

CTEQ Summer School 2013

Heavy Quarks

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CENTER



Administration Details

- If problems with wireless see me
- Note: Password must be 8-13 chars, must contain special character (Not all special characters are allowed)
- Recitation (19:30-21:00)
- Nightcap (21:00-22:30)



Lecture I

- History, motivation, what are heavy quarks?
- Review of Standard Model
- CKM matrix
- Effective Field Theory
- Flavor Changing Neutral Currents
- Minimal Flavor Violation

Lecture 2

- CP violation, more on CKM
- Tops



1200

photon

electron



proton

Didn't even have Bohr's model yet



Eightfold Way (1961)









The Quark Model (1964)



u, d, s

Three quarks for Muster Mark! Sure he has not got much of a bark And sure any he has it's all beside the mark. —James Joyce, *Finnegans Wake*

Bookkeeping device? Problems...





u, d, s

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The Quark Model (1964)

 $u\overline{u}+dd$ $u\overline{u}+d\overline{d}-2s\overline{s}$

us

ds

ud

 $d\overline{s}$

ūs

ūa

0 -

-1-



partor

proton

The Parton Model (1969)







Explained Bjorken Scaling



GIM Mechanism (1970)

$$\sum_{i=u,c} V_{is} V_{id}^* = 0$$

Suppression of FCNC \rightarrow Charm quark predicted





CKM matrix (1973)

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Need CP violation \rightarrow Bottom and top quark predicted



We have Charm (1974)!



In 1975 the τ was discovered and led to the search for other 3rdgeneration particles. In 1977 the Upsilon (a $b\bar{b}$ bound state) was observed at the Fermilab Tevatron.

We have Bottom (1977)!

We must have Top too!







18 long years later...



We have Top (1995)!





Why should we study heavy quarks?

• Measure and tests of SM parameters

• Search for new physics

Important to understand for backgrounds

• Can use large mass to our advantage

Why? (expanded)

Lots of interesting B physics

- Theory
 - Top loops no GIM & CKM suppressed
 - Large and clean CPV possible
 - Some hadronic physics is understandable model independent $m_b \gg \Lambda_{\rm QCD}$
- Experiment
 - Clean sources $\Upsilon(4S)$
 - Long B lifetime $\Delta m/\Gamma \sim \mathcal{O}(1)$

Why? (expanded)

• Low energy point of view (EFT)

At low energies anything that changes flavor is a local interaction (SM or NP)



Measure operator coefficients to constrain SM or see NP

Why? (expanded)

• New physics flavor problem

TeV scale (hierarchy problem) \ll flavor and CPV scale

Write down operators with ${\cal O}(1)$ coefficients



TeV-scale NP models typically have new sources of flavor and CP violation





Quark matter content (and Higgs)

$$q_{L} = \begin{pmatrix} u \\ d \end{pmatrix}_{L}, \begin{pmatrix} c \\ s \end{pmatrix}_{L}, \begin{pmatrix} t \\ b \end{pmatrix}_{L}, u_{R}, d_{R}; H: \langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$(3,2)_{1/6} (3,2)_{1/6} (3,1)_{1/6} (3,1)_{1/3} (3,1)_{1/3} (1,2)_{1/2}$$

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$$(3,2)_{1/6} (3,2)_{1/6} (3,1)_{2/3} (3,1)_{-1/3} (1,2)_{-1/2} (1$$

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$$(3,2)_{1/6} (3,2)_{1/6} (3,1)_{2/3} (3,1)_{-1/3} (1,2)_{-1/2} (1$$

Keep track of breaking using spurions $\tilde{H}\bar{u}_R\lambda^u q_L \to \tilde{H}\bar{u}_R U_u^{\dagger}\lambda^u U_q q_L \text{ so } \lambda^u \to U_u\lambda^u U_q^{\dagger}$

Quark matter content (and Higgs)

$$q_{L} = \begin{pmatrix} u \\ d \end{pmatrix}_{L}, \begin{pmatrix} c \\ s \end{pmatrix}_{L}, \begin{pmatrix} t \\ b \end{pmatrix}_{L}, u_{R}, u_{R}, d_{R}; H: \langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

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Keep track of breaking using spurions $\tilde{H}\bar{u}_R\lambda^u q_L \rightarrow \tilde{H}\bar{u}_R U_u^{\dagger}\lambda^u U_q q_L$ so $\lambda^u \rightarrow U_u\lambda^u U_q^{\dagger}$ $\bar{u}_R\lambda^u q_L \rightarrow \bar{u}_R U_u^{\dagger} (U_u\lambda^u U_q^{\dagger}) U_q q_L = \bar{u}_R\lambda^u q_L$

