Vector bosons and direct photons

Lecture 2

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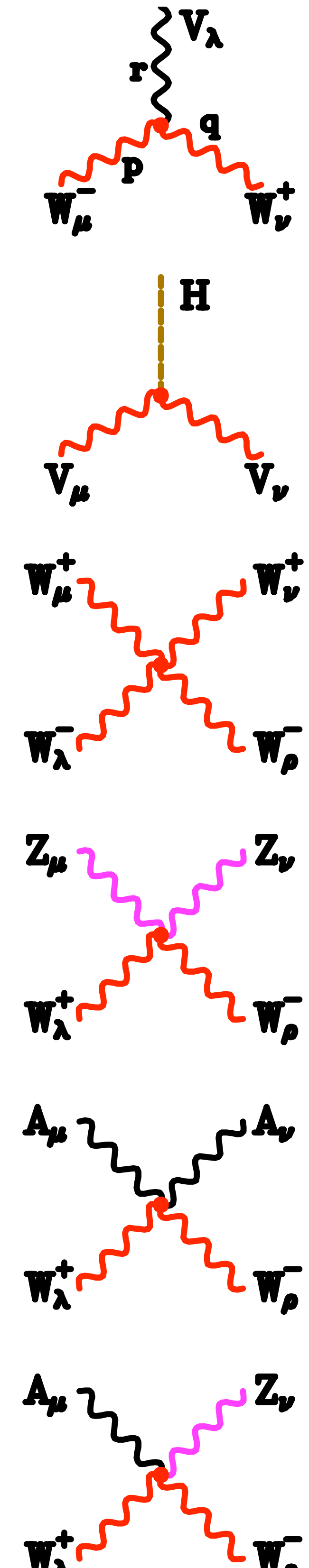
Outline of lectures

- Overview of vector boson basics.
- Underlying theory of W,Z production.
- Discussion of the direct photon process.
- Di-photon production.

- The importance of multi-boson production.
- Review of selected di-boson phenomenology.
- Beyond inclusive di-boson measurements.

Weak boson self-interactions

- Now turn to multiple production of vector bosons, with at least one W or Z.
- These have an essentially different character from di-photon production because of self-interactions.
- Probes of triple couplings:
  - di-boson production
  - single production through VBF
- Probes of quartic couplings:
  - tri-boson production (and beyond)
  - di-boson production in VBS
- Rich structure predicted by the SM Lagrangian to explicitly test in all these processes.



$$+ig_V\,[(p-q)_\lambda g_{\mu\nu}+(q-r)_\mu g_{\nu\lambda}+(r-p)_\nu g_{\lambda\mu}]$$
(all momenta incoming, $g_A=e$, $g_Z=g_W\cos\theta_W$)

$$+ig_{VH}M_W g_{\mu\nu}$$
($g_{WH}=g_W$, $g_{ZH}=g_W/\cos^2\theta_W$)

$$+ig_W^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$

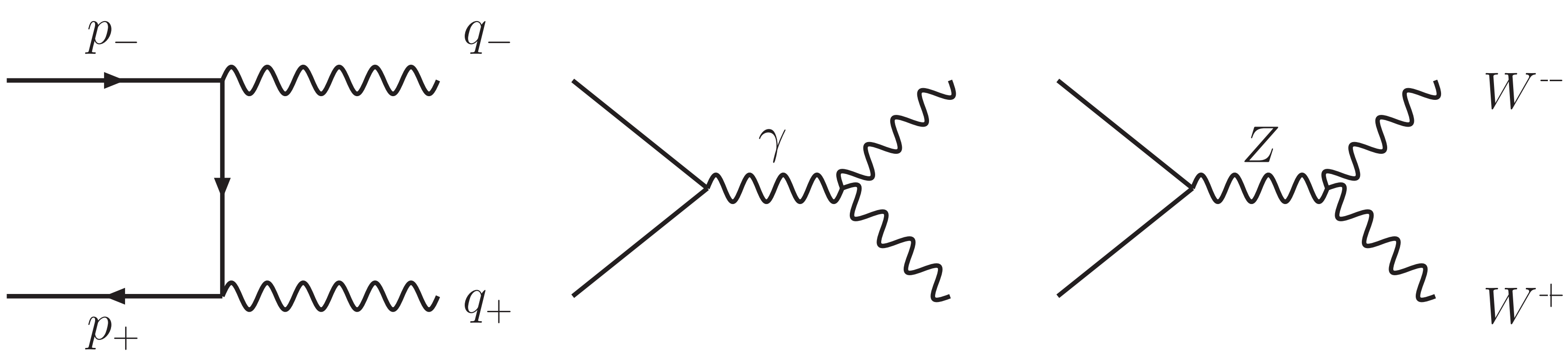
$$-ig_W^2\cos^2\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$

$$-ie^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$

$$-ieg_W\cos\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$$

The special role of self-interactions

- To illuminate the special role self-interactions play, consider the reaction $e^+e^-\to W^+W^-$ at a lepton collider.



- Choose frame in which the W 3-momenta are in the *z*-direction:

$$q_\pm = (E, 0, 0, \pm q)$$
$$E^2 - q^2 = m_W^2$$

- Polarization vectors of W ($\epsilon.q = 0$, $\epsilon^2 = -1$):

$$\epsilon^\mu = (0, 1, 0, 0), \quad \epsilon^\mu = (0, 0, 1, 0)$$

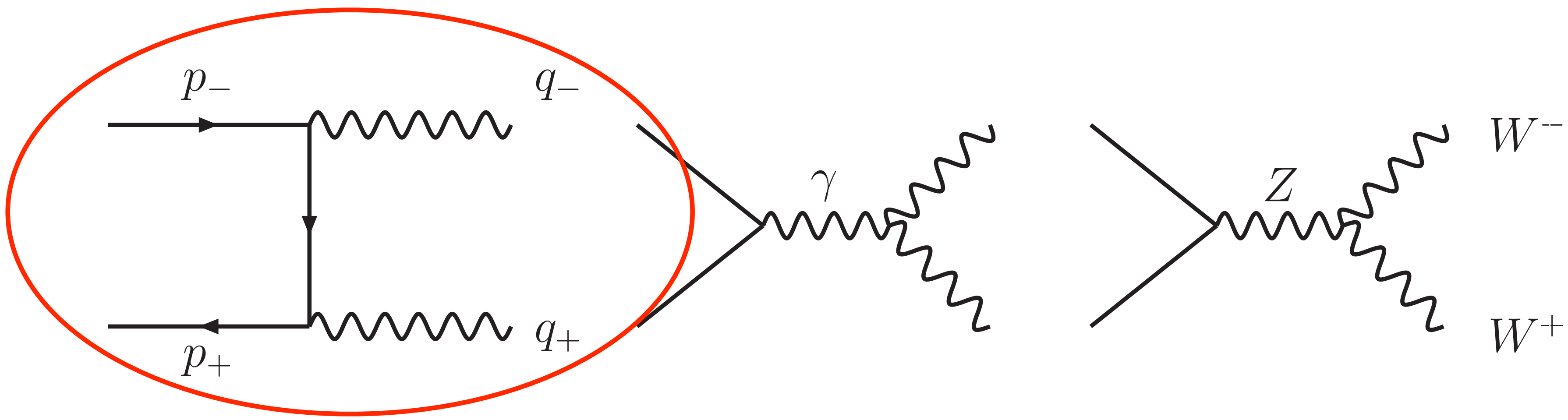
transverse (c.f. photon)

$$\epsilon^\mu_\pm = \frac{1}{m_W}(q, 0, 0, \pm E)$$

longitudinal (massive bosons)

- Longitudinal mode means diagrams grow as $E^2 \to$ focus on this limit.
  - in that case can study longitudinal modes by approximating $\epsilon^\mu_\pm \to \frac{1}{m_W} q_\pm$

Longitudinal contribution



- Contribution from first diagram:

$$M = \frac{(-ig_w)^2}{8} \bar{v}(-p_+) \not{\epsilon}(q_+)(1-\gamma_5) \frac{i}{\not{p}_- - \not{q}_-} \not{\epsilon}(q_-)(1-\gamma_5) u(p_-)$$

- Using longitudinal polarization and keeping only leading term:

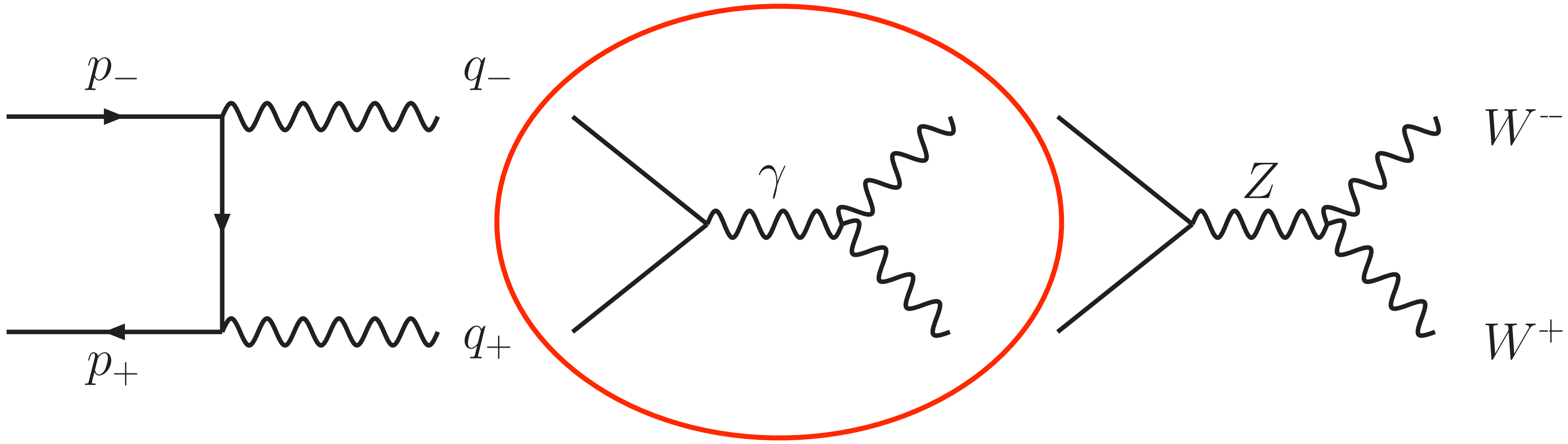
$$M = -i \frac{(-ig_w)^2}{4M_W^2} \bar{v}(-p_+) \not{q}_+ (1-\gamma_5) u(p_-)$$

(via equation of motion)

- Useful to rewrite using momentum conservation:

$$M = -i \frac{(-ig_w)^2}{8M_W^2} \bar{v}(-p_+)(\not{q}_+ - \not{q}_-)(1-\gamma_5) u(p_-)$$

