Vector bosons and direct photons

Lecture 1



John Campbell, Fermilab

Introduction

- I am a theorist interested in hadron-collider phenomenology.
- Main interest: higher order corrections in QCD.
- Author of next-to-leading order Monte Carlo code MCFM.
- Two lectures, today and tomorrow.
- For questions or comments:
 - discussion sessions tonight and tomorrow night;
 - or, email: johnmc@fnal.gov

Some material taken from "QCD for Collider Physics" by Ellis, Stirling, Webber
 excellent resource for further details on many subjects covered here.

Outline of lectures

- Overview of vector boson basics.
- Underlying theory of W,Z production.
- Discussion of the direct photon process.
- Di-photon production.
- The importance of multi-boson production.
- Review of selected di-boson phenomenology.
- Beyond inclusive di-boson measurements.

Setting the scene

- Cross sections for producing W, Z bosons and photons are huge.
 - radiating additional jets (approx. factor of α_s) still leaves large cross sections.
 - multiple boson production still significant versus BSM rates.
- Experimentally important:
 - clean final states good for calibration (leptons, photons).
 - leptons, missing energy (+jets) crucial backgrounds.
- Theoretically important:
 - expect well-understood cross sections, test of new calculations



Electroweak Feynman rules

• Electroweak interaction Lagrangian:

$$\mathcal{L} = \sum_{f} \bar{\psi}_{f} \left(i \not{\partial} - m_{f} - g_{W} \frac{m_{f} H}{2M_{W}} \right) \psi_{f}$$

$$- \frac{g_{W}}{2\sqrt{2}} \sum_{f} \bar{\psi}_{f} (\gamma^{\mu} (1 - \gamma_{5}) T^{+} W^{+}_{\mu} + \gamma^{\mu} (1 - \gamma_{5}) T^{-} W^{-}_{\mu}) \psi_{f}$$

$$- e \sum_{f} Q_{f} \bar{\psi}_{f} \mathcal{A} \psi_{f} - \frac{g_{W}}{2\cos\theta_{W}} \sum_{f} \bar{\psi}_{f} \gamma^{\mu} (V_{f} - A_{f} \gamma_{5}) \psi_{f} Z_{\mu}$$

- W boson couples to left-handed fermions
- Z boson couples to both, different strengths:

$$(V_f - A_f \gamma_5) = \frac{V_f + A_f}{2} (1 - \gamma_5) + \frac{V_f - A_f}{2} (1 + \gamma_5)$$

• vector and axial couplings in terms of weak isospin $T_{f}^{3} = \pm \frac{1}{2}$

$$V_f = T_f^3 - 2Q_f \sin^2 \theta_W$$
$$A_f = T_f^3$$

$$V_u \approx 0.2, \ V_d \approx -0.35, \ V_\nu = \frac{1}{2}, \ V_e \approx -0.04$$
$$A_u = \frac{1}{2}, \ A_d = -\frac{1}{2}, \ A_\nu = \frac{1}{2}, \ A_e = -\frac{1}{2}$$



W and Z decays

• Partial decay widths at leading order:



- W decays: 3 charged leptons (C=1), 2 open generations of quarks (C=3)
 ⇒ Br(W⁺ → e⁺v) = 1/9.
- Z decays: $V_e \approx 0$, $|A_e| = |A_v| = |V_v| \Rightarrow Br(Z \rightarrow vv) \approx 2 \times Br(Z \rightarrow e^+e^-)$



Large fraction of decays into difficult-to-measure modes.

W and Z production

- You have already seen a sketch of the NLO corrections to these processes and discussed some of the simple phenomenology such as rapidity distns.
 - you will also get more later on (NLO and matching, pdf fits)
- Instead, I will focus on different aspects of W and Z production and the underlying theory.
- Historically, these processes have provided an essential role in extending the perturbative description to higher orders, beyond NLO QCD.
 - they are the simplest non-trivial calculations, containing only a single scale
 - an electroweak final state, so QCD corrections only occur in production
- Of course, improving the accuracy of the predictions important in its own right
 - very large cross sections for basic physics objects
 - improved extractions of fundamental quantities, e.g. M_W, pdfs
- First up: going from NLO to NNLO QCD.

