An Introduction to the Parton Model and Perturbative QCD

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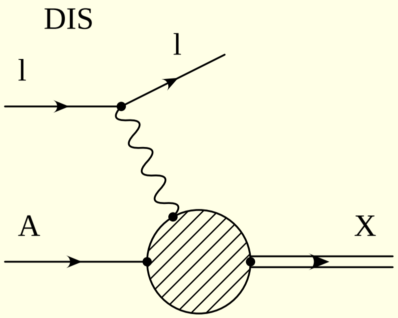
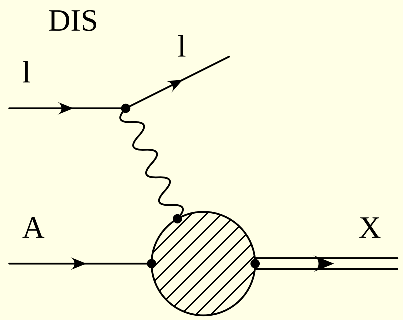
Lecture II - Deeply Inelastic Scattering

- Kinematics
- Cross sections and structure functions
- Lowest order results - parton model
- Parton distributions functions (PDFs)
- Higher order corrections
- Factorization schemes
- PDF scale dependence and evolution equations
- QCD-improved parton model

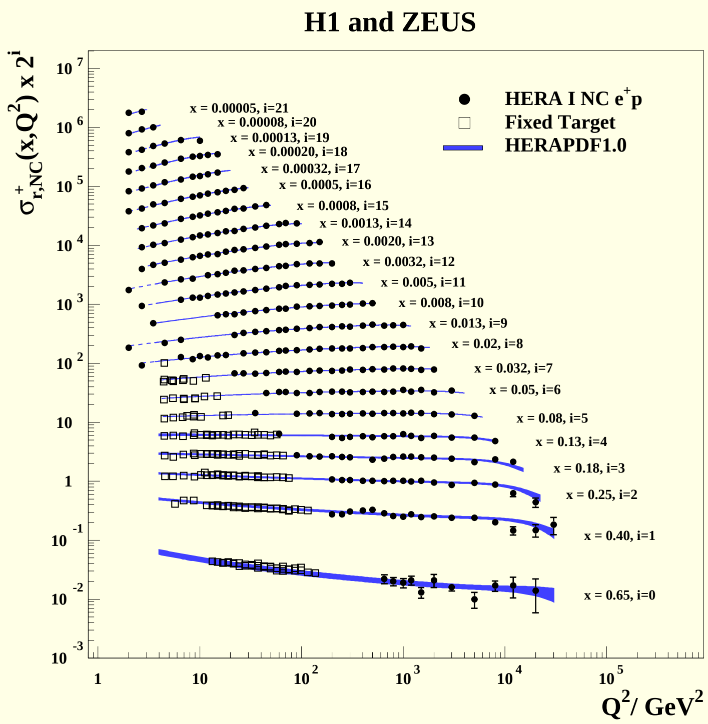
Brief Overview of DIS



- The basic idea is to use the known interaction of a photon to probe the structure of the target particle
- Elastic lepton scattering from a point-like target particle can be calculated using QED
- If the target is an extended object the cross section is modified by one or more form factors, e.g., one for a spinless target, two for a proton.
- These form factors depend on the squared four-momentum transfer Q^2
- The Fourier transform of the electric form factor gives the spatial dependence of the charge density



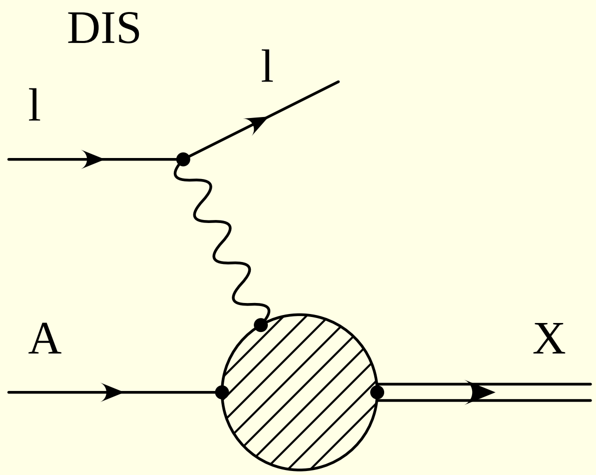
- The generalization to inelastic scattering from a proton introduces two "structure functions" (three if one is considering neutrino scattering)
- These structure functions depend on two kinematic variables - Q^2 and the energy transfer ν, for example.
- Early measurements at SLAC (1968) showed that for fixed values of Q^2/ν the structure functions showed no Q^2 dependence - that is, they only depended on one variable. This was called "scaling."
- Feynman's parton model (today's lowest order QCD) provided an intuitive description of scaling
- Higher order QCD corrections provide an excellent description of the observed deviations from exact scaling