Self-coupling contributions



- Contribution from second diagram:

write in terms of g_W using $e=g_W \sin\theta_W$

$$M = (-ig_w)(ig_w)Q_e \sin^2\theta_W \bar{v}(-p_+)\gamma^\rho u(p_-)\frac{-ig_{\rho\alpha}}{(q_+ + q_-)^2}$$
$$\times \quad V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+)\varepsilon_\beta(q_-)\varepsilon_\delta(q_+)\ .$$

triple-boson vertex

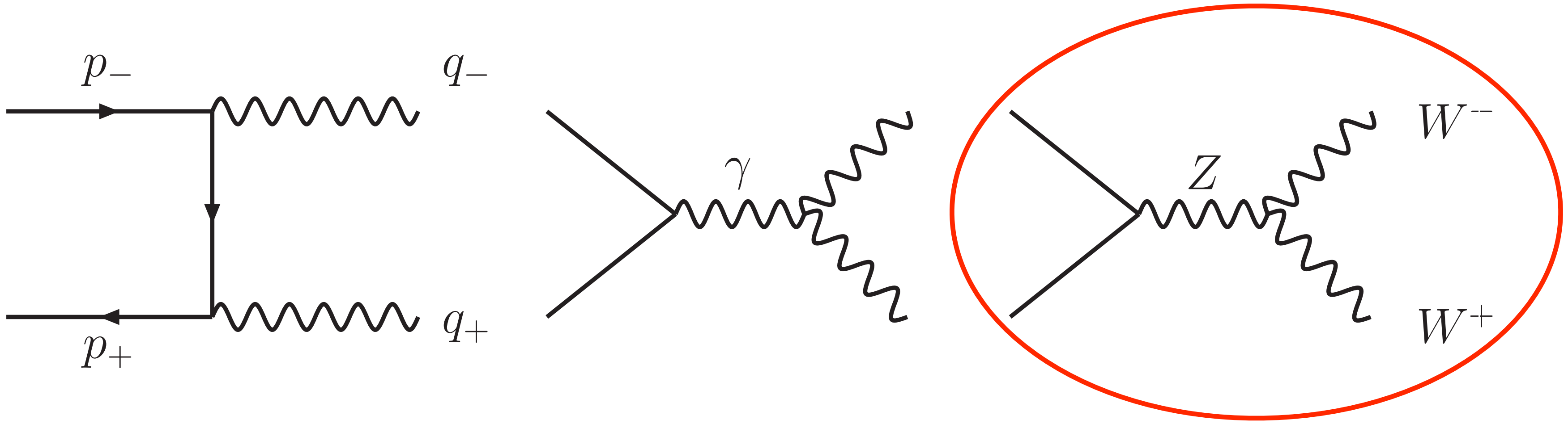
- Triple-boson vertex: $V^{\alpha\beta\delta}(p,q,r) = g^{\alpha\beta}(p^\delta - q^\delta) + g^{\beta\delta}(q^\alpha - r^\alpha) + g^{\delta\alpha}(r^\beta - p^\beta)$

- Contracted with longitudinal polarizations here:

(discard h.o. terms)

$$V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+)\varepsilon_\beta(q_-)\varepsilon_\delta(q_+) = -\frac{(q_+ + q_-)^2}{2M_W^2}\left[q_+^\alpha - q_-^\alpha\right] + O(1)$$

Self-coupling contributions



- Similar contribution from third diagram:

vector and axial couplings from before

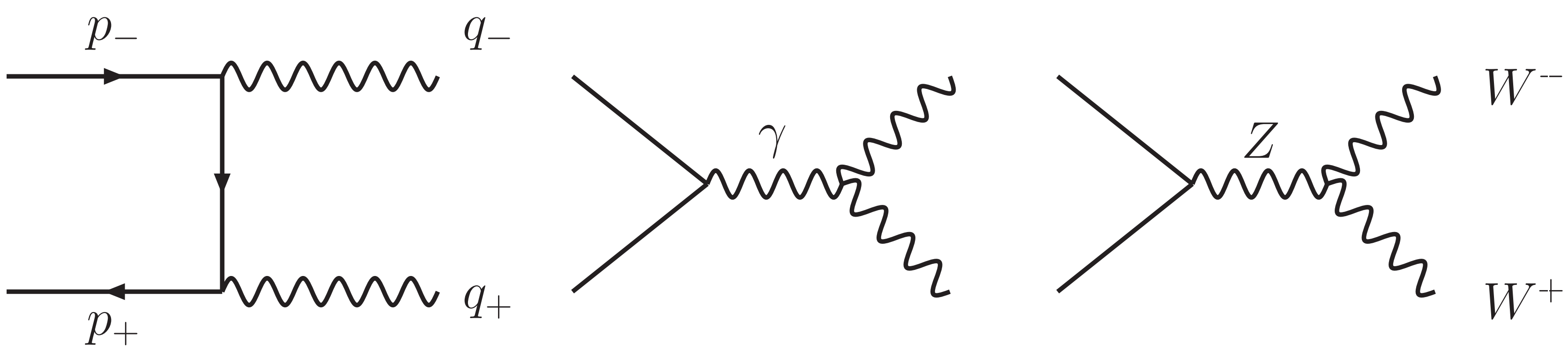
$$M = \frac{(-ig_w)(ig_w)}{2}\bar{v}(-p_+)\gamma^\rho(V_e - A_e\gamma_5)u(p_-)\frac{-ig_{\rho\alpha}}{(q_+ + q_-)^2 - M_Z^2}$$
$$\times \quad V^{\alpha\beta\delta}(q_+ + q_-, -q_-, -q_+)\varepsilon_\beta(q_-)\varepsilon_\delta(q_+)$$

- Hence, combining second and third diagrams:

$$M = -i\frac{(-ig_w)^2}{4M_W^2}\bar{v}(-p_+)(\not{q}_+ - \not{q}_-)\left[2Q_e\sin^2\theta_W + V_e - A_e\gamma_5\right]u(p_-)$$

(discarding non-leading terms)

Total in the high-energy limit



$$M = -i\frac{(-ig_w)^2}{8M_W^2}\bar{v}(-p_+)(\not{q}_+ - \not{q}_-)\left[1 + 4Q_e\sin^2\theta_W + 2V_e - (1+2A_e)\gamma_5\right]u(p_-)$$

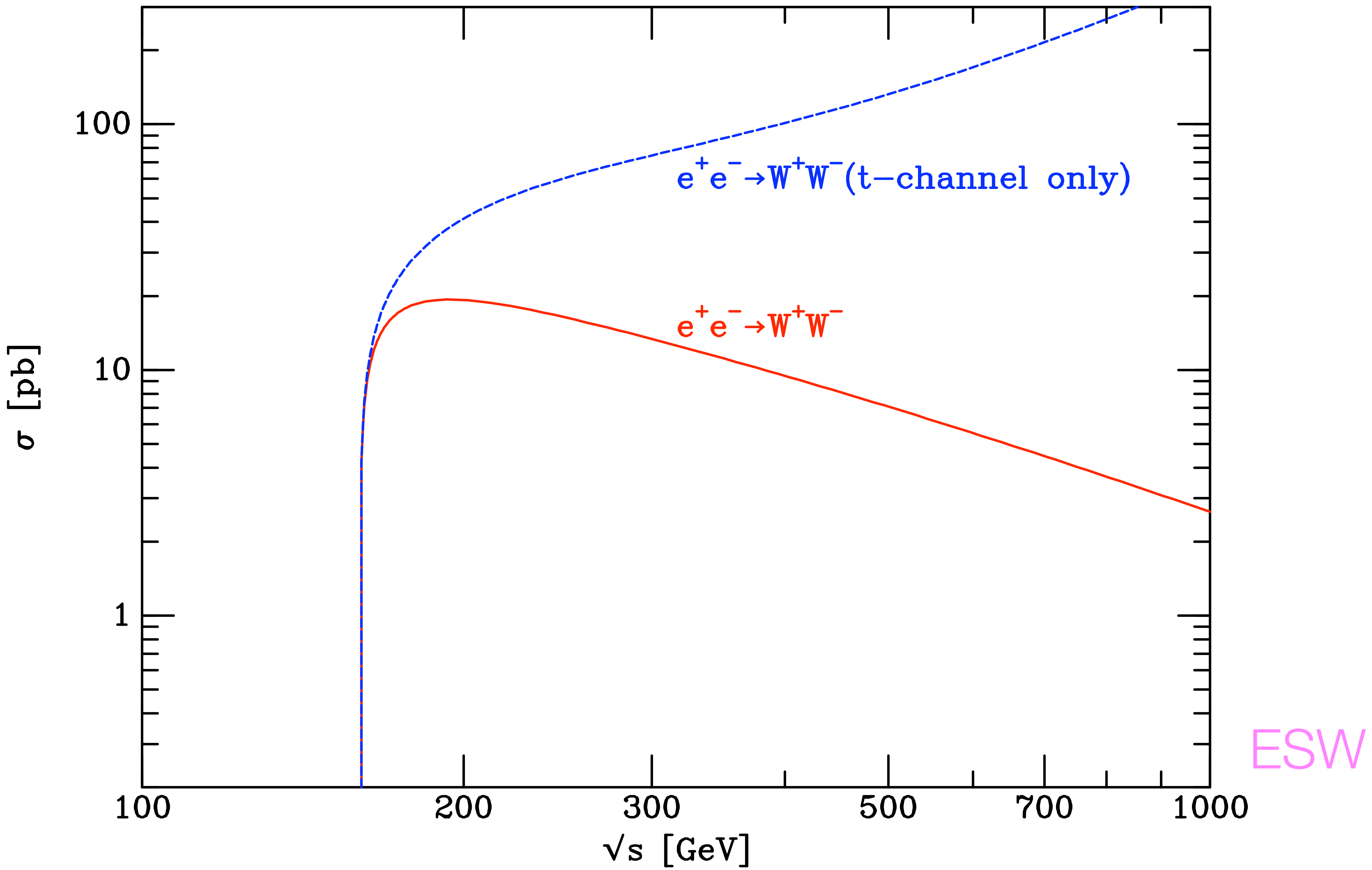
- Recall definitions of Z couplings: $V_e = -\frac{1}{2} - 2Q_e\sin^2\theta_W\,, \qquad A_e = -\frac{1}{2}$

  to see that the leading high-energy behaviour is cancelled.