$2 \rightarrow 1$ processes at NNLO

(out of my purview, but same story for $gg \rightarrow H$)

• Much more than just virtual/real combination at NLO - many ways of making gs⁴.



NNLO result



- Very large correction from LO to NLO does not repeat from NLO to NNLO \rightarrow stabilisation of the expansion.
- NNLO outside NLO error estimate, now has error of a few percent.

Beyond NNLO QCD

- Numerically, expect NNLO QCD (α_s^2) to be at the same order as NLO QED and electroweak effects (α).
 - virtual loops of photons, W, Z bosons;
 - real radiation of photons;
 - since W and Z bosons are massive and are explicitly reconstructed in the detector (and put in different event samples) no need to add their effects.
- EW effects especially important near the Z peak.
- Sensitivity to the definition of the lepton ("bare" or "dressed", recombined with photon).
- Should include photon-induced contributions $\gamma\gamma \rightarrow \ell + \ell$.
- Open issue: how to combine QCD and QED contributions.



Vector bosons and direct photons - John Campbell - 10

A different approach

- Extending the perturbative description order-by-order is one tactic.
- However there are limitations present at each order that can be better-handled with a different approach.
 - usually associated with the extra radiation present at higher orders.
- Start by looking at the transverse momentum distribution of the W as given by a NLO calculation of the total rate.

1) All of the genuine NLO corrections, from the 1-loop virtual diagrams, enter at $p_T=0$. In fact they are large and negative \rightarrow get any answer you want in the first bin, depending on the bin width.

2) prediction for any $p_T>0$ is really just a leading-order one, from real radiation diagrams.



One approach

- Easiest to tackle the second problem first: improving prediction at high p_T .
- Recognize that at large p_T we can just compute the corrections to the process,

$$pp \to W + \text{jet}$$

with the jet providing a non-zero recoil even at LO.

 The calculation requires the definition of a jet, specifying a minimum transverse momentum; W p_T with NLO accuracy above this cut (25 GeV here).





One approach

- Easiest to tackle the second problem first: improving prediction at high p_T .
- Recognize that at large p_T we can just compute the corrections to the process,

$$pp \to W + \text{jet}$$

with the jet providing a non-zero recoil even at LO.

 The calculation requires the definition of a jet, specifying a minimum transverse momentum; W p_T with NLO accuracy above this cut (25 GeV here).



 Below the jet cut, W p_T is populated by configurations with two almostbalancing jets → LO only.



• Prediction unreliable adjacent to jet cut.

What now?

- At high p_T the prediction does not depend on the cut
 - how small can we take it?
 - how can the behaviour at small p_T be fixed?
- In particular, how can a perturbative description produce turn-over like the one seen in data?
- Solution: need to account for all possible recoils against multiple partons in a systematic fashion.



Introduction to resummation

• To have a feeling for how resummation works, simplest to back off to a lepton collider: trade quarks for electrons and gluons for photons.

Parisi and Petronzio (1979)

 Look at the form of the cross section at small transverse momenta; consider virtual photon only (no Z).



Definition of kinematics:

$$Q = m_{\ell^+ \ell^-}, \ \hat{s} = m_{e^+ e^-}$$

• Close to partonic threshold differential cross section has the form:

$$\frac{\mathrm{d}\hat{\sigma}_R}{\mathrm{d}Q_{\perp}^2} = \hat{\sigma}_0 \frac{\alpha}{\pi} \frac{1}{Q_{\perp}^2} \left[\log \frac{\hat{s}}{Q_{\perp}^2} + \mathcal{O}(1) \right] \longrightarrow \mathrm{d}\hat{\sigma}_R \approx \hat{\sigma}_0 \frac{\alpha}{\pi} \frac{\mathrm{d}Q_{\perp}^2}{Q_{\perp}^2} \log \frac{\hat{s}}{Q_{\perp}^2} + \mathcal{O}(1) = 0$$

• Can integrate out Q_T up to given p_T :

$$\int \mathrm{d}\hat{\sigma}_R = \hat{\sigma}_0 \,\frac{\alpha}{\pi} \,\int_0^{p_\perp^2} \frac{\mathrm{d}Q_\perp^2}{Q_\perp^2} \,\log\frac{\hat{s}}{Q_\perp^2}$$