- Reduced cross section (a combination of structure functions) versus Q^2 at fixed values of $x = Q^2/2M\nu$
- Notice the near constant values for $x \approx 0.1 - 0.3$
- Cross section increases with Q^2 at low values of x and decreases at high values of x

- Could simply calculate the cross section in terms of the interaction of the virtual photon with the partons in the target
- Historical approach has been based on structure functions
- The basic idea is to remove as much of the known physics of the lepton vertex as possible, constrain the remaining hadronic piece using gauge invariance, current conservation, parity invariance (for the electromagnetic interaction) and time reversal invariance and then express what is left in terms of the hadronic structure functions F_1 and F_2 (plus F_3 for weak interactions)
- For some purposes it is often preferable to work directly with the cross sections since that avoids any model-dependent assumptions associated the extraction of the structure functions
- On the other hand, the structure functions are easy to interpret in terms of the parton structure of the target
- I will summarize here the structure function approach

Start with a few definitions for the process $e^-(k) + A(P) \to e^-(k') + X$ in the target rest frame where M denotes the target mass



$$
\begin{aligned}
q^2 &= -Q^2 = (k-k')^2 \qquad x = Q^2/2P\cdot q = Q^2/2M\nu \\
E &= k\cdot P/M \qquad E' = k'\cdot P/M \\
\nu &= P\cdot q/M = E - E' \qquad W^2 = (P+q)^2 = M^2 + Q^2\left(\frac{1}{x} - 1\right) \\
y &= \nu/E = 1 - E'/E \quad \text{(evaluated in the lab frame where } P^\mu = (M,0,0,0))
\end{aligned}
$$

The cross section can be written as

$$\sigma = \frac{1}{4ME} \int \frac{d^3k'}{(2\pi)^3 2E'} \frac{1}{4} \sum_{spins} \sum_X \prod_{n=1}^{N_X} \int \frac{d^3p_n}{(2\pi)^3 2E_n} |T_{fi}|^2 (2\pi)^4 \delta(k+P-k'-p_X)$$

- The leptonic and hadronic parts have been written separately
- Can simplify this by being differential in the scattered lepton energy and scattering solid angle.
- Can also express T_{fi} as

$$T_{fi} = e^2 \overline{u}(k') \gamma_\mu u(k) \frac{1}{q^2} J^\mu$$

- Here J^μ is the matrix element of the electromagnetic current operator between the initial and final hadronic states

Exercise: show that

$$
\begin{aligned}
\frac{d\sigma}{dE' d\Omega'} &= \frac{\alpha^2}{MQ^4} \left(\frac{E'}{E} \right) L_{\mu\nu} W^{\mu\nu} \\
L_{\mu\nu} &= 2 \left(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k \cdot k' \right) \\
W^{\mu\nu} &= \frac{(2\pi)^3}{4} \sum_{spins} \sum_X \prod_{n=1}^{N_X} \frac{d^3 p_n}{(2\pi)^3 2E_n} J^{\mu\dagger} J^\nu
\end{aligned}
$$

- Gauge invariance, current conservation, and parity conservation give

$$
W^{\mu\nu} = F_1 \left(g^{\mu\nu} + \frac{q^\mu q^\nu}{Q^2} \right) + \frac{F_2}{M\nu} \left(P^\mu + q^\mu \frac{P \cdot q}{Q^2} \right) \left(P^\nu + q^\nu \frac{P \cdot q}{Q^2} \right)
$$

- The structure functions F_1 and F_2 contain information on the structure of the hadronic target
- Both depend on the 4-vectors P and q through Lorentz scalars
- Since $P^2 = M^2$ and $q^2 = -Q^2$, they can depend on Q^2 and $P \cdot q = M\nu = Q^2/2x$, for example

Interpretation of F_1 and F_2

Real photons have only transverse polarizations, but virtual photons can also have longitudinal polarization:

$$\epsilon^\mu(x) = (0,1,0,0) \quad \epsilon^\mu(y) = (0,0,1,0) \quad \text{but we need an expression for } \epsilon(0)$$

