psi}_a\psi_b\Phi + h.c$$

with ϕ is charged with $Q_\Phi = Q_b - Q_a$. The above term is invariant under $U(1)$ gauge symmetry.

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$$\textbf{Vertex :} \qquad h(x)\overline{\psi}_a\psi_b \quad\rightarrow\quad i\frac{Y}{\sqrt{2}} = -i\frac{m_{\psi,ab}}{v}$$

Goldstone theorem

Consider a set of real fields Φ_i that transform according to some representation of the gauge symmetry group G that has n generators.

$$\Phi_i \to U_{ij}(\zeta(x))\Phi_i(x), \qquad U(\zeta(x)) = \exp\left(i\mathbf{T}\cdot\zeta(\mathbf{x})\right),$$

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$$\delta V = \frac{\partial V}{\partial \Phi_i}\delta\Phi_i = i\epsilon^a \frac{\partial V}{\partial \Phi_i} T^a_{ij}\Phi_j \quad = \quad 0$$

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Defferenting with respect to Φ_k

$$\frac{\partial^2 V}{\partial \Phi_k \partial \Phi_i} T^a_{ij}\Phi_j + \frac{\partial V}{\partial \Phi_i} T^a_{ik} = 0$$

Goldstone theorem

If V develops minima at $\Phi_j = v_j$, second term vanishes

$$\left.\frac{\partial^2 V}{\partial \Phi_k \partial \Phi_i}\right|_{\Phi_j = v_j} T^a_{ij} v_j \quad = \quad 0$$

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$$V(\Phi) = V(v_j) + \frac{1}{2} \left.\frac{\partial^2 V(\Phi)}{\partial \Phi_k \partial \Phi_i}\right|_{\Phi_j = v_j} (\Phi_k - v_j)(\Phi_i - v_j) + \cdots$$

The mass square matrix M^2_{ki}:

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Suppose G has a subgroup G' with n' generators which leaves the vacuum invariant:

$$\begin{aligned} T^b_{ij} v_j &= 0, \quad \text{for } b = 1, .., n' &\Longleftarrow \quad \text{unbroken generators} \\ T^c_{ij} v_j &\neq 0, \quad \text{for } c = n' + 1, ..., n &\Longleftarrow \quad \text{broken generators} \end{aligned}$$

- If T^a are linearly independent, it is clear that M^2 has $n - n'$ zero eigen values.
- Goldston theorem: spontaneous symmetry breaking implies existence of massless spinless particle. The number of spontaneously broken generators = number of massless fields.
- These spinless,massless particles are called Goldstone bosons.

Gauge fields in the Standard Model

$SU(2)_L \times U(1)_Y$ invariant gauge field Lagrangian:

$$\mathcal{L}_1 = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

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$$\begin{aligned} \frac{\tau \cdot \mathbf{A}_\mu}{2} &\rightarrow U(\theta)\left(\frac{\tau \cdot \mathbf{A}_\mu}{2}\right)U^{-1}(\theta) - \frac{i}{g}\Big(\partial_\mu U(\theta)\Big)U^{-1}(\theta) \\ B_\mu &\rightarrow B_\mu - \frac{i}{g'}\Big(\partial_\mu U(\theta)\Big)U^{-1}(\theta) \end{aligned}$$

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- for $SU(2)_L$,

$$U(\theta) = \exp(-i \frac{\tau}{2} \cdot \theta(\mathrm{x})) \quad \text{with} \quad \tau^i (i = 1, 2, 3) \quad \text{are pauli matrices}$$

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$$\mathcal{L}_2 \quad = \quad \overline{\psi} i \gamma_\mu \mathcal{D}^\mu \psi \qquad \psi \quad : \quad \left\{ L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, e_R, Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R \right\}$$

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Since $\hat{\mathbf{T}}\, L = \frac{\tau}{2}\, L$, $\quad \frac{\hat{\mathbf{Y}}}{2}\, L = -\frac{1}{2}\, L$, $\quad \hat{\mathbf{T}}\, e_R = 0$, $\quad \frac{\hat{\mathbf{Y}}}{2}\, e_R = -e_R$,

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$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}, \qquad SU(2) \ \ \text{doublet}, \qquad Y(\Phi) = 1$$

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- The $SU(2)_L \times U(1)_Y$ invariant Yukawa interaction Lagrangian is given by

$$\mathcal{L}_4 = Y_e \overline{L} \Phi e_R + Y_u \overline{Q}_L \tilde{\Phi} u_R + Y_d \overline{Q}_L \Phi d_R + h.c$$

where $\tilde{\Phi} = i\tau_2 \Phi^*$ with $Y(\tilde{\Phi}) = -1$

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For $\mu^2 > 0$, the vacuum of this theory is spontaneously broken and the complex scalar field acquires vacuum expectation value:

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If we parametrize $\Phi(x)$ in terms of four real fields ($\Phi(x)\ :\ h(x), \zeta^1(x), \zeta^2(x), \zeta^3(x)$) as

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$$\Phi^U = U(\zeta)\Phi = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} = \frac{v+h(x)}{\sqrt{2}}\ \chi \qquad \chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- $\zeta_i(x)$ are called Goldstone bosons (massless,spinless)
- $h(x)$ is called the Higgs boson.

Fermion and Gauge fields in the Unitary gauge

The fermion fields in the unitary gauge are given by

$$\begin{aligned} \boldsymbol{L^U} &= \boldsymbol{U(\zeta)L}, \qquad \boldsymbol{e_R^U = e_R} \\ \boldsymbol{Q_L^U} &= \boldsymbol{U(\zeta)Q_L}, \qquad \boldsymbol{u_R^U = u_R}, \qquad \boldsymbol{d_R^U = d_R} \end{aligned}$$

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$$\begin{aligned} F_{\mu\nu}^{Ui} &= \partial_\mu A_\nu^{Ui} - \partial_\nu A_\mu^{Ui} + g\epsilon^{ijk} A_\mu^{Uj} A_\nu^{Uk} \\ G_{\mu\nu}^U &= \partial_\mu B_\nu^U - \partial_\nu B_\mu^U \end{aligned}$$