Vector bosons and direct photons - John Campbell - 14

Sketch of DLLA

- As it stands we cannot analyse the behaviour as Q_T→0: problem caused by the usual collinear divergence.
- But we know that in a NLO calculation this divergence is cancelled by the virtual loop contribution at exactly Q_T=0
 - result is then finite, giving an (α/π) correction to σ_0
- Dropping this term since it is not logarithm-enhanced, we thus have:

$$\hat{\sigma}_0 = \int \left(\mathrm{d}\hat{\sigma}_R + \mathrm{d}\hat{\sigma}_V \right) = \int_0^{p_\perp^2} \left(\mathrm{d}\hat{\sigma}_R + \mathrm{d}\hat{\sigma}_V \right) + \hat{\sigma}_0 \frac{\alpha}{\pi} \int_{p_\perp^2}^{\hat{s}} \frac{\mathrm{d}Q_\perp^2}{Q_\perp^2} \log \frac{\hat{s}}{Q_\perp^2}$$

full result with correction dropped

what we wanted on previous slide $(+\sigma_V)$

an integral we can do!

• Rearrange and do the integral:

$$\int_{0}^{p_{\perp}^{2}} \left(\mathrm{d}\hat{\sigma}_{R} + \mathrm{d}\hat{\sigma}_{V} \right) = \hat{\sigma}_{0} \left(1 - \frac{\alpha}{\pi} \int_{p_{\perp}^{2}}^{\hat{s}} \frac{\mathrm{d}Q_{\perp}^{2}}{Q_{\perp}^{2}} \log \frac{\hat{s}}{Q_{\perp}^{2}} \right)$$
$$= \hat{\sigma}_{0} \left(1 - \frac{\alpha}{2\pi} \log^{2} \frac{\hat{s}}{p_{\perp}^{2}} \right)$$

This is the double leadinglog approximation (DLLA) - all single log and constant terms have been dropped.

Multiple emission

- Single photon contribution to cross section: $\hat{\sigma}_0 \left(1 + \epsilon^{(1)}\right) \equiv \hat{\sigma}_0 \left(1 \frac{\alpha}{2\pi} \log^2 \frac{\hat{s}}{p_\perp^2}\right)$
- Factorization in the soft limit leads to an (approximate) simple form for n-photon contribution:

$$\epsilon^{(n)} = \frac{1}{n!} \left(-\frac{\alpha}{2\pi} \log^2 \frac{\hat{s}}{p_\perp^2} \right)^n$$

• At this point straightforward to account for multiple photon emissions:

$$\Sigma(p_{\perp}^2) \equiv \hat{\sigma}_0 \sum_{n=0}^{\infty} \epsilon^{(n)} = \hat{\sigma}_0 \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{\alpha}{2\pi} \log^2 \frac{\hat{s}}{p_{\perp}^2} \right)^n = \hat{\sigma}_0 \exp\left(-\frac{\alpha}{2\pi} \log^2 \frac{\hat{s}}{p_{\perp}^2} \right)$$
Sudakov form factor: no emissions harder than no

Sudakov form factor: no emissions harder than p_T

• Recover differential distribution by taking derivative:

$$\frac{\mathrm{d}\Sigma(p_{\perp}^2)}{\mathrm{d}p_{\perp}^2} = \hat{\sigma}_0 \,\frac{\alpha}{\pi} \,\left(\frac{1}{p_{\perp}^2}\log\frac{\hat{s}}{p_{\perp}^2}\right) \exp\left(-\frac{\alpha}{2\pi}\,\log^2\frac{\hat{s}}{p_{\perp}^2}\right) \qquad \begin{array}{l} \text{finite as } \mathsf{p}_{\mathsf{T}} \to \mathsf{0} \\ \text{(tends to zero)} \end{array}$$

Comments

- This is far from the end of the story:
 - although there is no divergence, the cross section is now too suppressed.
 - the behaviour is modified by sub-leading logarithms.
 - the treatment of multiple emission is also over-simplified, since emissions are all considered independent with no accounting for mom. conservation.
 - a proper treatment of this is beyond the scope of these lectures, but involves Fourier-transforming from momentum to impact parameter space.
- Recipe to get back to QCD:
 - remember effect of colour, so additional factor of C_{F}
 - go from e.m. to strong coupling, remembering dependence on scale

$$\longrightarrow \exp\left(-\frac{\alpha_s(p_\perp^2)C_F}{2\pi}\,\log^2\frac{\hat{s}}{p_\perp^2}\right)$$

(additional complications for very small p_T due to Landau pole)

Resummation in action

- More complicated than presented here:
 - accounts for momentum conservation
 - matching onto fixed order form at high p_T





- Collins-Soper-Sterman ("CSS") resummation formalism, as implemented in RESBOS code.
- effect of leading logs also accounted for in standard MC procedure, e.g. Pythia.