Consider a frame where the proton and virtual photon four-vectors are as follows:

$$P \longrightarrow \;\; \longleftarrow q$$

$$q^\mu = (0,0,0,-Q) \quad q^2 = -Q^2 \qquad P^\mu = (P_0, 0, 0, P_z)$$

Use $P \cdot q = M\nu = P_z Q$ and $P^2 = M^2$ to get

$$P^\mu = M\left(\sqrt{\frac{\nu^2 + Q^2}{Q^2}}, 0, 0, \frac{\nu}{Q}\right)$$

We need $\epsilon^\mu(0)$ such that $\epsilon(0) \cdot q = 0, \;\; \epsilon(0) \cdot \epsilon(x) = 0, \;$ and $\epsilon(0) \cdot \epsilon(y) = 0$

Can choose $\epsilon^\mu(0) = (1, 0, 0, 0)$

Next, recall

$$W_{\mu\nu} = F_1(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) + \frac{F_2}{P \cdot q}(P_\mu + \frac{q_\mu P.q}{q^2})(P_\nu + \frac{q_\nu P \cdot q}{q^2})$$

Transverse cross section: $\sigma_T \propto F_1$

Longitudinal cross section: $\sigma_L \propto -F_1 + F_2 M^2 \frac{\nu^2 + Q^2}{P \cdot q Q^2}$

Exercise: Derive these and rewrite the last result as

$$-F_1 + \frac{F_2}{2x}\left(1 + \frac{4M^2x^2}{Q^2}\right)$$

Sometimes see the ratio

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_2\left(1 + \frac{4M^2x^2}{Q^2}\right) - 2xF_1}{2xF_1}$$

Interpretation:

- F_1 measures the interaction of transverse photons
- Up to corrections of $\mathcal{O}(1/Q^2)$, $F_2 - 2xF_1$ measures the interaction of longitudinal photons

- In terms of these structure functions, one can write the cross section as

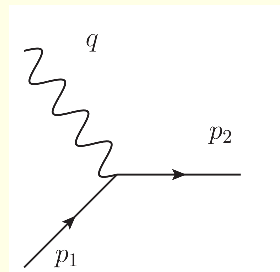
$$\frac{d\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1+(1-y)^2\right)F_1 + \frac{(1-y)}{x}\left(F_2 - 2xF_1\right)\right]$$

- Alternatively, using $F_L = F_2 - 2xF_1$ one can write

$$\frac{d\sigma}{dxdQ^2} = \frac{2\pi\alpha^2}{Q^4}\left[\left(1+(1-y)^2\right)F_2 - y^2 F_L\right]$$

- To separate F_2 and F_L one needs to have data at fixed values of x and Q^2, but different values of y.
- Since $Q^2 = 2MExy$ this requires data from different beam energies

- With these definitions, we can now examine the form of the structure functions in the parton model
- Start with the basic definition of $W_{\mu\nu}$ using a parton target of charge e_q



$$W_{\mu\nu} = \frac{(2\pi)^3}{2}\frac{1}{2}\sum_{spins} N \int \frac{d^3p_2}{(2\pi)^3 2E_2}\delta^4(p_2-q-p_1)e_q^2\left(\overline{u}(p_2)\gamma_\mu u(p_1)\right)^\dagger\left(\overline{u}(p_2)\gamma_\nu u(p_1)\right.$$

- N is a normalization factor to be defined below

- Use $\frac{d^3p_2}{2E_2} = d^4p_2\delta(p_2^2)$ to get

$$W_{\mu\nu} = Ne_q^2\delta(p_2^2)\left(2p_{1\mu}p_{1\nu} + p_{1\mu}q_\nu + p_{1\nu}q_\mu + \frac{q^2}{2}g_{\mu\nu}\right)$$

- Next, assume that the parton carries a fraction η of the target's 4-momentum and neglect target mass effects. Thus, $p_1 = \eta P$
- With this definition,

$$\begin{aligned} \delta(p_2^2) &= \delta[(p_1+q)^2] = \delta(q^2 + 2p_1\cdot q) \\ &= \frac{1}{2M\nu}\delta(\eta - \frac{Q^2}{2M\nu}) = \frac{1}{2M\nu}\delta(\eta - x) \end{aligned}$$

- So, to this order, x is a measure of the momentum fraction carried by the struck parton
- The normalization factor N corrects for the flux factor being that of the parton, not the target hadron: $N = 1/\eta$

- The end result is

$$W_{\mu\nu} = \frac{\eta}{2M\nu} e_q^2 \delta(\eta - x) \left[2P_\mu P_\nu + \frac{P_\mu q_\nu + P_\nu q_\mu}{\eta} + \frac{q^2}{2\eta^2} g_{\mu\nu} \right]$$

- From this expression one can read off the results

$$\hat{F}_2 = \eta e_q^2 \delta(\eta - x) \quad \hat{F}_1 = \frac{e_q^2}{2} \delta(\eta - x) \quad \Rightarrow \hat{F}_2 = 2x\hat{F}_1$$

- I have used the ˆ symbol to denote the contributions to the structure functions at the parton level.
- The last relation above is called the Callan-Gross relation which says that $F_L = 0$ for quarks at lowest order
- To calculate the *hadronic* structure function introduce a parton distribution function (PDF) defined such that $G_{a/A}(x)dx$ gives the probability of finding a parton a in a hadron A with a momentum fraction between x and $x + dx$

PDF Sum Rules

PDFs are inherently non-perturbative and so can not be calculated using perturbative QCD. But we do know some properties they must satisfy.