Masses of the Higgs boson and the fermions

Consider the scalar field part of the Lagrangian

$$\mathcal{L}_3 \quad = \quad (\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}^\mu \Phi - V(\Phi)$$

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The fermions masses and their interaction with $h(x)$

$$\boxed{m_i = \frac{Y_i v}{\sqrt{2}}}, \qquad i = e, u, d, \qquad h(x)\overline{Q}Q \Longrightarrow i\,\frac{m_Q}{v} \qquad Q = \tau, b, t$$

Masses of $W^{\pm}$ bosons

$$|\mathcal{D}_\mu \Phi|^2 \quad \Longrightarrow \quad \frac{v^2}{2}\left(1+\frac{h(x)}{v}\right)^2 \chi^\dagger \left(\frac{g}{2}\tau \cdot A^U_\mu + \frac{g'}{2}B^U_\mu\right)\left(\frac{g}{2}\tau \cdot A^{U\mu} + \frac{g'}{2}B^{U\mu}\right)\chi$$

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where the mass of $W^{\pm}$ boson is given by

$$\boxed{\frac{g^2v^2}{4} \equiv M_W^2}$$

The vertices are

$$h(x)W^+_\mu W^{-\mu} \implies 2i\frac{M_W^2}{v}g_{\mu\nu}, \qquad h(x)h(x)W^+_\mu W^{-\mu} \implies 2i\frac{M_W^2}{v^2}g_{\mu\nu}$$

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$$\frac{v^2}{8}\left(1+\frac{h(x)}{v}\right)^2 (gA_\mu^{U3} - g' B_\mu^U)^2 = \frac{v^2}{8}\left(1+\frac{h(x)}{v}\right)^2 \begin{pmatrix} A_\mu^{U3} & B_\mu^U \end{pmatrix} \begin{pmatrix} g^2 & -gg' \\ -gg' & g^2 \end{pmatrix} \begin{pmatrix} A^{U3\mu} \\ B^{U\mu} \end{pmatrix}$$

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Counting number of degrees of freedom

- $SU(2)_L \times U(1)_Y$ gives 4 massless gauge fields ($A^i_\mu, B_\mu,\ \ i = 1, 2, 3$).
- Couplex scalar doublet had four real scalar fields ($\zeta^i, h(x), i = 1, 2, 3$).
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- Charged and neutral current interactions gives:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \implies v = 2^{-\frac{1}{4}} G_F^{-\frac{1}{2}} \approx 246 \text{ GeV}$$
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Bound on Higgs mass from Unitarity

- Consider $W_L^+ W_L^- \to W_L^+ W_L^-$ scattering process. The amplitude of the process in terms of spin-l partial wave is

$$\mathcal{M} = \sum_{l=0}^{\infty} \mathcal{M}_l, \qquad \text{where} \quad \mathcal{M}_l = 16\pi(2l+1)P_l(\cos\theta)a_l$$

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$$|Re(a_l)| \leq \frac{1}{2}$$

- When $s \gg M_W^2, m_h^2, \qquad a_0 \to -m_h^2/(8\pi v^2)$

$$\left| -\frac{m_h^2}{8\pi v^2} \right| \leq \frac{1}{2} \quad \Longrightarrow \quad m_h < 2v\sqrt{\pi} = 870GeV$$

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$$\left| -\frac{m_h^2}{8\pi v^2} \right| \leq \frac{1}{2} \quad \Longrightarrow \quad m_h < 2v\sqrt{\pi} = 870GeV$$

- Combined analysis with similar longitudinal scattering processes gives the upper bound on higgs mass $m_h < 710$ GeV.

Bound on Higgs mass from Landau pole

- Finiteness of λ coupling upto a cut off scale of the theory can give useful information on higgs mass through Renormalisation group equation. Droping gauge and Yukawa contributions,

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- If $\Lambda_P = 10^{19}$ GeV, the higgs has to be light $m_h \leq 145$ GeV.
- If $\Lambda_P = 10^3$ GeV, the higgs has to be heavy $m_h \leq 750$ GeV.

Bound on the Higgs mass from Perturbativity

- Including gauge and Yukawa couplings, the RG equation for λ is given by

$$\mu_R^2 \frac{d\lambda}{d\mu_R^2} = \frac{1}{16\pi^2}\left(12\lambda^2 + 12\frac{m_t^2}{v^2}\lambda - 12\frac{m_t^4}{v^4} - \frac{3}{2}\lambda(3g'^2 + g^2) + \frac{3}{16}(2g'^4 + (g'^2 + g^2)^2)\right)$$

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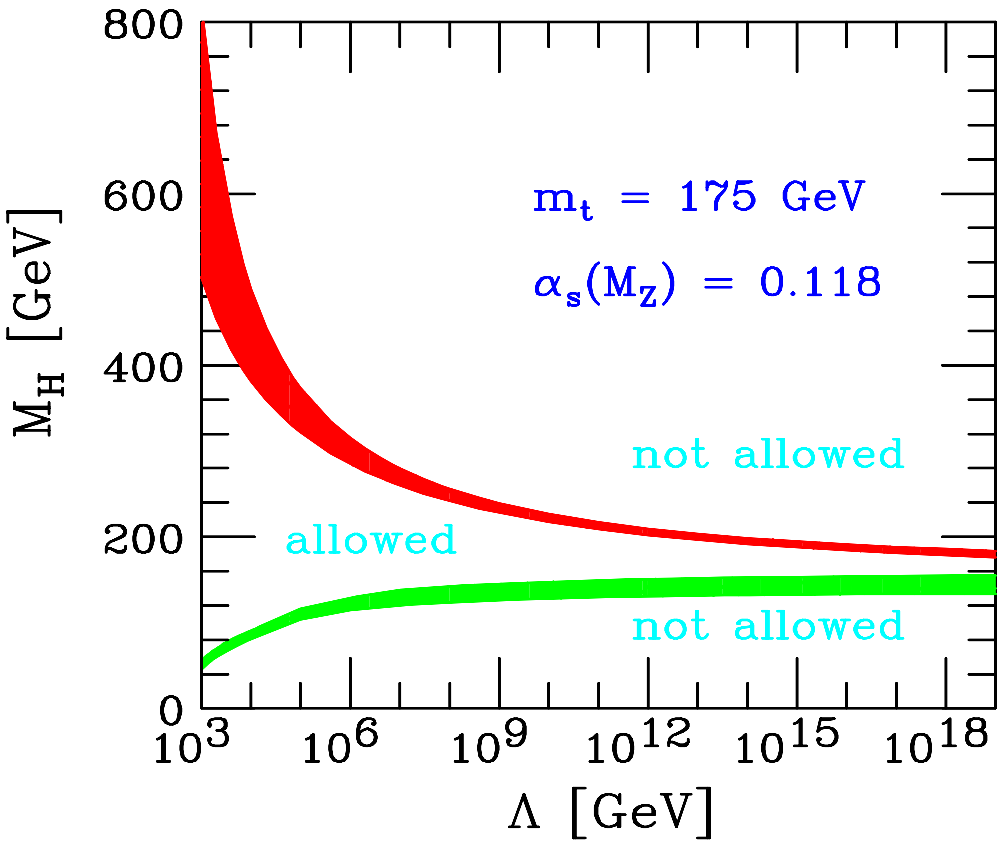
- Choosing $Q = \Lambda_S$,