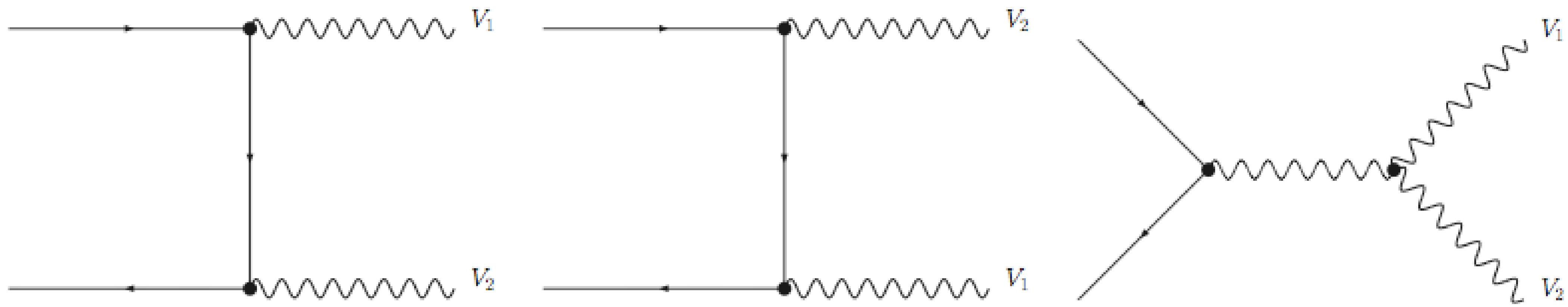
  - due to the relationship between the coupling of the W,Z and photon to fermions and the triple-boson couplings
  - equivalently, due to the underlying gauge structure of the weak sector of the Standard Model.
  - imperative to test at hadron colliders.

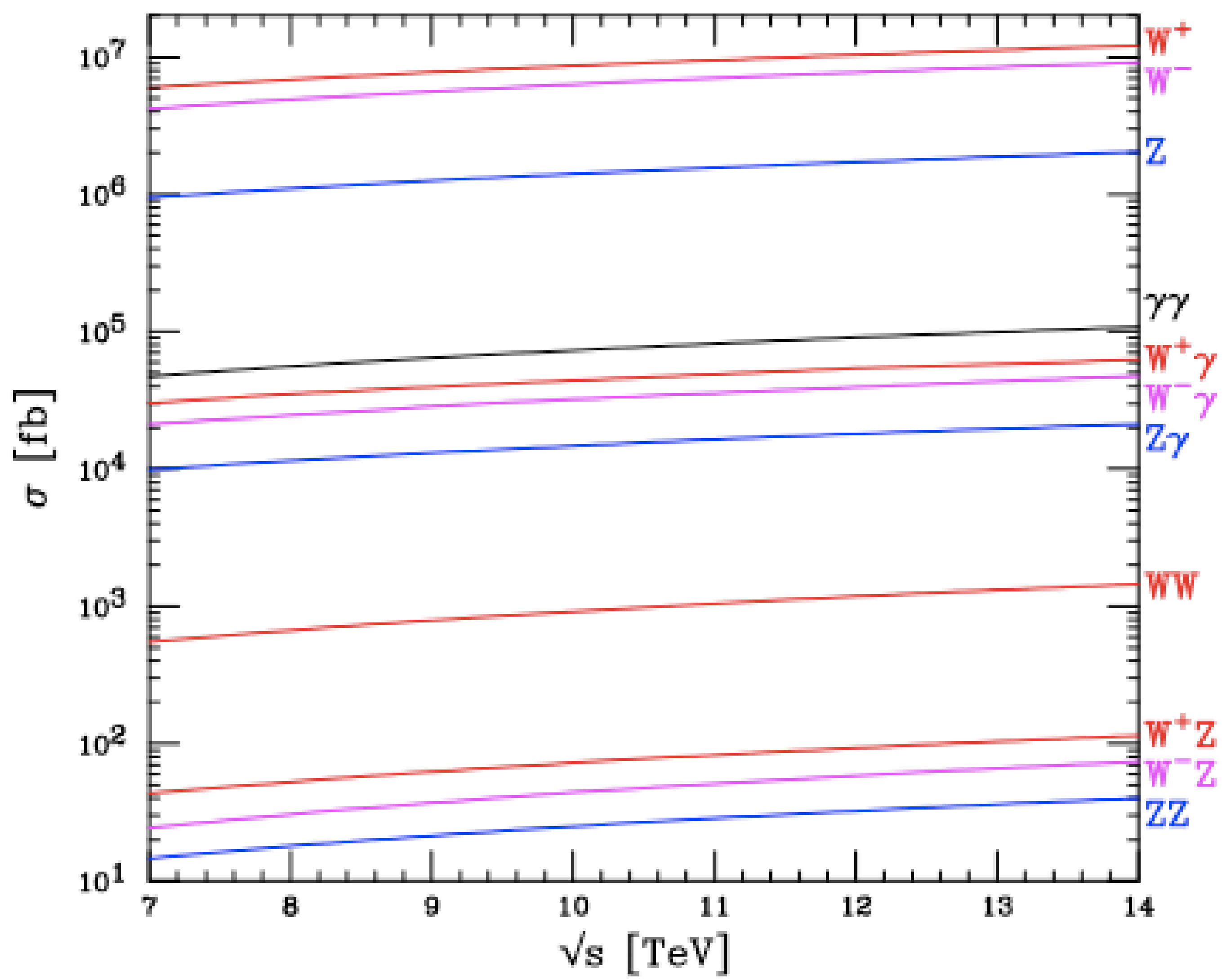
Strength of high-energy cancellation

- Full result including sub-leading terms.



Di-boson production at hadron colliders

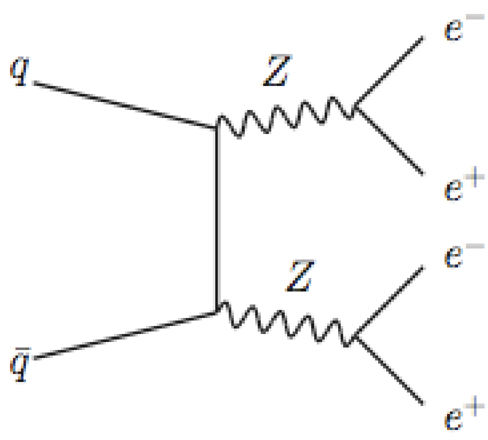




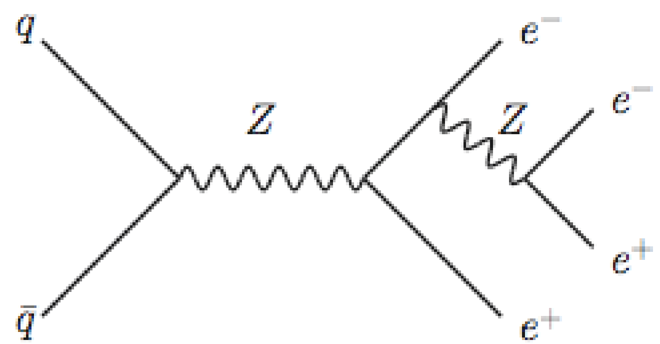
- Triple gauge coupling present for all processes except Zγ.
- Processes involving photons strongly-dependent on photon p_T (and rapidity) cut.
- Further suppression by BRs once decays are included.
- Next-to-leading order corrections known analytically, included in MCFM, VBFNLO (also POWHEG NLO MC).

Single-resonant diagrams

- Modern calculations of di-boson processes include effects of decays; in that case, EW gauge invariance requires that additional diagrams are included.