- The number of quarks (or antiquarks) in a proton is indeterminate since quantum fluctuations can create $q\bar{q}$ pairs which subsequently annihilate
- But the net number of u quarks should be two:

$$\int_0^1 dx\,(u(x) - \bar{u}(x)) = 2$$

- The net number of d quarks should be one:

$$\int_0^1 dx\,\left(d(x) - \bar{d}(x)\right) = 1$$

- The net number of s quarks should be zero:

$$\int_0^1 dx\,(s(x) - \bar{s}(x)) = 0$$

  with similar relations for c and b quarks

- Note: This does not mean that $s(x) \equiv \bar{s}(x)$ – the s and $\bar{s}$ PDFs can have different x dependences

- Momentum must be conserved:

$$\int_0^1 dx x \left[\sum_q (q(x) + \bar{q}(x)) + g(x) \right] = 1$$

  where g(x) denotes the gluon PDF

- The hadronic structure functions are given by weighting the partonic structure function by the appropriate PDFs:

$$\begin{aligned} F_2(x,Q^2) &= 2xF_1(x,Q^2) \\ &= \sum_q e_q^2 \int d\eta q(\eta)\eta\delta(\eta - x) = \sum_q e_q^2 x q(x) \end{aligned}$$

- One can see that to this order the structure functions are independent of Q^2, which is the scaling result discussed earlier