$$\Lambda_S \approx 10^3 GeV \quad \Longrightarrow \quad m_h \geq 70 GeV$$
$$\Lambda_S \approx 10^{16} GeV \quad \Longrightarrow \quad m_h \geq 130 GeV$$

Bounds on the mass of Higgs boson



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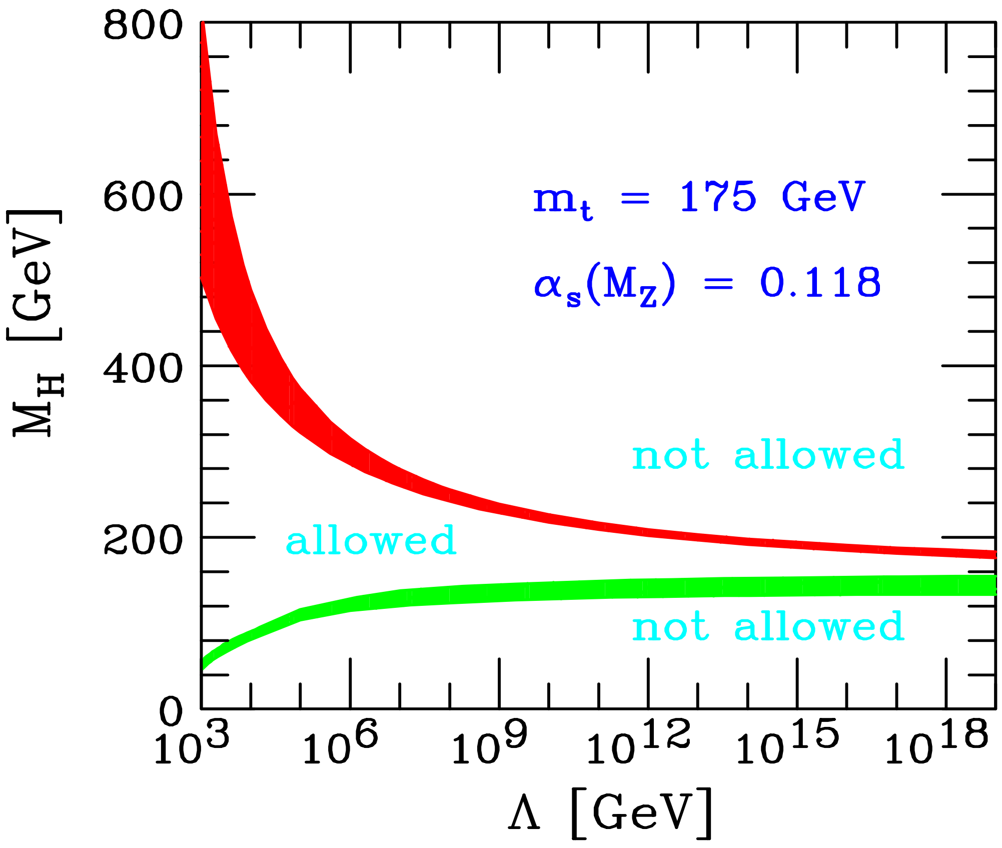

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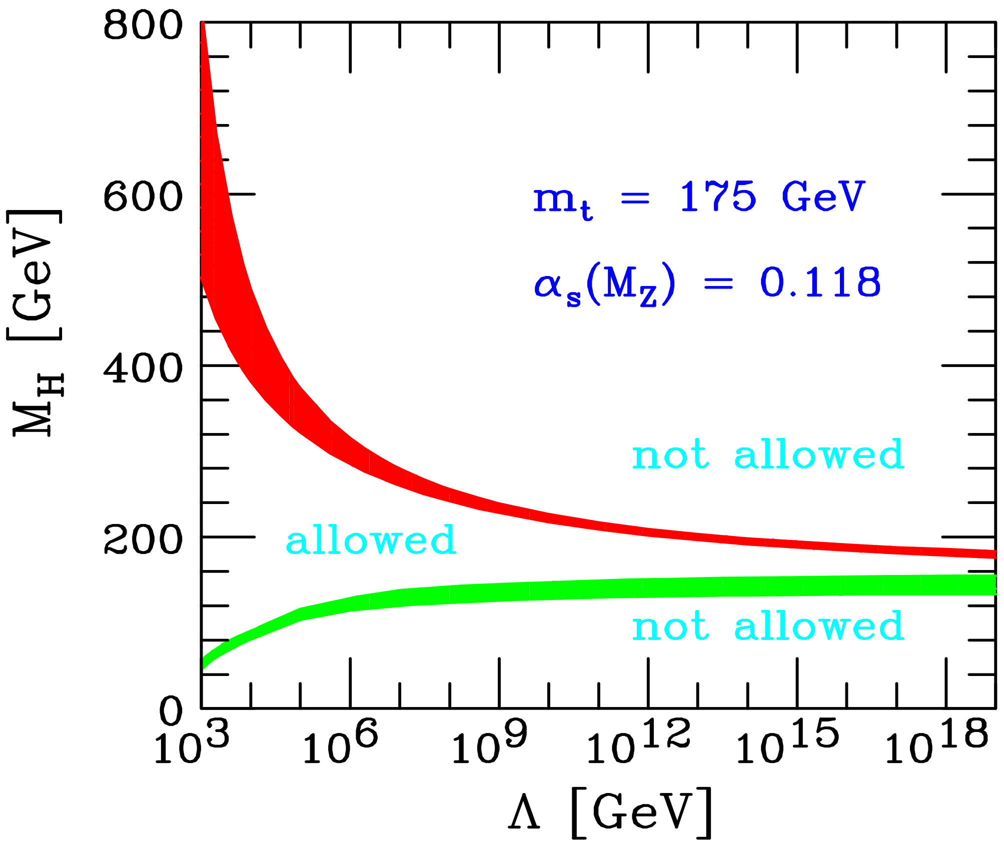