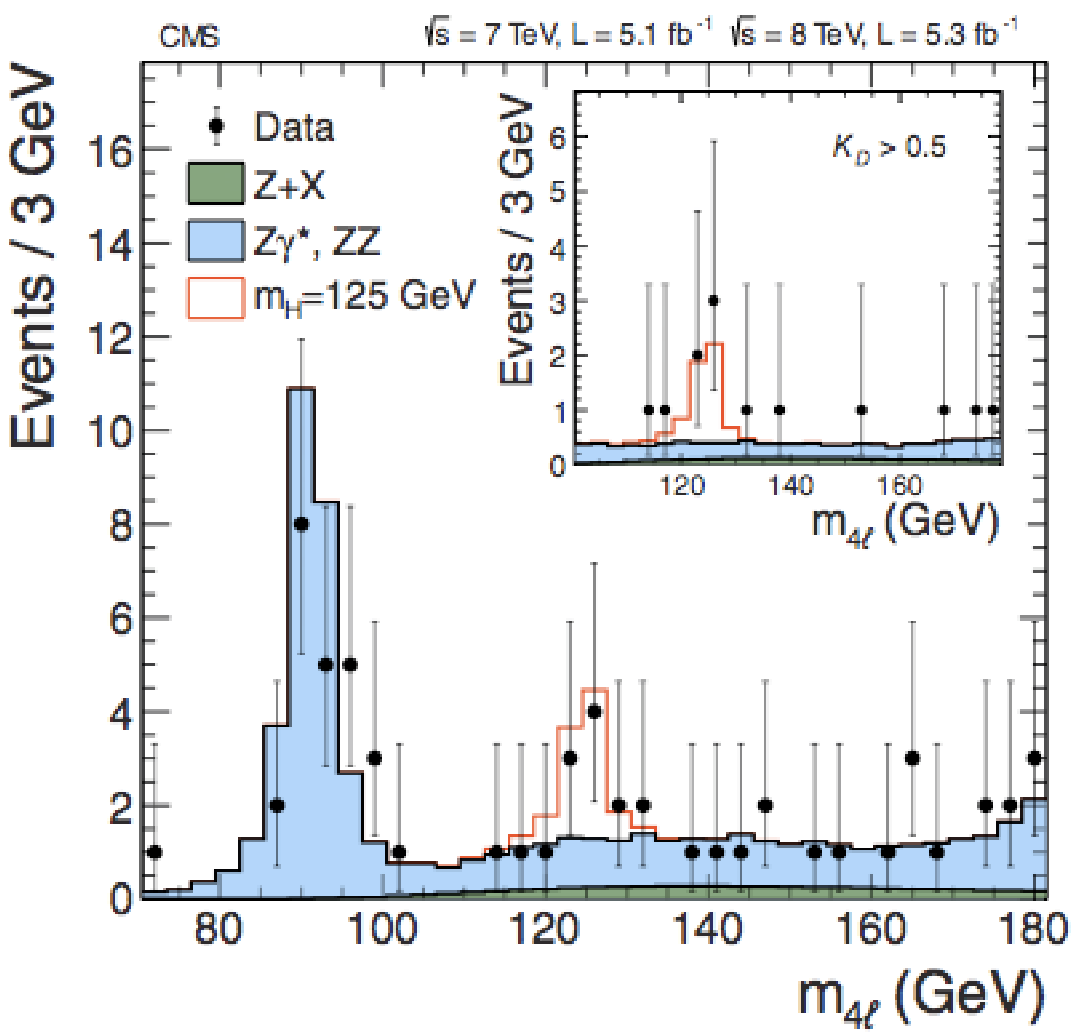
“double”-



“single”-resonant

$q\bar{q} \rightarrow ZZ \rightarrow e^+e^-e^+e^-$

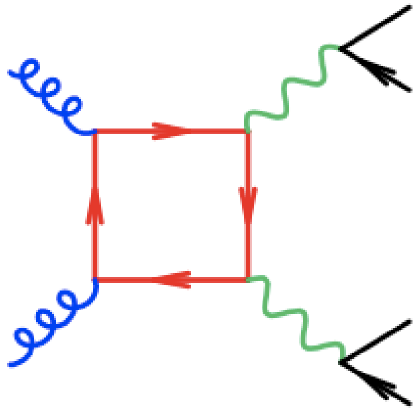
CMS-HIG-12-028

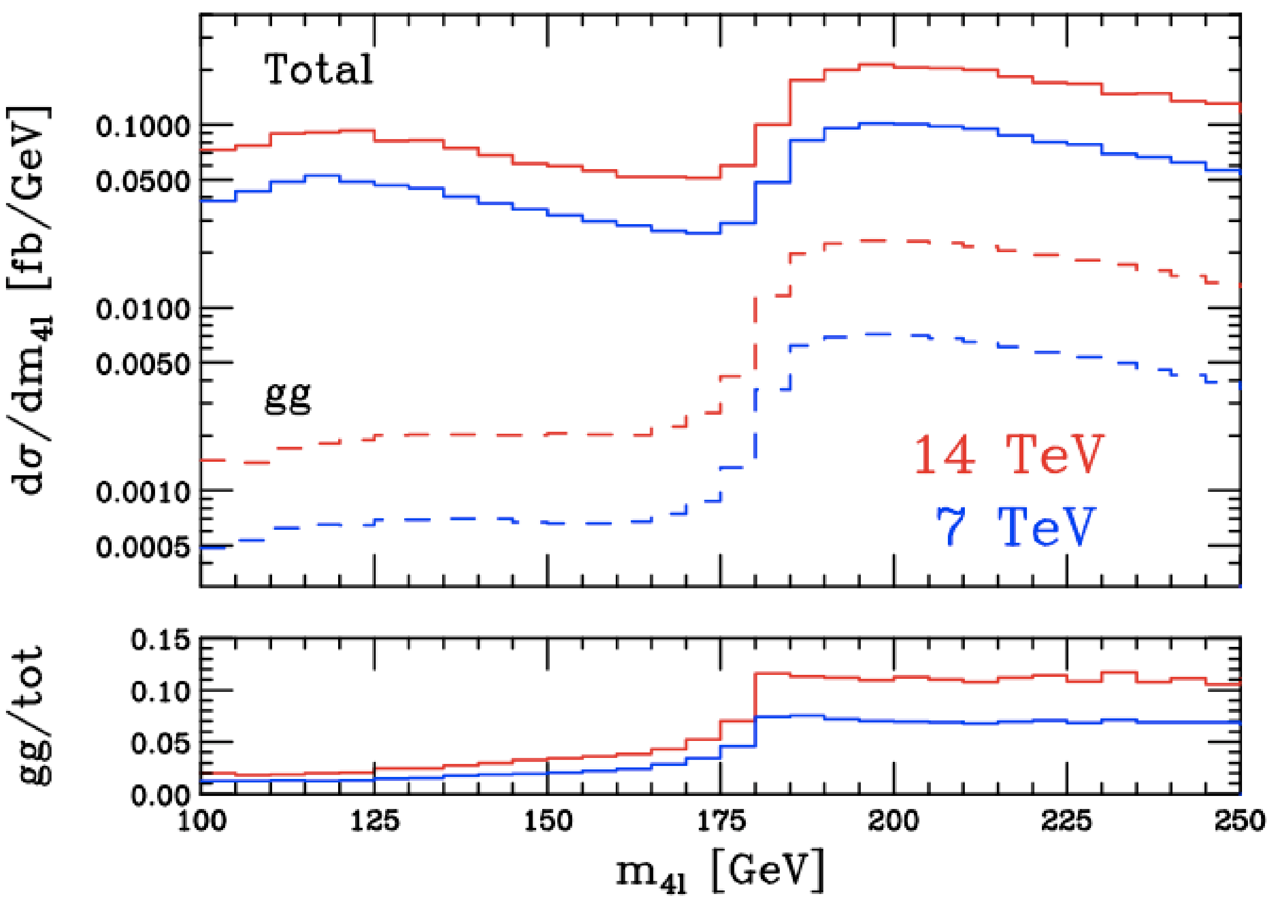


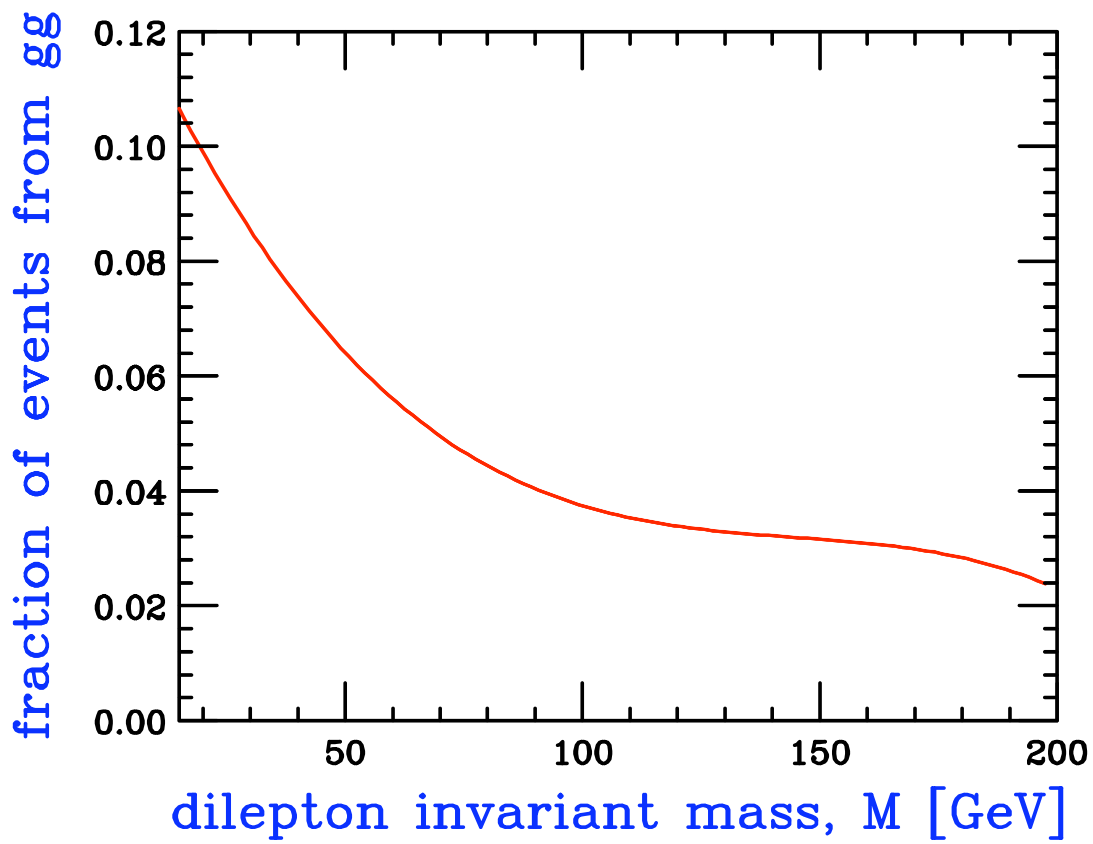
- Inclusive cross section is dominated by the double-resonant contribution, but other distributions can be sculpted.
- Notably: invariant mass of 4 leptons.
- Useful cross-check of analysis in Higgs search.

Gluon-induced contributions

- Just like di-photon production, part of NNLO contribution to WW and ZZ production is numerically relevant at the LHC.





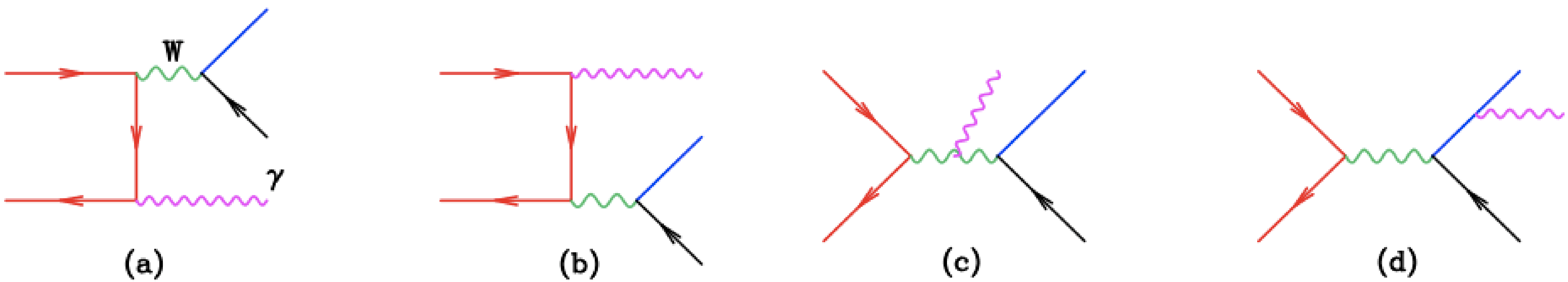


ZZ: small below Z pair threshold (e.g. H search), but large above; will be bigger at 14 TeV.

WW: impact of gg contribution enhanced by H analysis cuts such as low dilepton invariant mass

Photon radiation in decays

- For Wγ and Zγ production it is essential to account for the effect of photon radiation from the products of the W or Z decay.