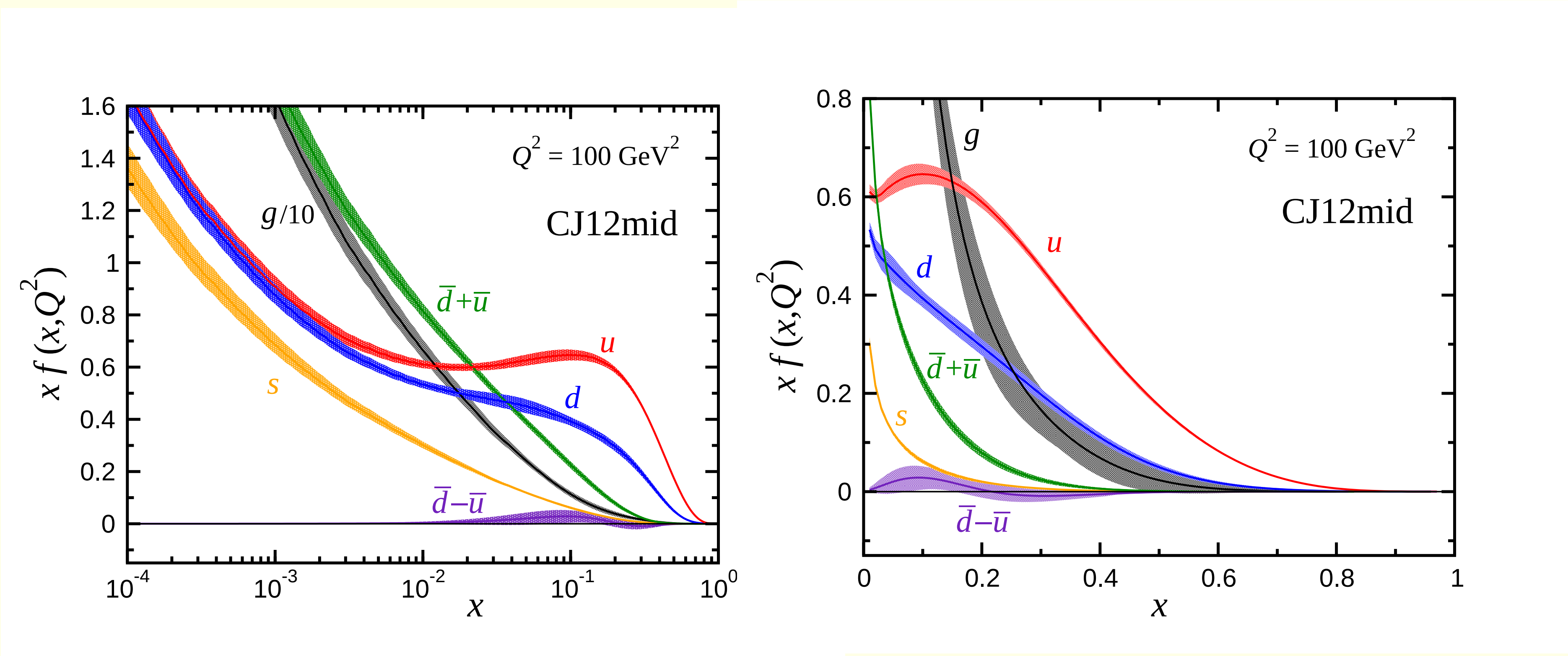


Figure 1: Uncertainty bands for the u, d, $\bar{d}+\bar{u}$, $\bar{d}-\bar{u}$, s and g PDFs for the CJ12mid fit at $Q^2 = 100$ GeV^2, shown on logarithmic (left) and linear (right) scales in x. Note that in the left panel the gluon is scaled by $1/10$.

Parton model, quarks, and DIS

- Simple parton model calculation shows scaling behavior
- Early SLAC data showed F_L was small - consistent with the partons having spin 1/2
- Early data also showed

$$\int_0^1 dx F_2^{ep}(x) \approx \frac{4}{9} < x >_u + \frac{1}{9} < x >_d \approx 0.18$$

$$\int_0^1 dx F_2^{en}(x) \approx \frac{1}{9} < x >_u + \frac{4}{9} < x >_d \approx 0.12$$

(I used $G_{u/n} = G_{d/p}, G_{d/n} = G_{u/p}$, and neglected antiquarks)

These values suggested that

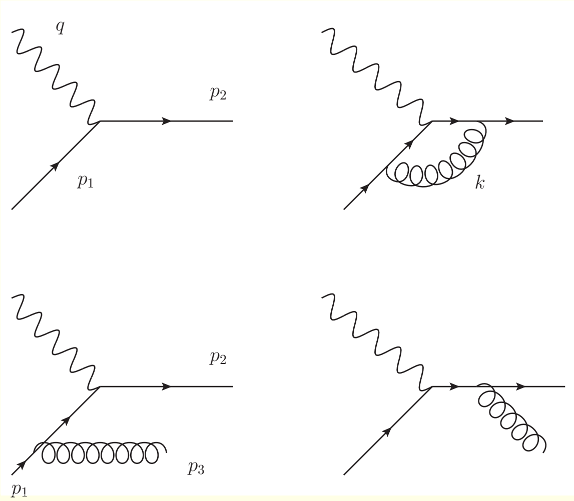
$$< x >_u \approx 0.36 \text{ and } < x >_d \approx 0.18$$

Only about half the proton's momentum is carried by the quarks!

$\Rightarrow$ neutral partons carry about half the proton's momentum

First evidence based on the parton model for gluons

- We are now prepared to consider the higher order corrections to this result, starting with corrections involving quarks in the initial state
- By now the procedure (if not the details) should be familiar
  - Write the cross section expression in n dimensions to determine the expression for the cross section in terms of the hadronic tensor
  - Write the n-dimensional expression for the hadronic tensor at the parton level for both the one-loop results and the real gluon radiation graphs
  - Add the results, cancelling the ϵ^{-2} contributions and some of the ϵ^{-1} terms, as well
  - Isolate the residual collinear singularities associated with the initial state partons
  - Factorize these collinear singularities and absorb them into the bare quark and gluon PDFs



- I will summarize the results of the steps outlined above. In the following, let $\mathcal{F}_2(x) = F_2(x)/x$. This will simplify the convolution notation. Then, the full structure function can be written in terms of contributions from quarks and gluons as

$$\mathcal{F}_2(x) = \int \sum_q e_q^2 \left[\hat{\mathcal{F}}_2^q(z) q(y) + \hat{\mathcal{F}}_2^g(z) g(y)\right] \delta(x - zy) dz\, dy$$

- With this notation, the lowest order result is $\hat{\mathcal{F}}_2^q(z) = \delta(1 - z)$

- Using this same notation, the one-loop vertex correction to the lowest order quark result is

$$\hat{\mathcal{F}}_2^{q,v}(z) = -\frac{\alpha_s}{2\pi} C_F \left(\frac{Q^2}{4\pi\mu^2}\right)^{-\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \delta(1-z) \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + 8 + \frac{\pi^2}{3}\right)$$