Diagonalize

$$u_{L} \to V_{u_{L}} u_{L}, \ d_{L} \to V_{d_{L}} d_{L}, \ u_{R} \to V_{u_{R}} u_{R}, \ d_{R} \to V_{d_{R}} d_{R}$$
$$V_{u_{R}}^{\dagger} \lambda^{u} V_{u_{L}} = \lambda^{u'}, \ V_{d_{R}}^{\dagger} \lambda^{d} V_{d_{L}} = \lambda^{d'}$$
$$\overbrace{\text{Diagonal, real}}$$

Gives diagonal masses, leaves everything else unchanged except

$$\bar{q}_L \left(\frac{1}{2}g_2 \sigma^a W^a\right) q_L \to \bar{u}_L \frac{1}{\sqrt{2}} g_2 V_{u_L}^{\dagger} V_{d_L} W^+ d_L + \text{h.c.}$$

 $V_{CKM} = V_{u_L}^{\dagger} V_{d_L}$

CKM matrix

Mass and flavor eigenstates do not line up $N \times N$ unitary matrix has N^2 real parameters Rotate relative phases of quarks reduces to $(N-1)^2$ For 3 generations: 4 parameters = 3 angles, 1 phase

Wolfenstein parameterization: shows hierarchy

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

 $\lambda\approx 0.22$ while $A,\rho,\eta\sim 1$

CKM matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

• One complex phase \rightarrow CP violation

$$\bar{u}_L \gamma^\mu d_L \xrightarrow{CP} \bar{d}_L \gamma_\mu u_L, W^{+\mu} \xrightarrow{CP} W^-_\mu \Longrightarrow V^\dagger = V$$

if CP conserved

- Precise measurement needed to
 - test Standard Model
 - constrain new physics

Unitarity Triangles

Unitarity puts constraints on CKM matrix

$$\sum_{k} V_{ik} V_{jk}^* = \sum_{k} V_{ki} V_{kj}^* = \delta_{ij}$$

Vanishing of product of first and third columns:

 $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \text{ scales as } A\lambda^3 + A\lambda^3 + A\lambda^3$

 (ρ,η)

 $\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$

(0.0)

 α

 $\frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*}$

(1.0)

Represent graphically as a triangle

Total of six triangles

Kaon: $0 = V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* \sim \lambda + \lambda + A^2\lambda^5$

Unitarity Triangles



• Angles $\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \quad \alpha = \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad \gamma = \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$

Invariant under phase transformation of quarks \rightarrow physical

• Area

$$\begin{aligned} \operatorname{area} &= -\frac{1}{2} \operatorname{Im} \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} = -\frac{1}{2} \frac{1}{|V_{cd} V_{cb}^*|^2} \operatorname{Im} (V_{ud} V_{ub}^* V_{cd}^* V_{cb}) \\ &\swarrow \end{aligned} \\ J &= \operatorname{Im} (V_{ud} V_{ub}^* V_{cd}^* V_{cb}) \quad \text{Jarlskog parameter} \end{aligned}$$

 $\operatorname{Im}(V_{ij}V_{kl}V_{il}^*V_{kj}^*) = J(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}) \text{ common area of all triangles}$

Normalized area $\frac{J}{2 \text{ largest} |V_{ij}V_{kl}^*|^2}$

Previous Unitarity Triangle Bounds



Unitarity Triangle Today



Unitarity Triangle Today





An Effective Field Theory is an approximation to the true underlying theory, with enough in it to describe the physics of interest.

> i.e., true theory = Standard Model? EFT = depends on question

Particularly useful when there are multiple well-separated scales

Effective Field Theory

- When there are well-separated scales, can find small dimensionless numbers
- Goal is to try to expand in one of these small numbers
- Equivalently, shrink large energies (small distances) to a point

-Think multipole expansion

• In Field Theory, remove heavy d.o.f.

Reep light (long wavelength) modes

-Effects turn into coefficients

What is needed?

The "light" degrees of freedom

If EFT contains correct d.o.f., get the IR correct
If missing d.o.f., get problems

Don't know this



(high energies)

Extra particles whose propagation not relevant for low energies

EFT gets here down

Want to describe



(low energies)



If $m_Q \to \infty$ just sits in center as color source



If $m_Q \to \infty$ just sits in center as color source For finite mass, momentum will be order $\Lambda_{\rm QCD}$ Heavy Quark Effective Theory $m_Q \gg \Lambda_{\rm QCD}$