  - required by EM gauge invariance unless dileptons confined to resonance region → not always easy to enforce experimentally
  - effect can be dramatic: $\sigma(e^+\nu\gamma) \neq \sigma(W^+\gamma) \times \mathrm{Br}(W^+ \to e^+\nu)$

W on-shell (no FSR)

W off-shell (includes FSR)

Decay	Cuts	$\sigma^{LO}(e^+\nu\gamma)$	$\sigma^{NLO}(e^+\nu\gamma)$
No FSR	Basic γ	4.88	8.74
	M_T cut	1.99	3.78
	Lepton cuts	1.49	2.73
Full	Basic γ	23.0	30.1
	M_T cut	2.12	3.94
	Lepton cuts	1.58	2.85

difference reduced by transverse mass cut $M_T(\ell\gamma,\nu)$>90 GeV

→ little room left to radiate in decay

W+photon amplitude

- Consider the lowest order partonic process (4-momenta in brackets):

$$\bar{u}(p_1) + d(p_2) \to W^+ + \gamma(p_3)$$

- The helicities of the quarks are fixed by the W coupling but we can choose a positive helicity photon. Up to an overall factor amplitude is:

$$Q_u \frac{[2\,3]}{\langle 1\,3\rangle} + Q_d \frac{[1\,3]}{\langle 2\,3\rangle}$$

$$Q_u = 2/3 \text{ and } Q_d = -1/3$$

(and we have used $Q_e = Q_d - Q_u$ to simplify)

- Convert back to more-familiar dot products by extracting overall spinor factor:

$$\frac{[2\,3]}{\langle 1\,3\rangle}\left(Q_u + Q_d \frac{p_1 \cdot p_3}{p_2 \cdot p_3}\right)$$

(recall, $\langle i\ j\rangle[j\ i] = 2p_i \cdot p_j$)

- Can now evaluate in the partonic c.o.m. Assume the down quark has a positive *z* component and denote the angle between it and the photon by θ*.
- Amplitude thus proportional to: $Q_u(1 + \cos\theta^\star) + Q_d(1 - \cos\theta^\star)$

Radiation amplitude zero

- Amplitude vanishes at the scattering angle given by:

$$\cos\theta^\star = \frac{Q_u + Q_d}{Q_d - Q_u} = -\frac{1}{3}$$

(independent of parton energies)

- This feature is characteristic of all helicity amplitudes for the emission of photons in multi-boson processes.
- "Radiation amplitude zero" (RAZ) the result of interference between diagrams.
- Easy to calculate the corresponding photon rapidity:

$$y_\gamma^\star = \frac{1}{2}\log\left(\frac{1+\cos\theta^\star}{1-\cos\theta^\star}\right) \approx -0.35$$

- Rather than reconstructing all objects and trying to boost back to c.o.m, easiest to construct a (boost invariant) rapidity difference: $\Delta y^\star = y_\gamma^\star - y_W^\star$.
- For small photon p_T relative to m_W the W rapidity in the c.o.m. is approximately: $y_W^\star \approx \frac{1}{2}\log\left(\frac{m_W - p_T^\gamma\cos\theta^\star}{m_W + p_T^\gamma\cos\theta^\star}\right)$

Position of zero

- Expanding for small p_T gives: $$y^\star_\gamma \approx \frac{p_T^{\gamma,\mathrm{min}}}{3m_W}$$

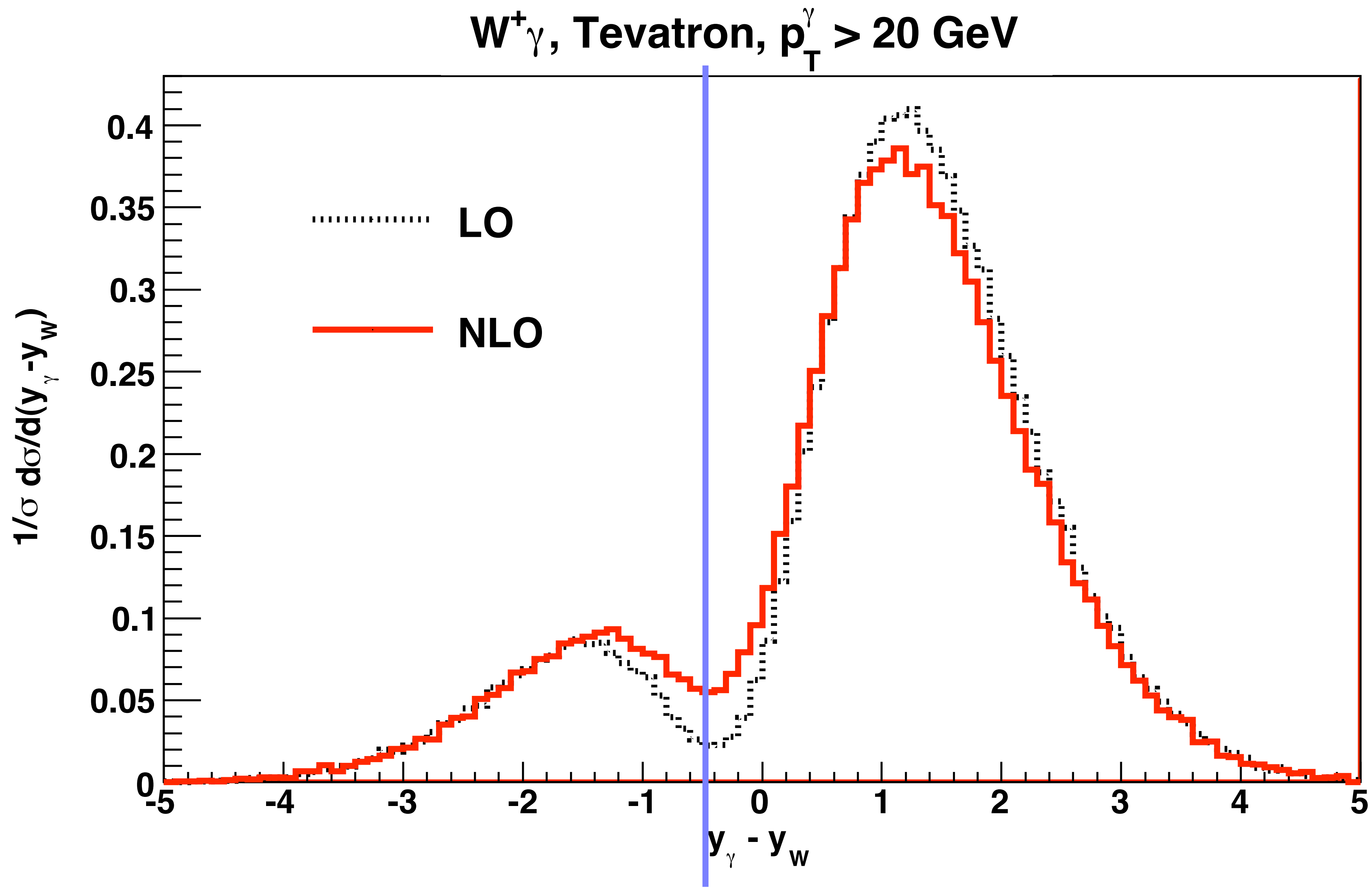
- Hence the corresponding zero in the W rapidity distribution is positive, but at a significantly smaller value.

- Rapidity difference, e.g. for typical experimental cuts at 20 GeV: $\Delta y^\star \approx -0.45$ (for the sub-process we looked at: $\bar{u}d \to W^+\gamma$).

- Tevatron: quark and anti-quark directions coincide with those of protons and anti-protons, to first approximation.
  - prediction for the radiation zero derived above should be reproduced approximately once pdfs are folded in;
  - however this pdf dilution means that we do not obtain exact vanishing of the distribution but instead a pronounced dip.

- LHC: no well-defined direction for protons, so RAZ should be at $\Delta y^*=0$.

Radiation zero with pdf effects



expected position of RAZ

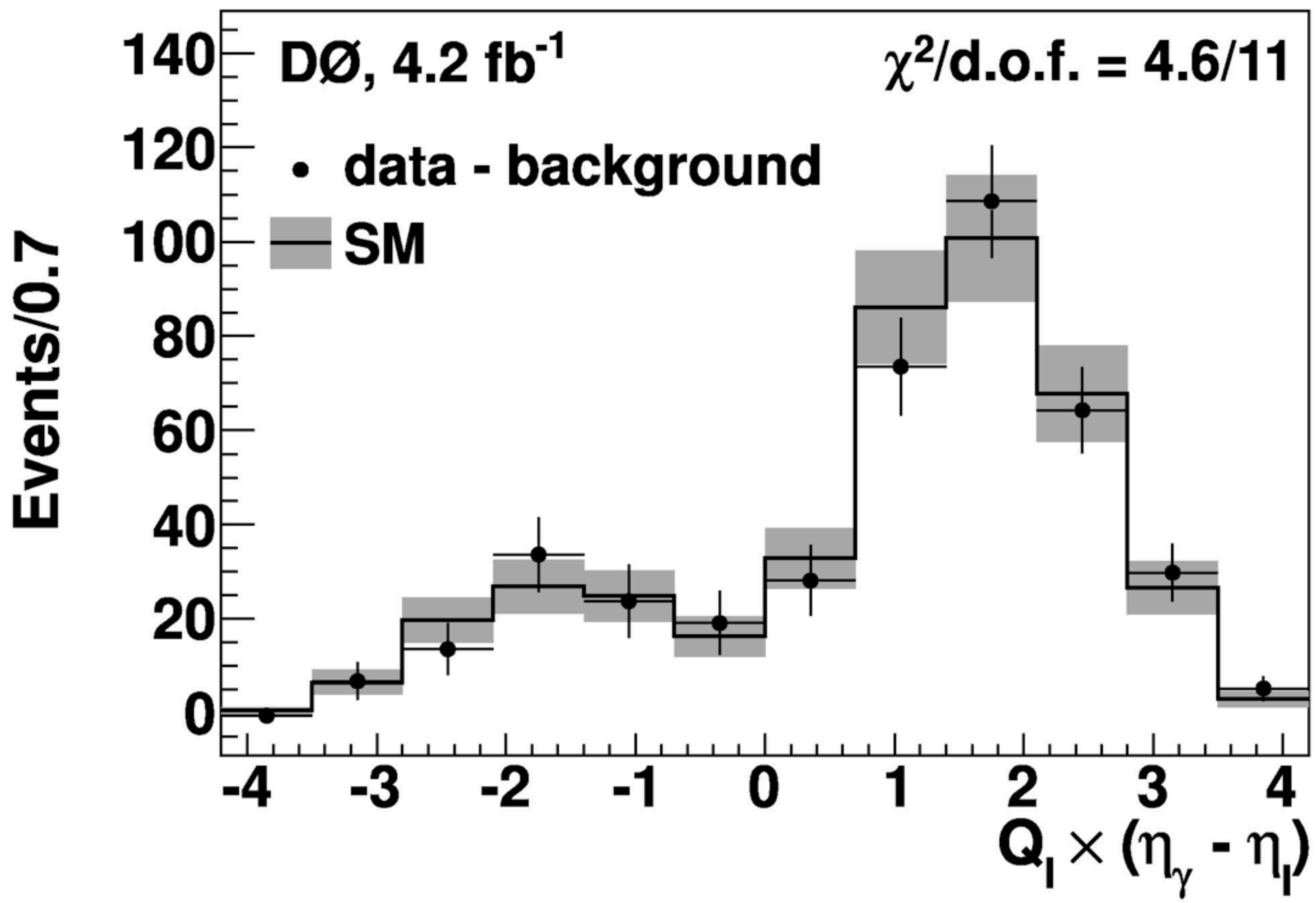
Amplitude zero a feature of the LO amplitude only