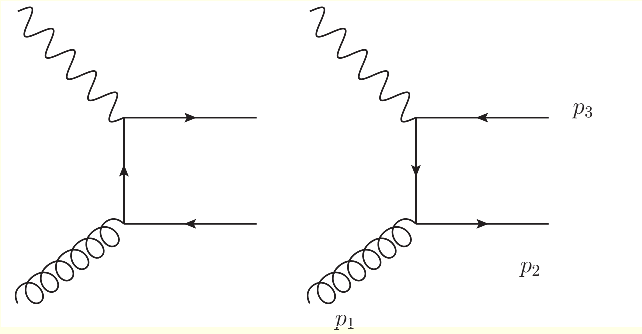
- The contribution from the real emission graphs is

$$\begin{aligned} \hat{\mathcal{F}}_2^{q,r}(z) &= \frac{\alpha_s}{2\pi} C_F \left(\frac{Q^2}{4\pi\mu^2}\right)^{-\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \\ & \left[\delta(1-z)\left(\frac{2}{\epsilon^2} + \frac{3}{2\epsilon}\right) - \frac{1}{\epsilon}\frac{1+z^2}{(1-z)_+} + \text{finite terms}\right] \end{aligned}$$

- Adding these two terms together yields the intermediate quark result

$$\hat{\mathcal{F}}_2^q(z) = \delta(1-z) + \frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{Q^2}{4\pi\mu^2}\right)^{-\epsilon}\left[-\frac{1}{\epsilon}P_{qq}(z) + \tilde{f}_2^q\right]$$

- $\tilde{f}_2^q$ represents a finite correction term which will be detailed shortly



The next contribution to consider is that from the photon gluon fusion process shown above. The sequence of steps is the same as that for the gluon radiation process with the result that

$$\hat{\mathcal{F}}_2^g(z) = \frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{Q^2}{4\pi\mu^2}\right)^{-\epsilon}\left[-\frac{2}{\epsilon}P_{qg}(z) + \tilde{f}_2^g\right]$$

- It is clear that there are uncancelled poles in ϵ in both the quark and the gluon contributions
- These are collinear divergences which result from configurations where two initial state partons are parallel to each other.
- These divergent terms represent long distance physics reflecting the evolution of the initial quark state before the hard scattering
- As such, they can be absorbed into the bare quark PDF using a procedure analogous to that used for the FFs in Lecture I

Define a scale dependent quark PDF as

$$q(x, M_f^2) = \int dy\, dz \delta(x - yz)$$

$$\left[q(y)\delta(1-z) + \frac{\alpha_s}{2\pi}\left(-\frac{1}{\epsilon}\right)\left(\frac{M_f^2}{4\pi\mu^2}\right)^{-\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left[P_{qq}(z)q(y) + P_{qg}(z)g(y)\right]\right]$$

With this definition, all the remaining collinear divergences have been absorbed into the definition of the scale-dependent PDFs. The finite parton-level structure functions have simple forms

$$\hat{\mathcal{F}}_2^q(z) = \delta(1-z) + \frac{\alpha_s}{2\pi}\left[\ln\left(\frac{Q^2}{M_f^2}\right)P_{qq}(z) + \tilde{f}_2^q\right]$$

$$\hat{\mathcal{F}}_2^g(z) = \frac{\alpha_s}{2\pi}\left[\ln\left(\frac{Q^2}{M_f^2}\right)P_{qg}(z) + \tilde{f}_2^g\right]$$

- The full structure is now given by

$$F_2(x) = \int dy\, dz\, \delta(x - yz) \sum_q e_q^2 \left[\hat{\mathcal{F}}_2^q(z) q(y, M_f^2) + \hat{\mathcal{F}}_2^g(z) g(y, M_f^2) \right]$$

- Now, it must be remembered that the $\hat{\mathcal{F}}$s in the above expression contain dependences on both Q^2 and M_f^2 in the form of $\ln Q^2/M_f^2$
- Suppose that M_f^2 was chosen to be Q^2? Then the log terms vanish
- In this case the result is rather simple:

$$\begin{aligned} F_2(x, Q^2) &= \sum_q e_q^2 x q(x, Q^2) \\ &\quad + \frac{\alpha_s}{2\pi} \sum_q e_q^2 x \int \frac{dz}{z} \left[q(\frac{x}{z}, Q^2) \tilde{f}_2^q(z) + g(\frac{x}{z}, Q^2) \tilde{f}_2^g(z) \right] \end{aligned}$$