- The upper curve results from demanding perturbativity of the coupling $\boldsymbol{\lambda}$ upto the scale $\boldsymbol{\Lambda}$
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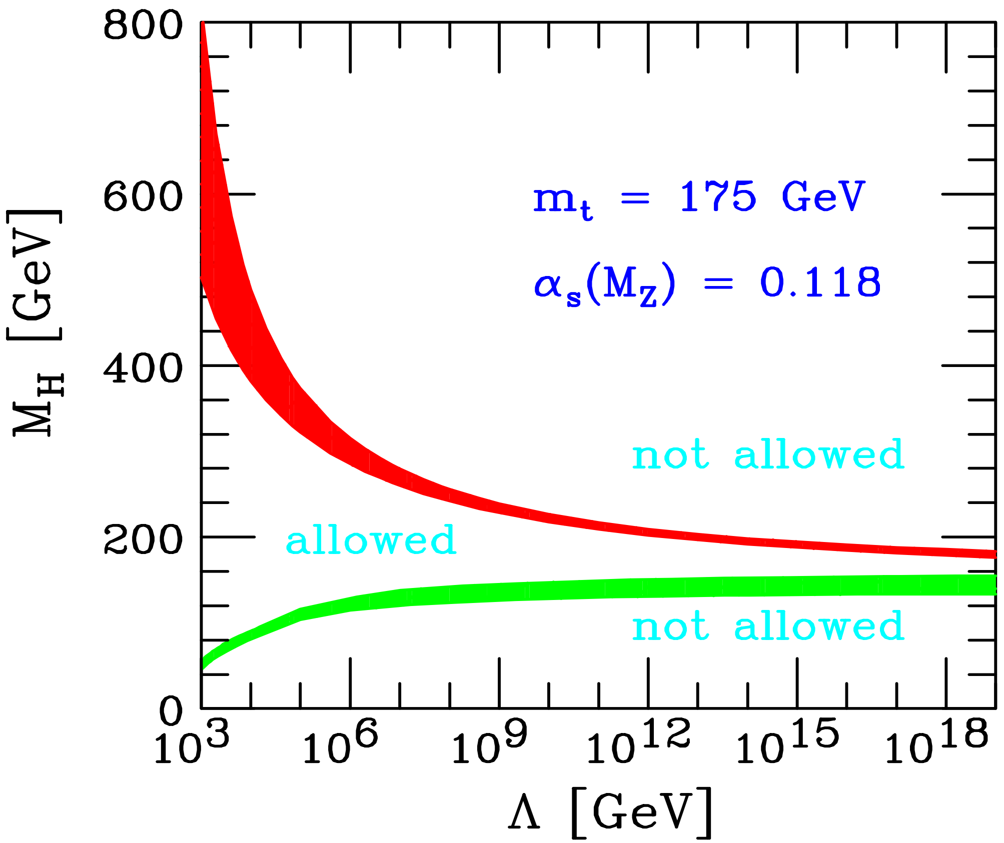
Bounds on the mass of Higgs boson



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Spread in the lines is due to theory uncertainities.

Higgs Mass

[Summer 2004, LEPEWWG]

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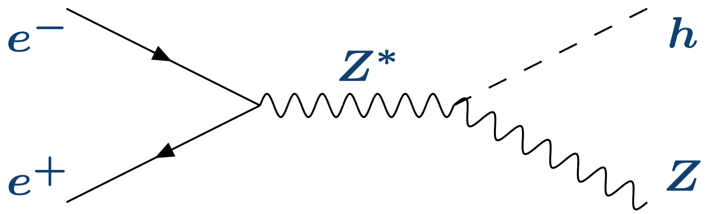
Direct:



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Higgs Mass

[Summer 2004, LEPEWWG]

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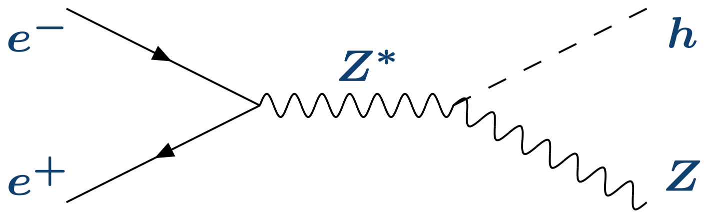

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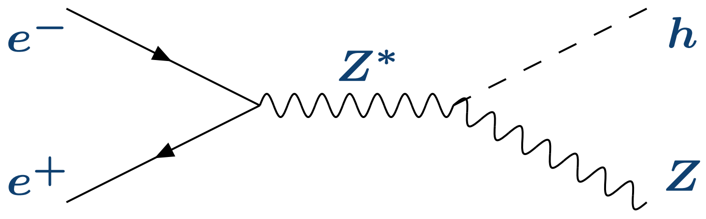

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Direct:

- LEP is a e^+e^- collider with $\sqrt{s} = 209$ GeV
- Primary search mode $e^+e^- \rightarrow hZ$
- On-shell higgs can be produced if the mass of the higgs is greater than $\sqrt{s} - M_Z = 118$ GeV
- Low statistics and insufficient energy available gives the lower bound $m_h > 114.4\ GeV$

Higgs Mass

[Summer 2004, LEPEWWG]