→ partially washed out at higher orders

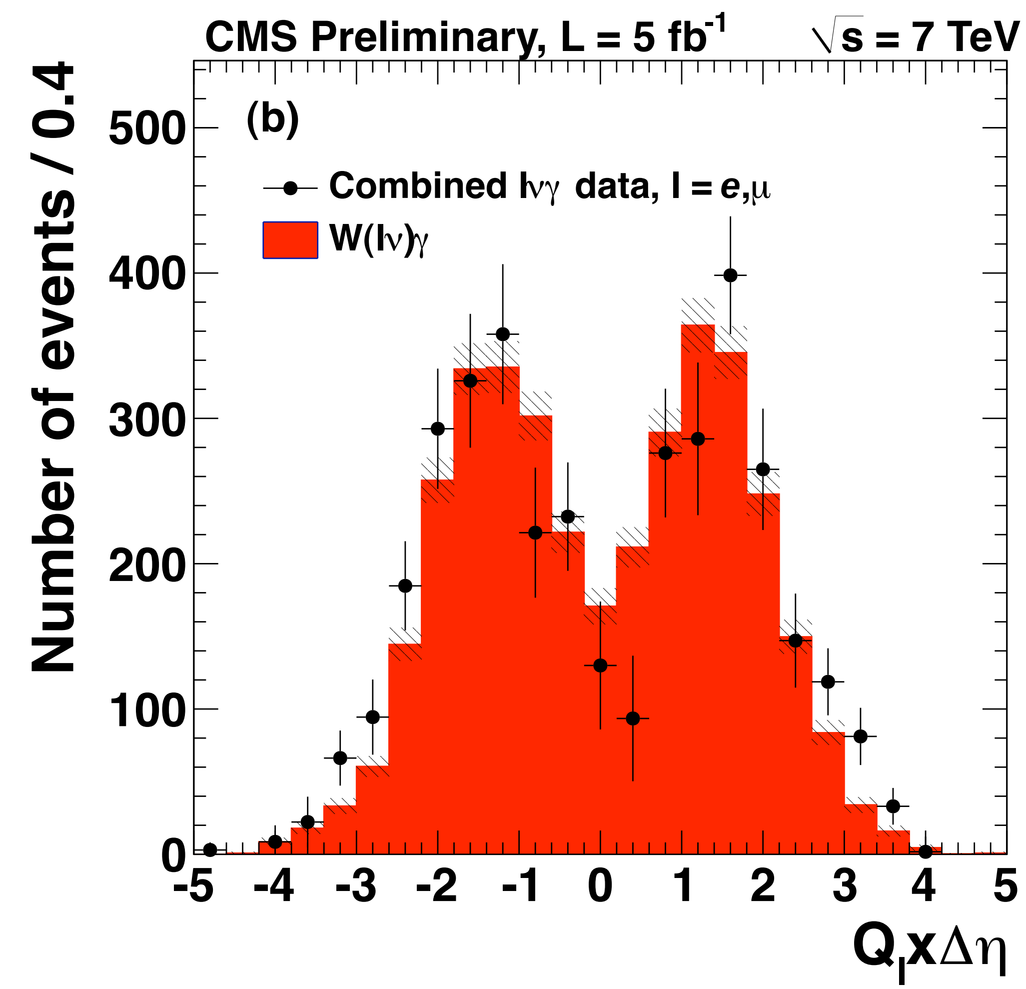
Experimental evidence for RAZ

- Experimental issues that wash out dip:
  - use of lepton rapidity rather than reconstructing W (retains most information)
  - contamination from photon radiation in W decay

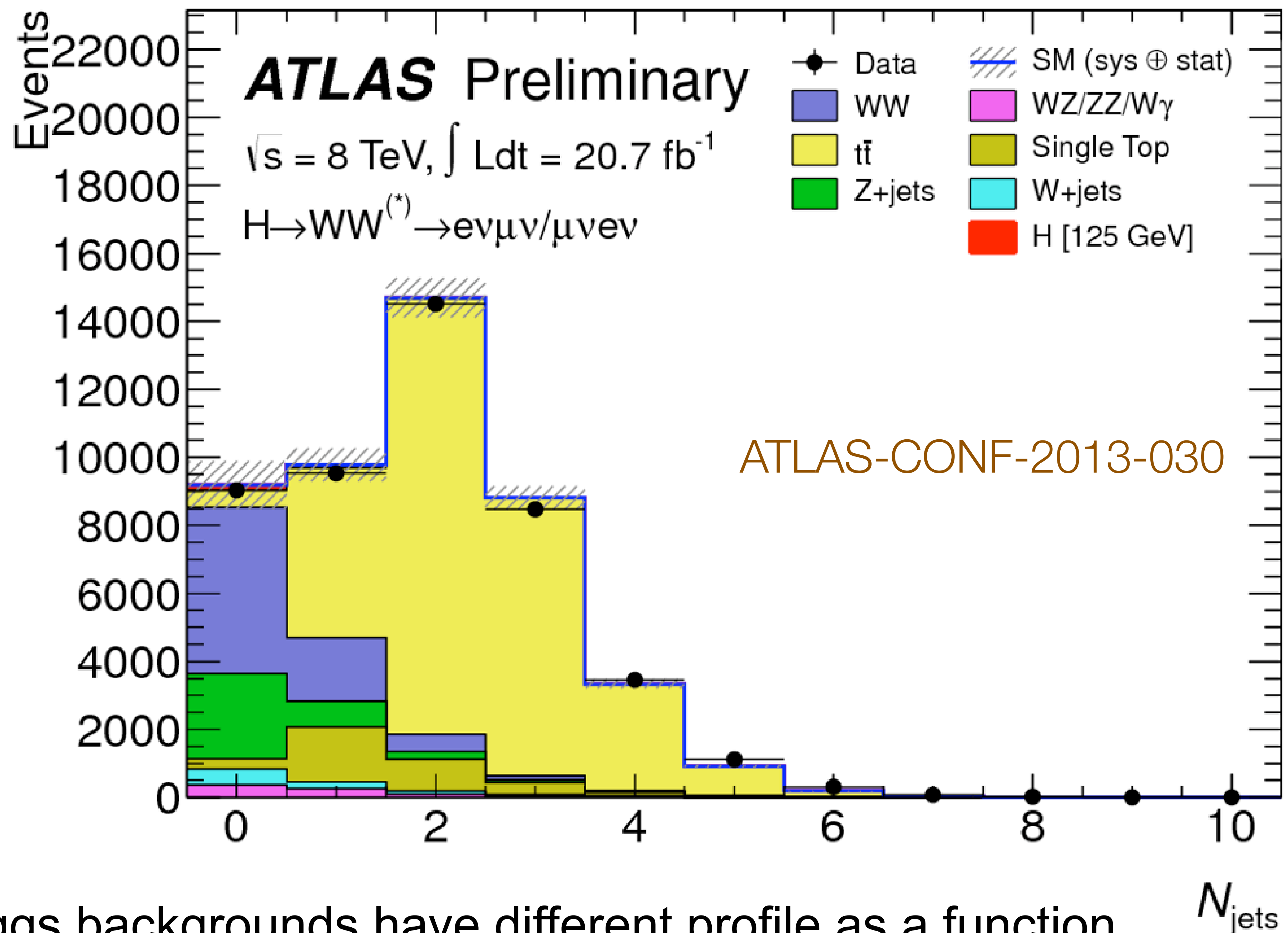
D0, arXiv 1109.4432



CMS, PAS-EWK-11-009



WW: the importance of jet-binning



- Higgs backgrounds have different profile as a function of the number of jets present in the event
  → important to understand theory the same way: notably, Higgs signal, top and WW backgrounds.

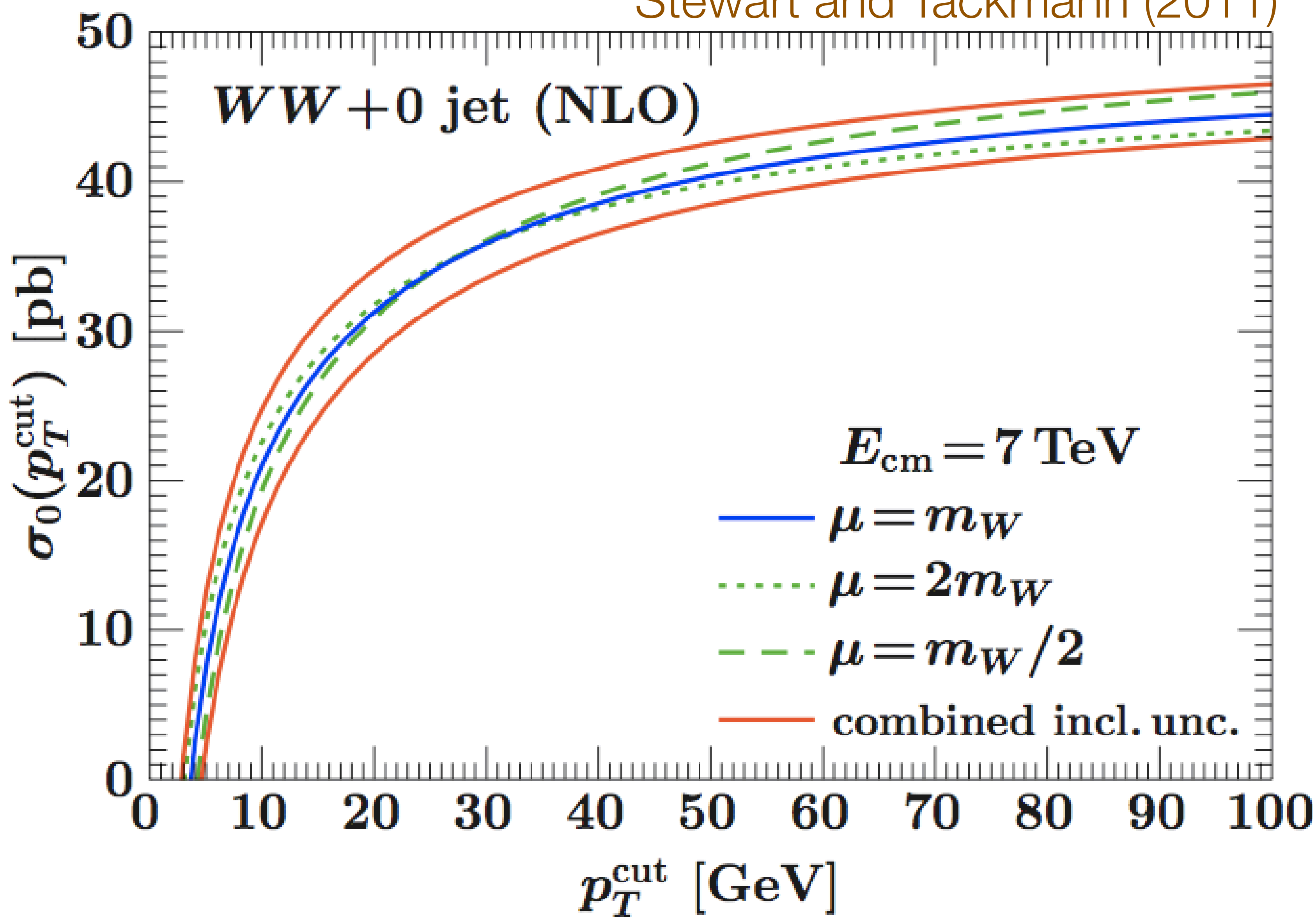
Jet vetoes

- Top backgrounds naturally contain jets: at least partly understood via well-known weak interaction.
- In contrast, WW process only produces jets through QCD.
  - jet-binned cross sections can be subject to larger uncertainties.
- The reason is that the veto is explicitly removing part of the real radiation that is responsible for ensuring that infrared divergences cancel.
  - the incomplete cancellation that results introduces a logarithm into the perturbative expansion;
  - consider an inclusive WW cross section at NLO; naively,
    vetoing jets to obtain 0-jet cross-section is removing a term of order α_s;
  - however, the derivation of the Sudakov factor we sketched earlier tells us that we're actually introducing a factor more like $\alpha_s \log^2[2m_W/p_T^{veto}]$;
    for typical values of the veto this factor is numerically large ~ 3.
  - we should therefore expect worse perturbative behaviour.