- The last expression shows that the potentially large logs of Q^2 have been absorbed into the quark PDFs leaving an $\mathcal{O}(\alpha_S)$ correction. But wait! It gets even better ...
- When we subtracted the collinear singularities we had the freedom to subtract additional finite terms - that was how we introduced the factorization scale.
- Suppose we also subtracted out the $\tilde{f}$s? Then the last term would be absent and the expression for F_2 would remain the same as for the parton model, but with Q^2 dependent PDFs
- This scheme is referred to as the "DIS scheme." It has seen some use when describing DIS data
- However, the down side is that the PDFs now contain the finite corrections from the $\tilde{f}$s and these must be subtracted out if the PDFs are to be used in any other process
- It is much more common today to use the $\overline{\mathrm{MS}}$ scheme as presented above. That way the finite corrections are calculated on a case-by-case basis for each process

$\overline{\text{MS}}$ DIS Corrections

For completeness, I give here the two finite DIS correction terms for F_2

$$\begin{aligned}
\tilde{f}_2^q(z) &= C_F \left[(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - \frac{3}{2} \frac{1}{(1-z)_+} \right. \\
&- \left. \frac{1+z^2}{1-z} \ln z + 3 + 2z - \left(\frac{9}{2} + \frac{\pi^2}{3} \right) \delta(1-z) \right]
\end{aligned}$$

and

$$\tilde{f}_2^g(z) = \frac{1}{2} \left[\left(z^2 + (1-z)^2 \right) \ln \frac{1-z}{z} + 8z(1-z) - 1 \right]$$

I will refer back to these when we discuss the lepton pair production process shortly

DGLAP Equations

- It is all well and good to have a simple expression for F_2 in terms of scale-dependent PDFs, but where do the PDFs come from and how do you calculate their dependence on the scale?
- Refer back to the definition I introduced for the scale-dependent PDFs
- The scale entered through a term

$$-\frac{1}{\epsilon}\left(\frac{M_f^2}{4\pi\mu^2}\right)^{-\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} =$$

$$-\frac{1}{\epsilon}+\ln\left(\frac{M_f^2}{\mu^2}\right)-\ln(4\pi)+\gamma_E+\ldots$$

- The partial derivative of this term with respect to $\ln M_f^2$ is just one, so the derivative projects out the coefficient of this term which is just the convolution of the splitting function and the appropriate PDF

- The result is known as the set of DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) Equations
- They take the form

$$
\begin{aligned}
\frac{\partial q(x,t)}{\partial t} &= \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{qq}(y)q(\frac{x}{y},t) + P_{qg}(y)g(\frac{x}{y},t)\right] \\
\frac{\partial g(x,t)}{\partial t} &= \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{gq}(y)q(\frac{x}{y},t) + P_{gg}(y)g(\frac{x}{y},t)\right]
\end{aligned}
$$

- Here $t = \ln M_f^2/\mu^2$ and I have introduced two additional splitting functions beyond the two we had already encountered.
- These coupled integro-differential equations may be solved iteratively by computer, given a set of initial boundary conditions at some scale
- The boundary conditions on the initial PDFs may be parametrized and then varied to fit a wide variety of data. This is the heart of the global fitting program for determining PDFs, about which more will be said in a later lecture

Splitting Functions

- The splitting functions, P_{ij}, can be expanded in a perturbative series
- The lowest order expressions are referred to as the one-loop splitting functions

$$
\begin{aligned}
P_{qq}^{(0)}(z) &= C_F\left[\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right]\\
P_{qg}^{(0)}(z) &= T_R\left[z^2+(1-z)^2\right]\\
P_{gq}^{(0)}(z) &= C_F\left[\frac{1+(1-z)^2}{z}\right]=P_{qq}^{(0)}(1-z), z<1\\
p_{gg}^{(0)}(z) &= 2C_A\left[\frac{z}{(1-z)_+}+\frac{1-z}{z}+z(1-z)\right]+\delta(1-z)\frac{11C_A-4n_fT_R}{6}
\end{aligned}
$$

- For SU(3) $C_F=4/3, C_A=3, T_R=1/2$ and n_f denotes the number of active flavors.

DGLAP Equations and Scaling Violations

Multiply the quark equation by xe_q^2 and sum over all flavors. Using the lowest order expressions for F_2 one has

$$\frac{\partial F_2(x,Q^2)}{\partial t} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 dy\, P_{qq}(y) F_2(\frac{x}{y}, Q^2) + \sum_q e_q^2 P_{qg}(y) \frac{x}{y} g(\frac{x}{y}, Q^2)$$