First step is to decompose $p^{\mu} = m_Q v^{\mu} + k^{\mu}$

Break quark field into large and small components

$$h_v(x) = e^{im_Q v \cdot x} \frac{1+\psi}{2} \psi \quad H_v(x) = e^{im_Q v \cdot x} \frac{1-\psi}{2} \psi$$
$$\psi = e^{-im_Q v \cdot x} [h_v(x) + H_v(x)]$$

and plug into QCD lagrangian

$$\mathcal{L} = \overline{\psi}(i D - m_Q) \psi$$

Heavy Quark Effective Theory $m_Q \gg \Lambda_{\text{QCD}}$ $\mathcal{L} = \bar{\psi}(i \not\!\!\!D - m_Q) \psi$ $= \bar{h}_v i v \cdot Dh_v - \bar{H}_v (i v \cdot D + 2m_Q) H_v$ $+ \bar{h}_v i \not\!\!\!D_\perp H_v + \bar{H}_v i \not\!\!\!D_\perp h_v$

Now what? Doesn't look improved However, integrate out large components

$$H_v = \frac{1}{2m_Q + iv \cdot D} i D\!\!\!/_\perp h_v$$

and, after some simplification, we get

$$\mathcal{L}_{\text{eff}} = \bar{h}_v iv \cdot Dh_v + \frac{1}{2m_Q} \bar{h}_v (iD_\perp)^2 h_v + \frac{g_s}{4m_Q} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v + O(1/m_Q^2)$$

Heavy Quark Effective Theory $m_Q \gg \Lambda_{\rm QCD}$

$$\mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot Dh_v + \frac{1}{2m_Q} \bar{h}_v (iD_\perp)^2 h_v + \frac{g_s}{4m_Q} \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v + O(1/m_Q^2)$$

Implications?

 $m_Q \to \infty \longrightarrow \mathcal{L}_{\text{eff}} = \bar{h}_v i v \cdot D h_v$

- Static source of color
- Independent of heavy quark mass
- Independent of spin [SU(2) symmetric]

Application: Extracting V_{cb} from $B \to D^{(*)} \ell \bar{\nu}$



ullet Measuring V_{cb} requires knowledge of hadron form factors

 $f_{B\to D^{(*)}}^V(q^2) = \langle D^{(*)} | \bar{c} \gamma_\mu \, b | B \rangle \qquad f_{B\to D^{(*)}}^A(q^2) = \langle D^{(*)} | \bar{c} \gamma_\mu \, \gamma_5 \, b | B \rangle$

- Heavy Quark Symmetry provides form factor relations, normalization $f_{B\to D}^V \sim f_{B\to D^*}^V \sim f_{B\to D^*}^A \sim \xi(v_B \cdot v'_D) \qquad \xi(1) = 1$
- Measure V_{cb} with 2% accuracy

 $b \to c \, \ell \, \bar{\nu} \, \propto V_{ch}$



Application: Extracting V_{cb} from $B \to D^{(*)} \ell \bar{\nu}$

Need to include perturbative correction Need to extrapolate to zero recoil

$$|V_{cb}|_{\rm exc} = (39.6 \pm 0.9) \times 10^{-3}$$

Inclusive measurements also use HQET

$$d\Gamma = \begin{pmatrix} b \text{ quark} \\ decay \end{pmatrix} \times \left[1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_B^2} + \dots + \alpha_s(\dots) + \alpha_s^2(\dots) + \dots \right]$$
$$|V_{cb}|_{\text{inc}} = (41.9 \pm 0.7) \times 10^{-3}$$

Another application of HEQT $|V_{ub}|_{inc}$



Need cuts on phase space → hard theoretically Need to resum large perturbative and non-perturbative corrections

Another application of HEQT $|V_{ub}|_{inc}$



Need to resum large perturbative and non-perturbative corrections Remove using $b \rightarrow s\gamma$ $|V_{ub}|_{inc} = (4.41 \pm 0.15^{+0.15}_{0.19}) \times 10^{-3}$

Problem?

 $|V_{ub}|_{\text{inc}} = (4.41 \pm 0.15^{+0.15}_{0.19}) \times 10^{-3}$

 $|V_{ub}|_{\rm exc} = (3.23 \pm 0.31) \times 10^{-3}$



Needs lattice too

Problem?



Other EFTs

• NRQCD: expansion in v

relevant for heavy quark pairs $m \gg mv \gg mv^2 \sim \Lambda_{\rm QCD}$ decay rate written as



• SCET: expansion in p_{\perp}/p_{-}

relevant for collinear physics (jets, corners of phase space)



Loop level FCNC are possible



In SM, FCNC suppressed relative to tree level by $\sim \frac{g_2^2}{16\pi^2} \sim \frac{\alpha}{4\pi\cos^2(\theta_W)}$



More FCNC suppression in SM: GIM-mechanism



(Not true for $s \to d\gamma$)

FCNC in top decays? $t \rightarrow cZ, t \rightarrow c\gamma$



Current bound < 3.2%SM prediction $\sim 10^{-13}$ \rightarrow lots of room for NP!