Vetoed uncertainties

- However, the usual method of scale variation results in uncertainties for vetoed cross sections smaller than for the inclusive case → too optimistic.
- The accidentally-small variation can be undone by assuming the scale uncertainties in the 0-jet and 1-jet bins are uncorrelated.

Stewart and Tackmann (2011)



$$\Delta^2_{0-\text{jet}} = \Delta^2_{\text{incl.}} + \Delta^2_{1-\text{jet}}$$

New uncertainty much larger across the range of p_T.

Some empirical evidence that this may be *too* conservative.

Real answer is to resum the logarithms → much work in case of H signal.

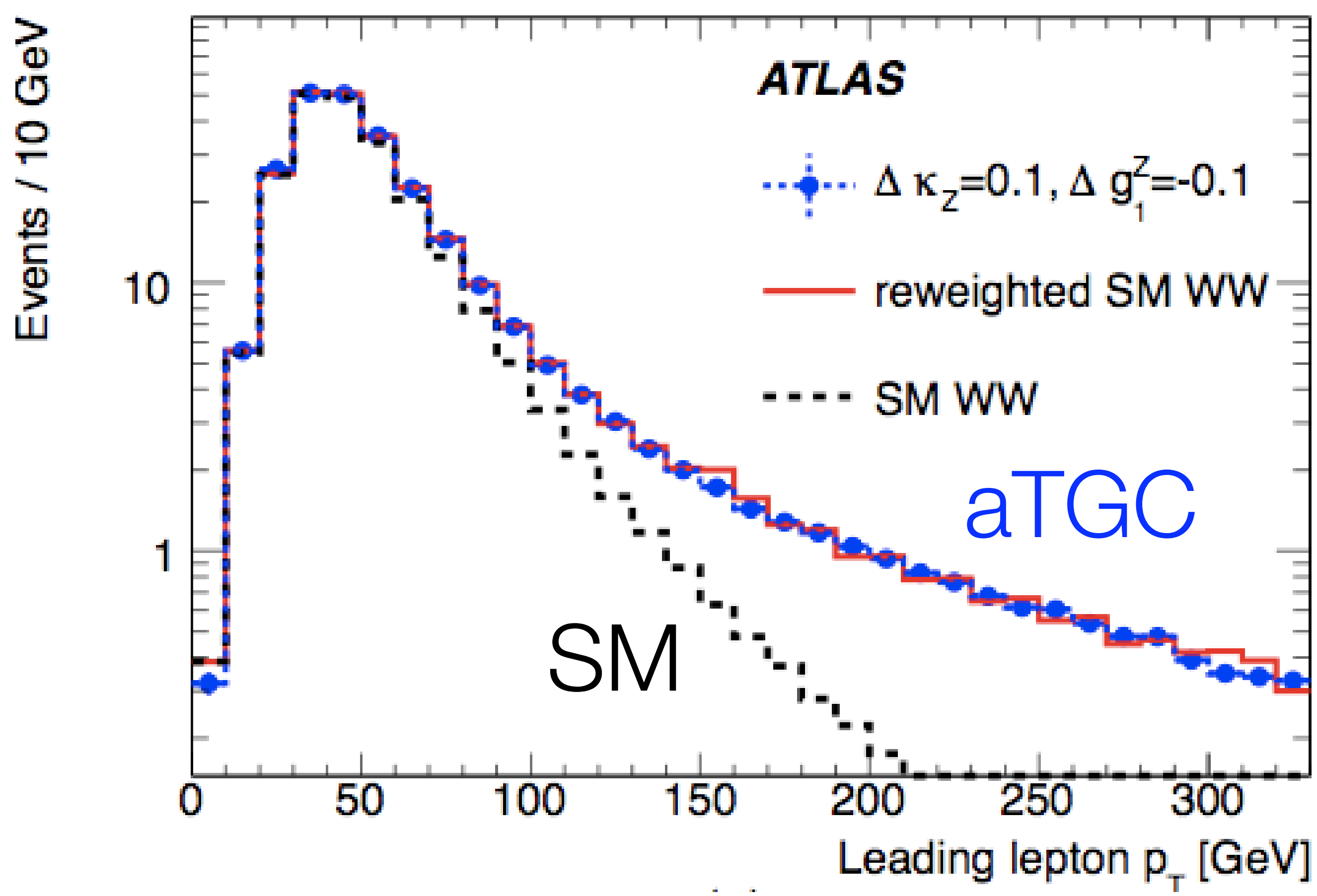
Anomalous triple gauge couplings

- aTGCs usually described in terms of additional interactions in the Lagrangian:

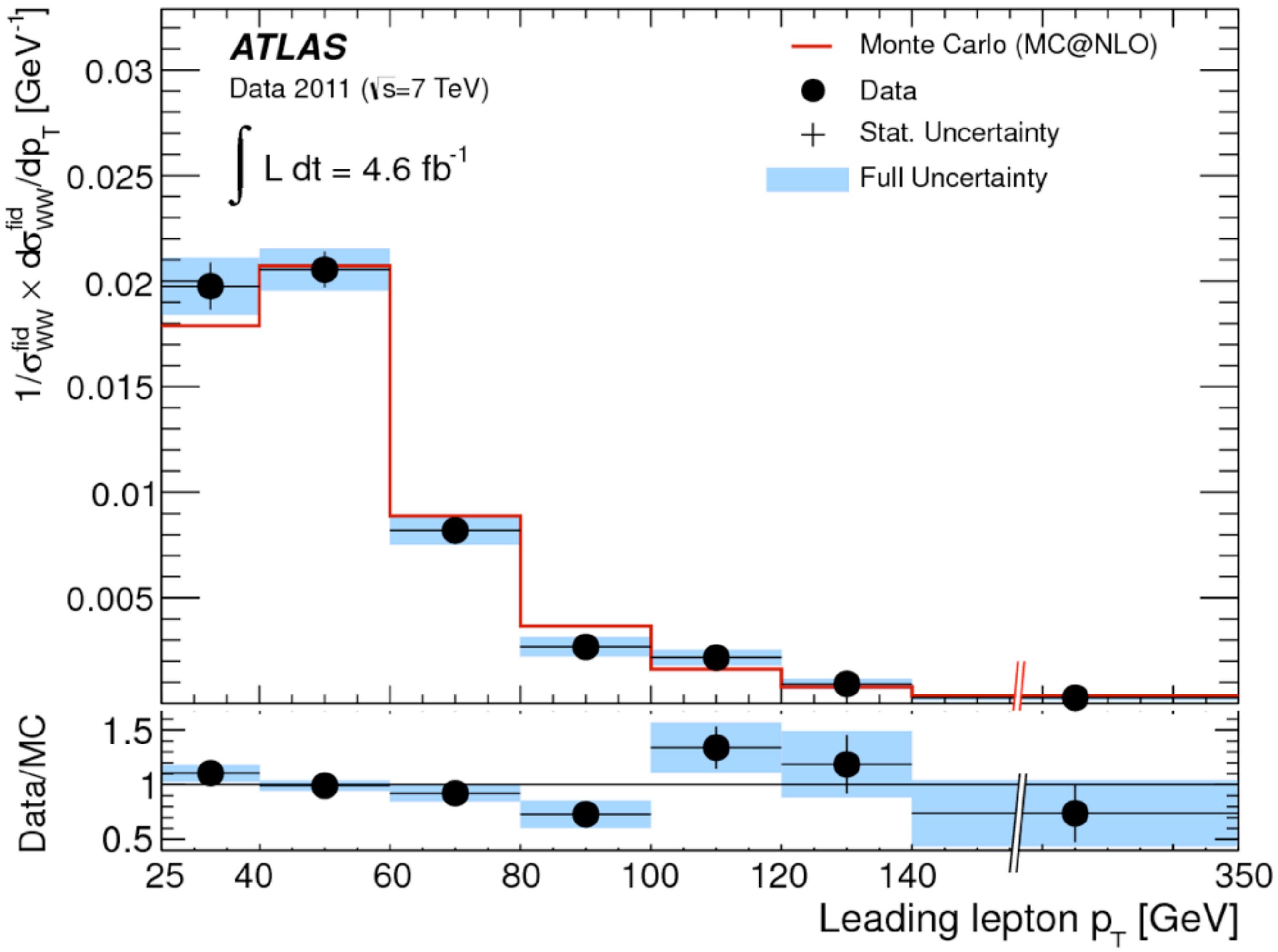
$$\mathcal{L}_{anom} = ig_{WWZ}\Big[\Delta g_1^Z \left(W^*_{\mu\nu}W^\mu Z^\nu - W_{\mu\nu}W^{*\mu}Z^\nu\right) + \Delta\kappa^Z W^*_\mu W_\nu Z^{\mu\nu}$$
$$+\frac{\lambda^Z}{M_W^2}W^*_{\rho\mu}W^\mu_\nu Z^{\nu\rho}\Big] + ig_{WW\gamma}\Big[\Delta\kappa^\gamma W^*_\mu W_\nu \gamma^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2}W^*_{\rho\mu}W^\mu_\nu\gamma^{\nu\rho}\Big]$$

- Most general contribution that separately conserves C and P.
- Operators do not change the predicted cross-section significantly, but instead alter distributions at high p_T, invariant mass, etc.
- This plot, for illustration, uses values of parameters outside current exclusion.
    - need to look for small deviation in tail.