W,Z + jets

- Go back to the higher order corrections we were considering before and now categorise by the number of jets in the final state, i.e. consider W+n jet, Z+n jet production.
- Motivation from both sides again:
 - final states with leptons, missing transverse momentum, jets
 - basic experimental signatures of New Physics, e.g. "MET+jets" SUSY
 - backgrounds to top production (W+jets) and Higgs studies
 - need to be understood to good precision
- At the forefront of developing theoretical tools on the "multiplicity frontier"
 - computation of amplitudes involving many jets, NLO corrections
 - systematic improvement of parton shower predictions matching, merging and the inclusion of higher-order corrections

LO predictions

- Instrumental in developing (very efficient) recursion relations for computing helicity amplitudes.
- Berends-Giele recursion implemented in VECBOS (1990).
 - first calculation of W+4 jets, leading background to top production.



W+jets, Tevatron Run I (LO, VECBOS)

Similar recursive techniques now used in ALPGEN, SHERPA, Madgraph

Useful observation about the scaling of the cross section with additional jets:

$$R_n = \frac{\sigma(W+n \text{ jets})}{\sigma(W+(n-1) \text{ jets})}$$

W/Z + jets at NLO

- Moving to jets (plural) requires evaluation of "pentagon" and higher-point loop integrals.
- V+2 jet case could be handled with usual technology but more than that required new methods.
- This inspired the rise of analytic and numerical on-shell unitarity techniques that form the basis of the loop calculations inside the latest theoretical tools
 - e.g. BlackHat, GoSam and aMC@NLO.
- In the arena of V+jets, BlackHat+SHERPA provides predictions for for up to 5 additional jets.
- Scale-dependence of cross-sections reduced from LO to NLO.



Vector bosons and direct photons - John Campbell - 21

(c.f. F. Krauss lectures)

Improved parton showers



Vector bosons and direct photons - John Campbell - 22

Recent comparison with data



Vector bosons and direct photons - John Campbell - 23

Direct photon production

 Unlike W and Z production, 2→2 process involving a jet, proceeding through quark- and gluon-initiated channels.



- Leading order kinematics: p_T(photon) = p_T(jet)
 - significance for calibrating detector performance -- well-measured photon probes response of hadronic calorimeters;
 - used to measure jet energy scale (JES) and its uncertainty.

JES determination

Recent ATLAS JES study: ATLAS-CONF-2013-004



Photon+jet most important for 100 < p_T(jet) < 600 GeV.

Amplitudes for photons and jets

- From theoretical point of view, much in common with jet production.
- For example, consider a helicity amplitude for the 2-jet (i.e. 4-parton) process:

$$0 \to \bar{q}^+(p_1) + q^-(p_2) + g^-(p_3) + g^+(p_4)$$

(this is a MHV amplitude - two partons of one helicity, remainder opposite)

• At leading order, amplitude has two color-ordered contributions

 $\mathcal{M}(\bar{q}_{1}^{+}, q_{2}^{-}, g_{3}^{-}, g_{4}^{+}) = ig^{2} \left[(T^{a_{3}}T^{a_{4}})_{i_{2}i_{1}} M(\bar{q}_{1}^{+}, q_{2}^{-}, g_{3}^{-}, g_{4}^{+}) + (T^{a_{4}}T^{a_{3}})_{i_{2}i_{1}} M(\bar{q}_{1}^{+}, q_{2}^{-}, g_{4}^{+}, g_{3}^{-}) \right]$

that can be written as simple expressions in terms of spinor products:

$$\begin{split} M(\bar{q}_{1}^{+}, q_{2}^{-}, g_{3}^{-}, g_{4}^{+}) &= \frac{\langle 1 \, 3 \rangle \langle 2 \, 3 \rangle^{3}}{\langle 1 \, 2 \rangle \langle 2 \, 3 \rangle \langle 3 \, 4 \rangle \langle 4 \, 1 \rangle}, \\ M(\bar{q}_{1}^{+}, q_{2}^{-}, g_{4}^{+}, g_{3}^{-}) &= -\frac{\langle 1 \, 3 \rangle \langle 2 \, 3 \rangle^{3}}{\langle 1 \, 2 \rangle \langle 2 \, 4 \rangle \langle 3 \, 4 \rangle \langle 3 \, 1 \rangle}, \end{split} \qquad \begin{aligned} \langle i \, j \rangle &= \langle i - |j + \rangle = \bar{u}_{-}(p_{i})u_{+}(p_{j}) \\ [i j] &= \langle i + |j - \rangle = \bar{u}_{+}(p_{i})u_{-}(p_{j}) \\ \langle i \, j \rangle \sim \sqrt{s_{ij}} \end{aligned} \qquad (up \text{ to a phase}) \end{split}$$

- Denominators signal soft and collinear divergences
 - recognise color ordering from <23><41> and <24><31>
 - <12> and <34> are remnants of triple-gluon vertex propagator.

Direct photon amplitude

- Simple prescription to obtain photon amplitudes:
 - replace corresponding color matrix in decomposition with identity matrix
 - change overall coupling

 $\begin{aligned} \mathcal{M}(\bar{q}_{1}^{+}, q_{2}^{-}, \gamma_{3}^{-}, g_{4}^{+}) &= ieQ_{q}g\left(T^{a_{4}}\right)_{i_{2}i_{1}}\left[M(\bar{q}_{1}^{+}, q_{2}^{-}, g_{3}^{-}, g_{4}^{+}) + M(\bar{q}_{1}^{+}, q_{2}^{-}, g_{4}^{+}, g_{3}^{-})\right] \\ &\equiv ieQ_{q}g\left(T^{a_{4}}\right)_{i_{2}i_{1}}M(\bar{q}_{1}^{+}, q_{2}^{-}, \gamma_{3}^{-}, g_{4}^{+}) , \end{aligned}$

• Performing combination analytically is useful:

- As it must, remnant of triple-gluon propagator cancels.
- Form of amplitude identical for 2-photon process.
- Very useful for recycling complicated amplitudes with more jets.

Photons in perturbative QCD

• In the presence of QCD radiation (i.e. beyond LO) the direct photon amplitudes develop additional singularities.



singular propagator when quark and photon are collinear

• Naive solution: remove collinear configurations with a cut.

no radiation in "isolation cone"



Cone problems

- Removing quark-photon singularities in this way would be acceptable (but only up to NLO, no all-orders definition like this).
- However, a physically meaningful prediction would also require the same cut on gluons.
- Enforcing such a cut would prohibit the emission of soft gluons inside the cone and be infrared-unsafe: cancellation of virtual/real singularities not complete.



Theorist solution

- Frixione (1998): allow soft partons, but remove collinear configurations.
- Enforced by a cut of the form:

$$\sum_{R_{j\gamma} \in R_0} E_T(\text{had}) < \epsilon_h p_T^{\gamma} \left(\frac{1 - \cos R_{j\gamma}}{1 - \cos R_0} \right)$$

- Parton required to be softer as it gets closer to photon.
- No contribution exactly at the collinear singularity.
- This is simple to apply to a theoretical calculation and results in a well-defined cross section.
 - with such a cut, higher order calculations with photons no more difficult than corresponding QCD ones
- Cannot be (exactly) implemented experimentally due to finite detector resolution.
 - could tweak parameters of the cut (ε_h, R₀) for good agreement with experimental data (ideally, universally)

Conventional approach

• Usually, isolation cone allows a small amount of hadronic energy inside.

$$\sum_{\in R_0} E_T(\text{had}) < \epsilon_h p_T^{\gamma} \quad \text{or} \quad \sum_{\in R_0} E_T(\text{had}) < E_T^{\max}$$

- Okay for QCD infrared-safety, but collinear quark-photon singularity again exposed.
- Singularities can be handled by usual higher-order machinery, e.g. dipole subtraction, and exposed:

$$-rac{1}{\epsilon}rac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}igg(rac{4\pi\mu^2}{M_F^2}igg)rac{lpha}{2\pi}\,e_q^2 P_{\gamma q}(z)$$

 Just like initial-state collinear singularities are absorbed into pdfs, these can be defined away.