If $x \ll 1$ then the gluon PDF term dominates. Since

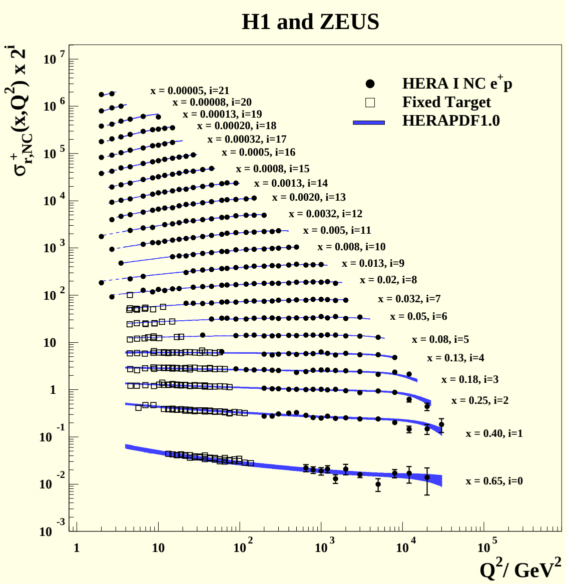
$$P_{qg}(y) = \frac{1}{2}\left(y^2 + (1-y)^2\right)$$

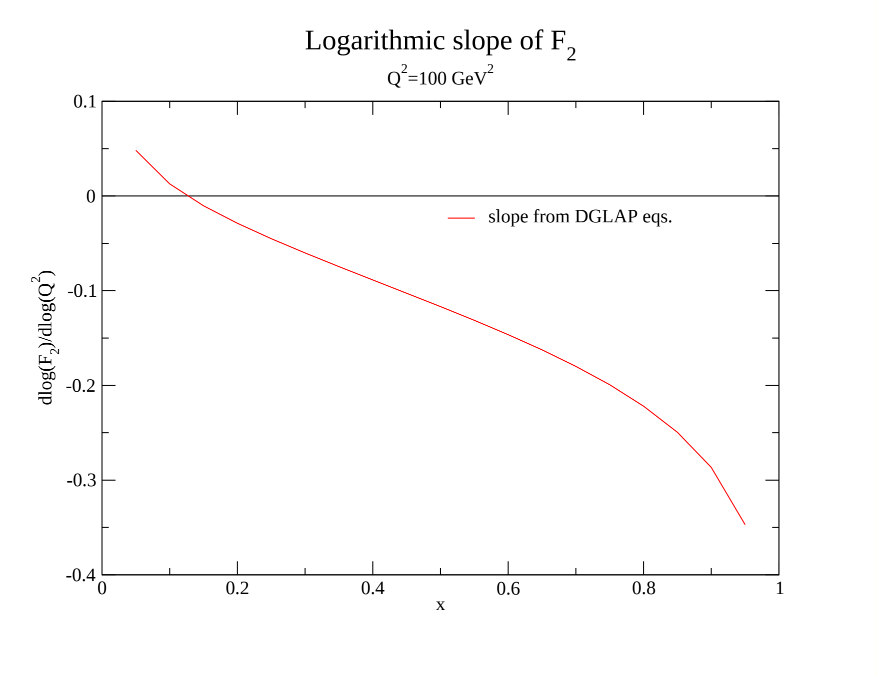
is positive definite, we see that the slope in $\ln Q^2$ is positive

For large x the first term dominates. Since

$$P_{qq}(y) = C_F \left[\frac{1+y^2}{(1-y)_+} + \frac{3}{2}\delta(1-y) \right]$$

we see the presence of $(1+y^2)F_2(\frac{x}{y}, Q^2) - 2F_2(x, Q^2) < 1$, so the slope turns negative as $x \to 1$.





Simple Interpretation

- In a hard collision quarks at high values of x radiate gluons
- This depletes the high x quark PDFs and builds them up at lower x
- The gluons can create $q\bar{q}$ pairs, thereby building up the quark PDFs at lower values of x

This explains the pattern of scaling violations seen in the data

QCD Improved Parton Model

- The predictions of the parton model are justified by lowest order QDC predictions.
- For processes with one large scale - call it Q^2 - these can be improved upon by using the techniques discussed in Lectures I and II to sum corrections from large leading logarithms
- Three steps
  - Replace α_s with the running coupling $\alpha_s(Q^2)$
  - Replace PDFs with scale-dependent PDFs which are solutions of the DGLAP equations
  - Replace FFs with scale-dependent FFs which are solutions of their DGLAP equations
- These three steps constitute the leading logarithm approximation (today usually labelled as LO for lowest order, but note the scale-dependent functions involved.)

Next Steps

For improved accuracy, as noted already in these lectures, one can go to NLO calculations (Here's another three-step plan)

- Include the next-to-leading-order hard scattering parton cross sections
- Use the two-loop running coupling (keep both b and b' in the QCD β function.)
- Use the two-loop splitting functions in the DGLAP equations for the PDFs and FFs

In Lectures III and IV we will investigate parton model predictions for hadron-hadron processes along with QCD improvements generated using these procedures.