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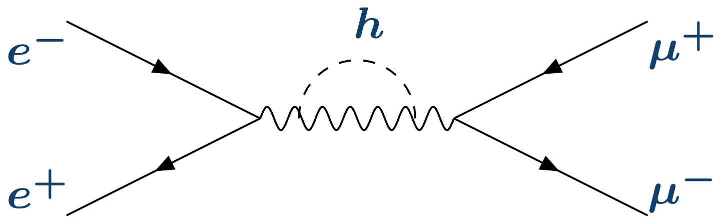

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[Summer 2004, LEPEWWG]

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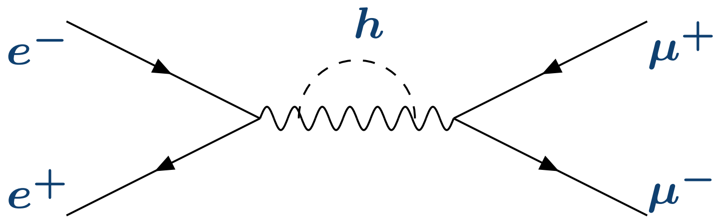

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- Higgs can contribute to many electroweak observables that are measured at LEP
- They can enter in W and Z self energies at one loop level.
- The mass of the higgs appears through its propagator and kinematics
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Higgs Mass

[Summer 2004, LEPEWWG]

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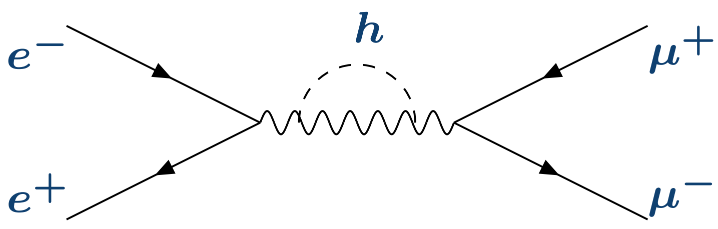

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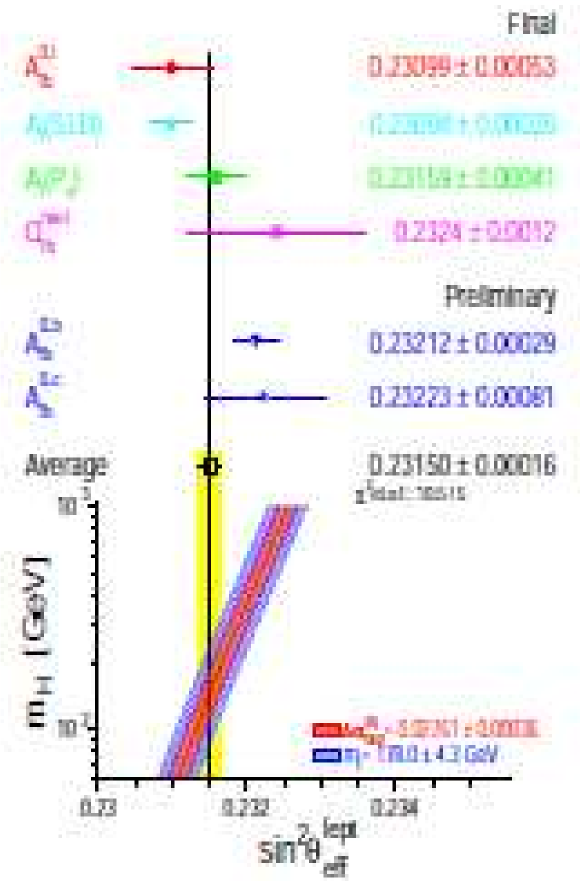
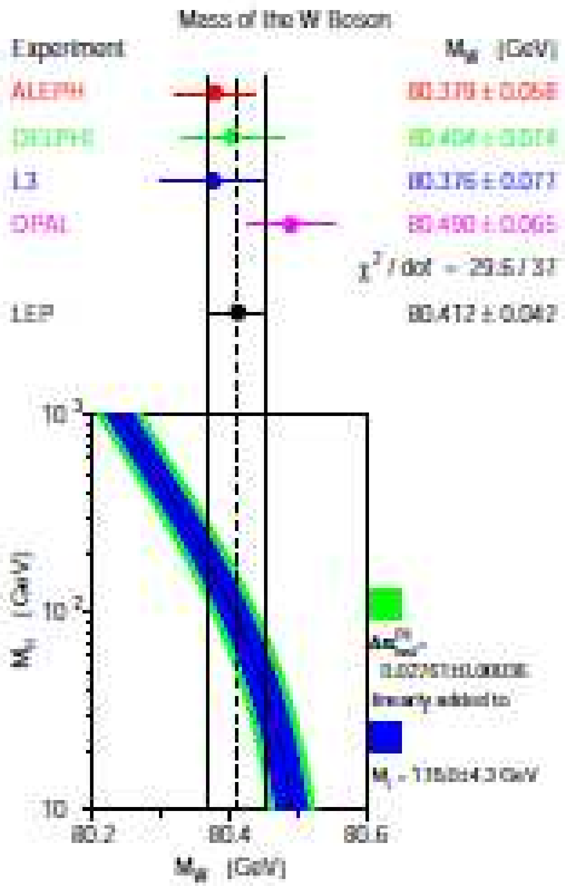
$W^{\pm}$ mass and $\sin^2\theta_W$





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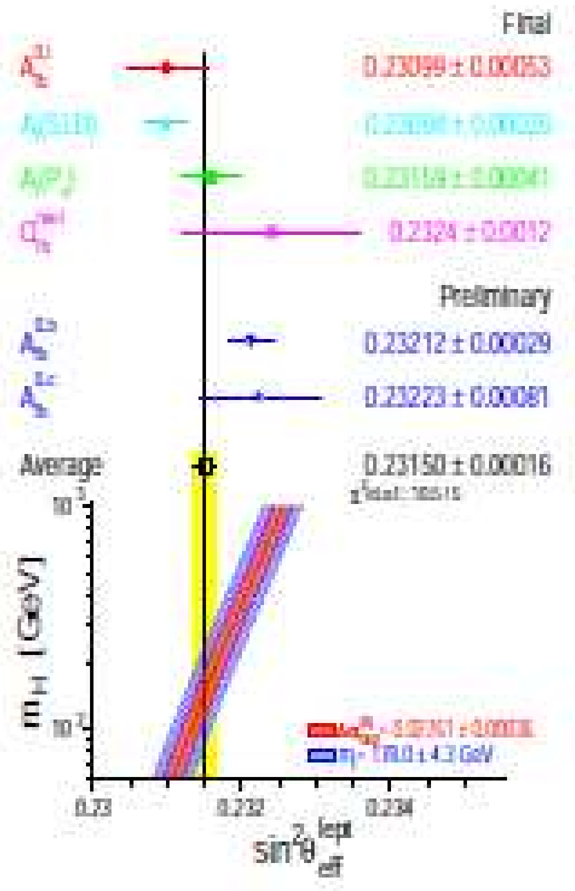