LHC is top factory \rightarrow get to $\sim 10^{-5}$ level

Can do operator analysis to estimate size of signal

Constraints from top and B decays

 $O^u_{LL} = (\bar{Q}_3 \gamma^\mu Q_2) (H^\dagger D_\mu H) + \text{h.c.} \longrightarrow \text{Br}(t \to cZ)_{\text{max}} \sim 10^{-7}$

 $O_{RR}^u = (\bar{t}_R \gamma^\mu c_R) (H^\dagger D_\mu H) + \text{h.c.} \longrightarrow \text{Br}(t \to cZ)_{\text{max}} \sim 0.1$

New physics flavor problem?



TeV-scale NP models typically have new sources of flavor and CP violation

How do we protect flavor? Back to flavor symmetry: $U(3)^3$

Extend SM by adding terms (local, Lorentz, gauge inv) that are invariant under $U(3)^3$ including spurions

 λ_u, λ_d

Minimal Flavor Violation (MFV)

Fields transform as: $Q_L(3, 1, 1), u_R(1, 3, 1), d_R(1, 1, 3)$

Spurions transform as: $\lambda_u(3, \overline{3}, 1), \ \lambda_d(3, 1, \overline{3})$

Go to basis $\lambda_d = \operatorname{diag}(y_d, y_s, y_b), \lambda_u = V_{\mathrm{CKM}}^{\dagger} \operatorname{diag}(y_u, y_c, y_t)$

EFT analysis possible

To get non-diagonal terms, need at least:

$$\begin{split} \bar{Q}_L \lambda_u^{\dagger} \lambda_u Q_L, \\ d_r \lambda_d^{\dagger} \lambda_u \lambda_u^{\dagger} Q_L, \\ \bar{d}_r \lambda_d^{\dagger} \lambda_u \lambda_u^{\dagger} \lambda_d d_r \end{split}$$

(or more insertions of λ)

Minimal Flavor Violation (MFV) Extensions of SM with $U(3)^3$ breaking by $\lambda_{u,d}$ satisfy MFV Examples

I. $B \to X_s \gamma$ $\Delta \mathcal{L} = \frac{X}{\Lambda_{\rm ND}} \mathcal{O} = \frac{X}{\Lambda_{\rm ND}} (\bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R)$ $\bar{s}_L b_R$ not invariant under $U(3)^3$ $Q_L \lambda_d d_R$ is flavor diagonal $\bar{Q}_L \lambda_u \lambda_u^{\dagger} \lambda_d d_R \rightarrow \bar{s}_L V_{ts}^* V_{tb} y_t^2 y_b b_R$ If MFV then $X \propto V_{ts}^* V_{tb} y_t^2 y_b$

Minimal Flavor Violation (MFV) Extensions of SM with $U(3)^3$ breaking by $\lambda_{u,d}$ satisfy MFV Examples 2. SUSY: Without SUSY breaking, MFV $\mathcal{L} = \int d^4\theta \left[\bar{Q}e^V Q + \bar{U}e^V U + \bar{D}e^V D \right] + \dots + \left(\int d^2\theta W + \text{h.c.} \right)$ $W = H_1 U \lambda_u Q + H_2 D \lambda_d Q + \dots$

Add soft SUSY breaking

 $\Delta \mathcal{L}_{\text{SUSY-break}} = \phi_q^* M_q^2 \phi_q + \phi_u^* M_u^2 \phi_u + \phi_d^* M_d^2 \phi_d + (\phi_{h_1} \phi_u g_u \phi_q + \phi_{h_2} \phi_d g_d \phi_q + \text{h.c.})$

Unless $M_{u,d,q}^2 \propto \mathbb{I}$ and $g_{u,d} \propto \lambda_{u,d}$ large flavor-changing interactions \rightarrow Motivation for gauge-mediated SUSY breaking

Minimal Flavor Violation (MFV) Predictions

- I. Spectra: $y_{u,d,s,c} \ll 1$, so approx $U(2)^3$ remains Ex: in gauge-mediated SUSY breaking, first two generations of squarks near-degenerate
- 2. Mixing: Only source is $V_{\rm CKM}$

$$V_{\rm CKM}^{\rm (LHC)} \sim \left(\begin{array}{cccc} 1 & 0.2 & 0 \\ -0.2 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right)$$

New particles decay to either 3rd or non-3rd generation (not both)

Summary

- Review of SM, CKM, FCNC
- Effective field theories useful for multi-scale problems → Useful for heavy quarks
- MFV → constraints on new physics (but may not be true)