Photon fragmentation

- The analogous quantity to pdfs is the photon fragmentation function: defined for each flavour of parton.
- Inclusion of fragmentation function introduces an additional scale to the problem: fragmentation scale, M_F.
- Just like pdfs: non-perturbative input required, but perturbative evolution.
 Defined order-by-order in pQCD.



• Using conventional isolation, cross-section now has two components:

$$d\sigma = d\sigma_{\gamma+X}(M_F) + \sum_i d\sigma_{i+X} \otimes D_{i \to \gamma}(M_F)$$

direct/prompt fragmentation

- separation well-defined only for a given M_F.
- After isolation, the finite remainder from the fragmentation contribution is typically small.

Size of contributions



- Direct photon production at NLO in MCFM and JETPHOX (shown here).
- In the inclusive case fragmentation contribution is large, even at high p_T .
- After isolation, both fragmentation and annihilation contributions small.
 - this process is therefore domination by the Compton mode and thus can potentially provide a useful probe of gluon pdf.

Pdf improvements from direct photons

- Study using NNPDF with up to 7 TeV LHC data only
 - shows slight improvement in gluon uncertainty
 - potential for improvement with more data, subject to some caveats: only NLO (NNLO becoming the standard), non-perturbative corrections need to be better-understood.



Vector bosons and direct photons - John Campbell - 34

Di-photon production

- Clear interest as the principal background to the Higgs process, $gg \rightarrow H \rightarrow \gamma\gamma$.
 - even though the background is subtracted with a fitting procedure, we should also have some control of this process ab initio.
- Experimentally, significant contamination of this partonic process from the production of jets, or photon+jet, where jets are mis-identified as photons.
 - the cross-sections for these strong processes are so much larger that mis-identification rates as small as 10⁻⁴ must be handled with care.
- Here, just focus on a few aspects of the true partonic process: $q\bar{q} \to \gamma\gamma$



Higher order corrections

- NLO corrections included in DIPHOX and MCFM.
- A particular class of NNLO contributions is separately gauge-invariant and numerically important at the LHC due to the large gluon flux:



Since there is no tree level $gg\gamma\gamma$ coupling, this loop contribution is finite \rightarrow can add separately.

(in fact, finite nature means that one can compute corrections to it, i.e. part of N³LO, using just NLO technology)

- Contributes approximately 15-25% of the NLO total, depending on exact choice of photon cuts, scale choice, etc.
- Interesting behaviour of perturbative calculation in the case of photon cuts favoured by the experiment - "staggered" p_T cuts where second photon not required to be as hard as first
 - useful for purity of the signal or rejection of fake backgrounds

Staggered photon cuts

- Consider typical cuts of the form: $p_T^{\gamma_1} > 40 \text{ GeV}$, $p_T^{\gamma_2} > 40 + \delta \text{ GeV}$
- Exposes a weakness in the pertubative calculation because at leading order photons are produced back-to-back with equal p_T
 - sensitivity to staggered cut only begins at NLO.

Rather sensitive to value of δ , NLO correction becomes very large if cuts are too far apart.

Cusp at δ =0 due to emission of soft gluons and presence of $\delta \log \delta$ enhancement (candidate for resummation). Frixione, Ridolfi (1997)

Lesson: perturbative stability in the threshold region requires "moderate" δ.





NNLO results

 A full NNLO calculation has recently been performed, in the "Frixione" scheme, i.e. no need for fragmentation contributions.

Catani et al (2012)

- Better description of kinematic regions that are poorly described or inaccessible at NLO.
- Good example: azimuthal angle between photons only non-trivial at NLO in the total cross-section.
- Even better description would require either higher orders or inclusion in parton shower → not yet feasible.



Summary

- Overview of vector boson basics.
 - importance, Feynman rules, decays
- Underlying theory of W,Z production.
 - NNLO QCD, NLO EW, DLLA, resummation, W/Z+jets
- Discussion of the direct photon process.
 - isolation, fragmentation, sensitivity to pdfs
- Di-photon production.
 - subtleties of higher orders in pQCD.