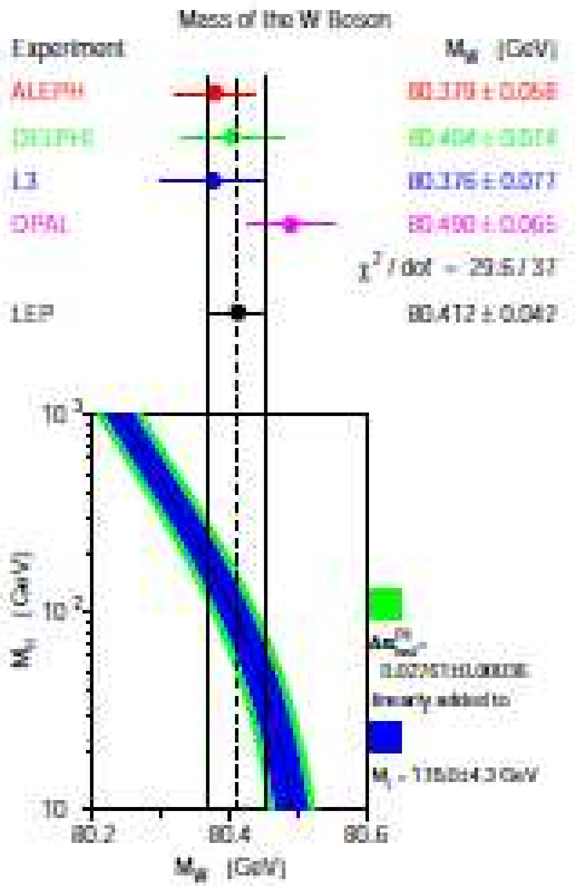

- Precise meausurement of mass of the W boson
- $\sin^2\theta_W$ from Forward back asymmetry and charge asymmetries
- Lower bound $m_h < 260\ GeV$ ($95\%\ CL$)

Higgs Mass

[Summer 2004, LEPEWWG]

Higgs Mass

[Summer 2004, LEPEWWG]

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01646
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1482
R_b	0.21629 ± 0.00066	0.21579
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$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0743
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A_c	0.670 ± 0.027	0.668
$A_l(SLD)$	0.1513 ± 0.0021	0.1482
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.399 ± 0.023	80.378
Γ_W [GeV]	2.085 ± 0.042	2.092
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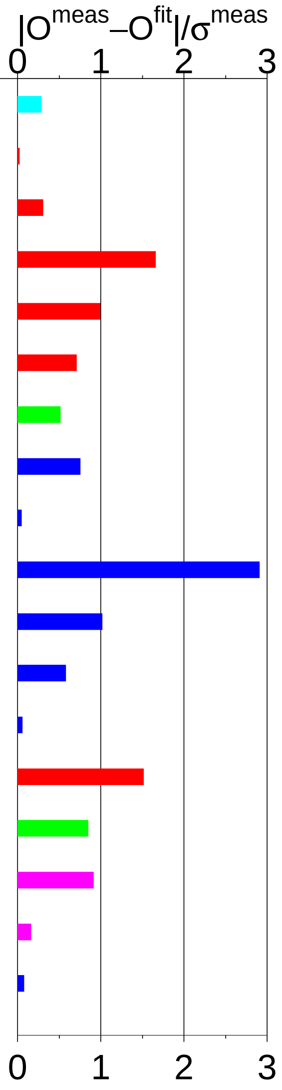

July 2011

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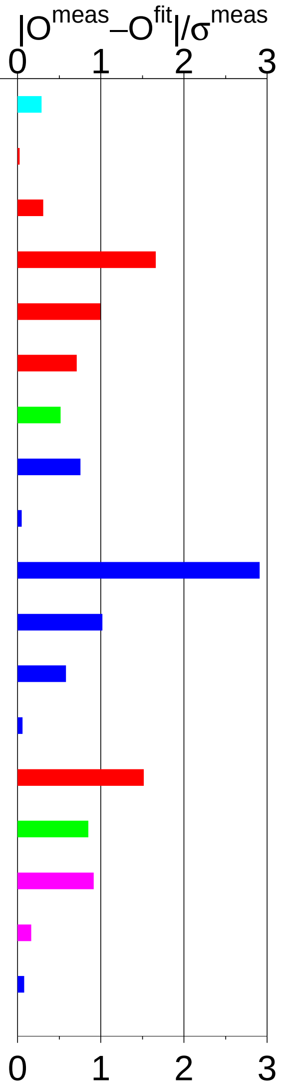

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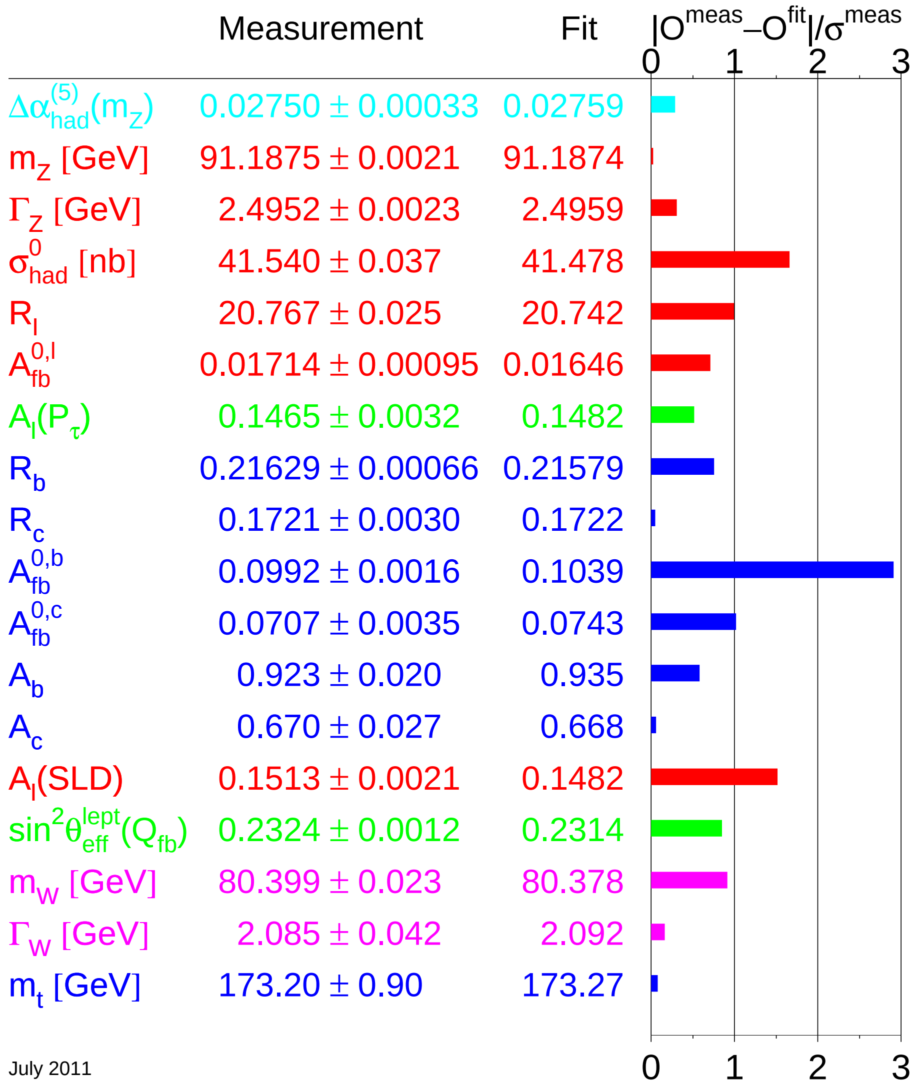

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$$\chi^2 = \sum_i \frac{(\mathcal{O}_i^{th}(m_h, x) - \mathcal{O}_i^{expt})^2}{(\Delta\mathcal{O}_i^{expt})^2}$$

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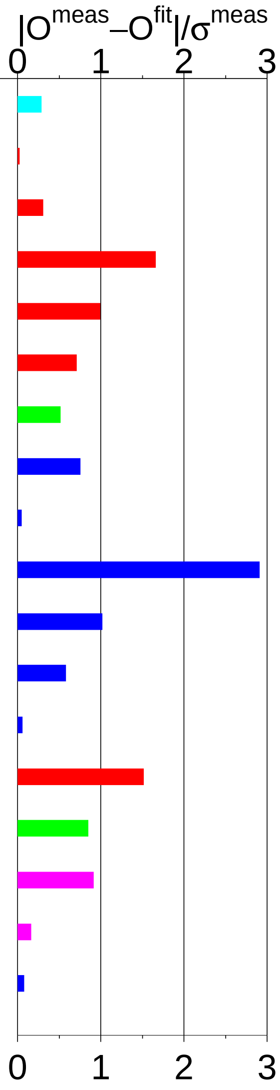

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Higgs Mass

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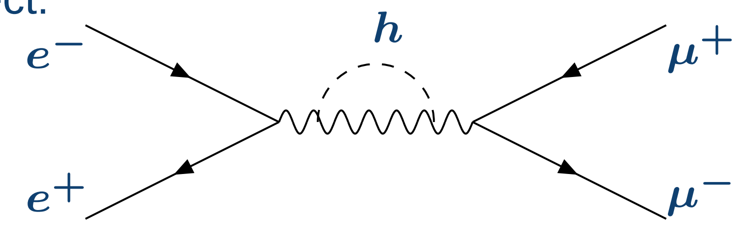

$$m_h < 260\ GeV\ (\ 95\%\ CL)$$

$$\Delta\chi^2(m_h, x) = \chi^2(m_h, x) - \chi^2_{min}$$

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Higgs Mass

[Summer 2004, LEPEWWG]

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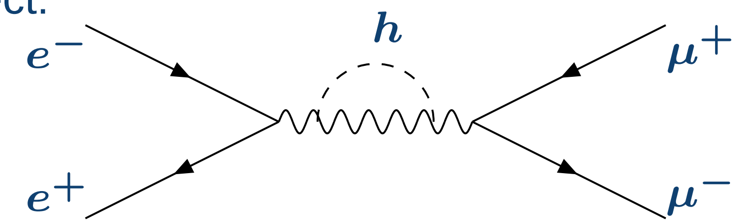

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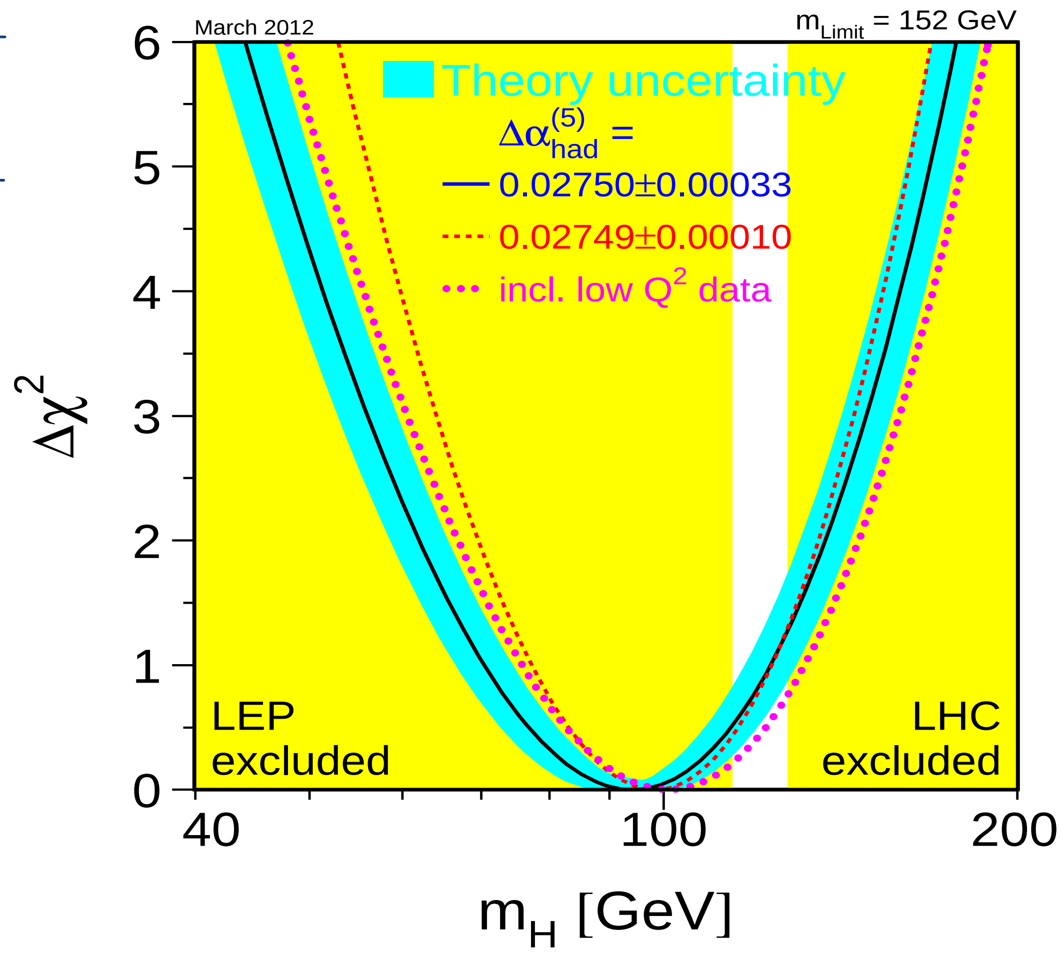

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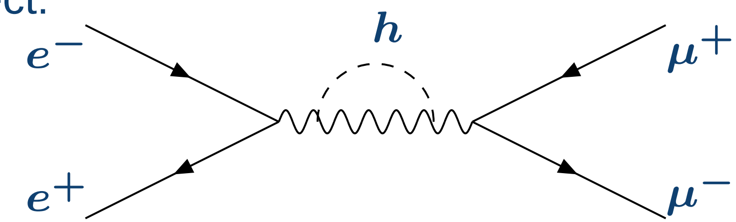

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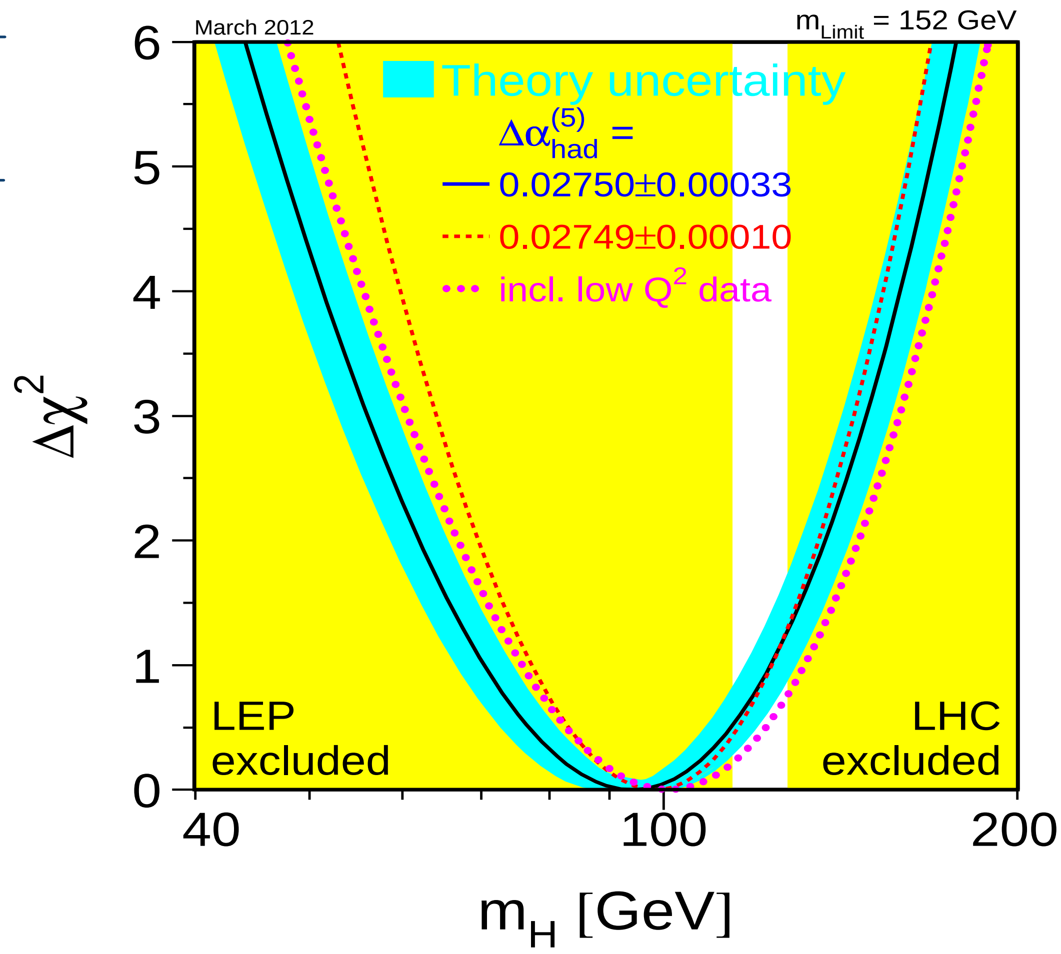

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$114.4 < m_h < 260$ GeV at 95% CL.