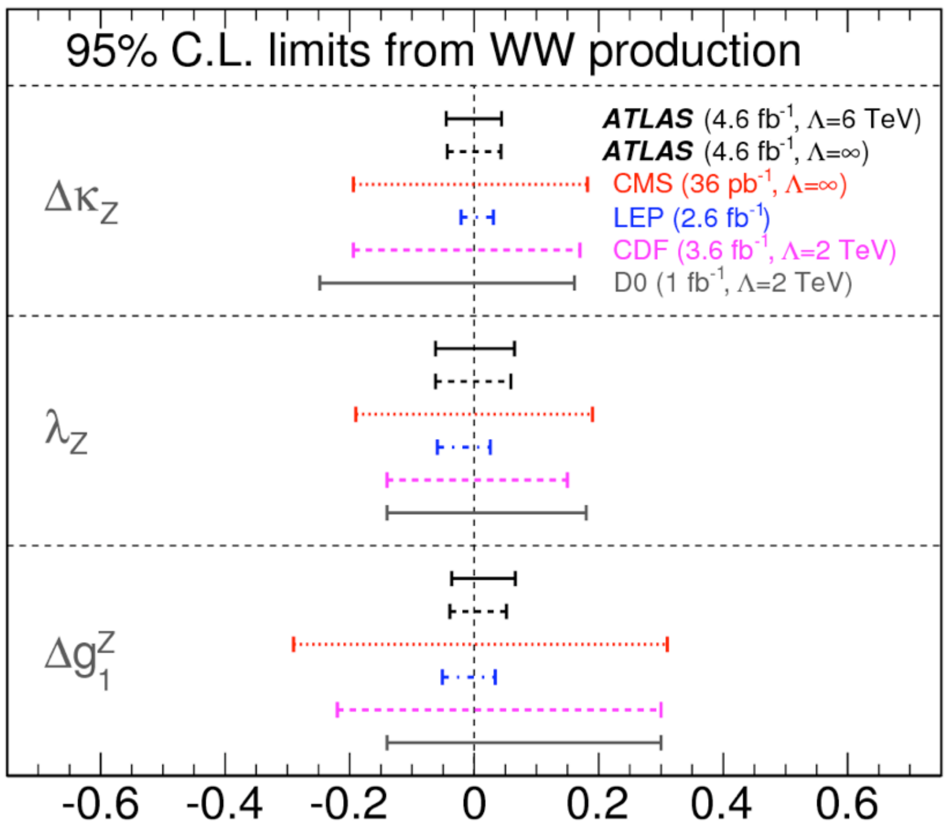
CERN-PH-EP-2012-242



Example of result



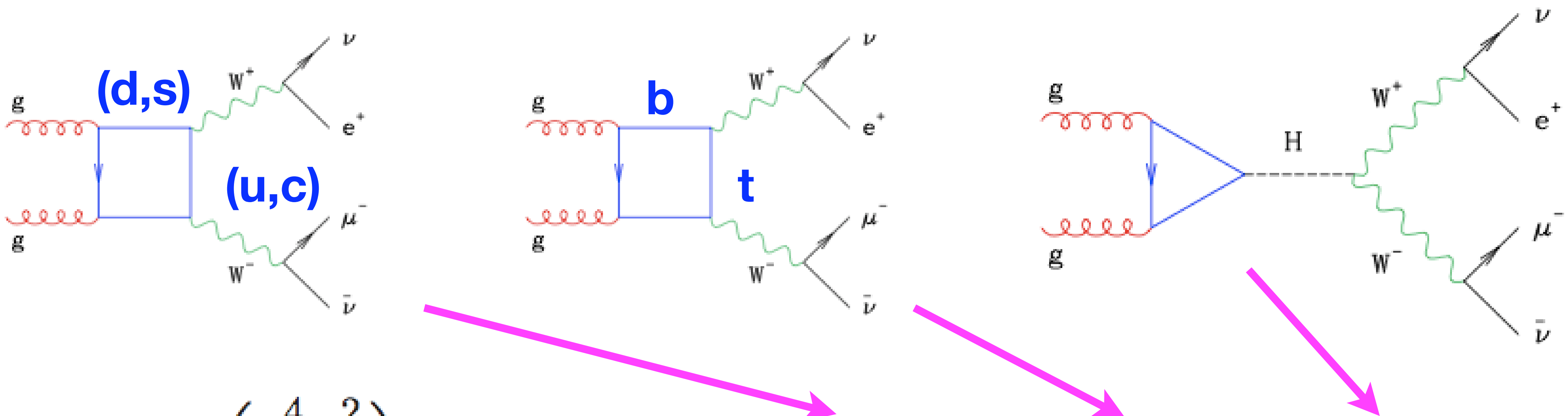
ATLAS WW analysis



Comparison with CMS, Tevatron and LEP

A bit beyond vector bosons ...

- The gluon-initiated WW contribution has the same external particles as the Higgs production process. Should calculate full amplitude before squaring:



$$\mathcal{A}_{\rm full} = \delta^{a_1 a_2}\left(\frac{g_w^4 g_s^2}{16\pi^2}\right)\mathcal{P}_W(s_{34})\mathcal{P}_W(s_{56})\ [2\,\mathcal{A}_{\rm massless} + \mathcal{A}_{\rm massive} + \mathcal{A}_{\rm Higgs}]$$

- Is the interference important? Need to check in view of importance to extracting couplings.
- How do we define signal and background?
  - at what point is the Higgs boson just another SM contribution?

Notation

$$\mathcal{A}_{\text{full}} = \delta^{a_1 a_2} \left(\frac{g_w^4 g_s^2}{16\pi^2} \right) \mathcal{P}_W(s_{34}) \mathcal{P}_W(s_{56}) \; \underbrace{[2\,\mathcal{A}_{\text{massless}} + \mathcal{A}_{\text{massive}}}_{\mathcal{A}_{\text{box}}} + \mathcal{A}_{\text{Higgs}}]$$

- Background only: $\sigma_B \longrightarrow |\mathcal{A}_{\text{box}}|^2$
- Signal only: $\sigma_H \longrightarrow |\mathcal{A}_{\text{Higgs}}|^2$

- This is the usual approach. To include the effect of interference define:

$$\sigma_i \longrightarrow 2\text{Re}\left(\mathcal{A}_{\text{Higgs}} \mathcal{A}^*_{\text{box}}\right)$$

- Cross section in the presence of the Higgs, i.e. including also the interference:

$$\sigma_{H,i} \;=\; \sigma_H + \sigma_i$$

- Can then compare results for σ_H and $\sigma_{H,i}$.

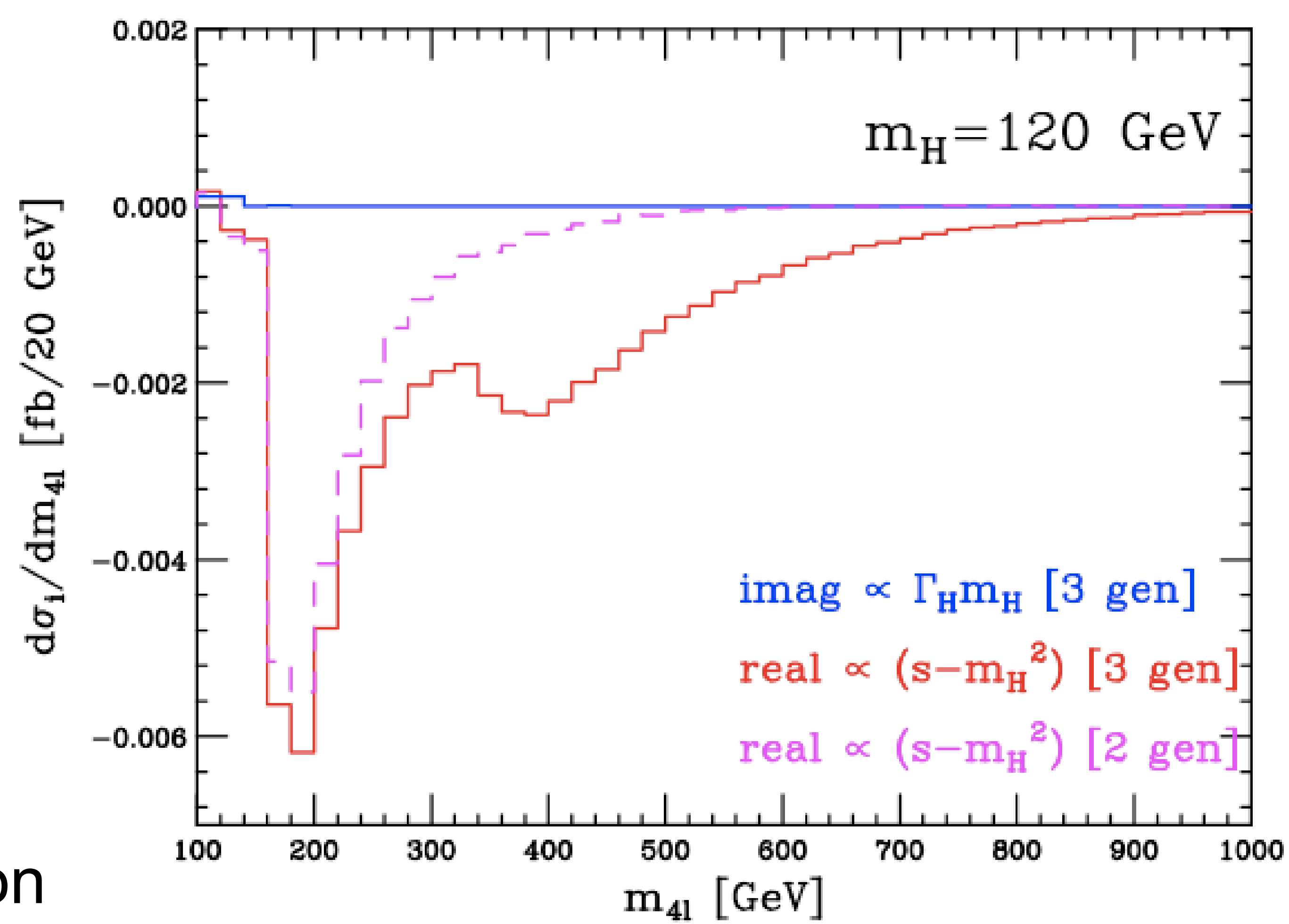
Analyzing the interference

- Separate interference by Re and Im parts of propagator:

$$\delta\sigma_i = \frac{(\hat{s} - m_H^2)}{(\hat{s}-m_H^2)^2 + m_H^2\Gamma_H^2} \,\mathfrak{Re}\left\{2\widetilde{\mathcal{A}}_{\rm Higgs}\mathcal{A}^*_{\rm box}\right\} + \frac{m_H\Gamma_H}{(\hat{s}-m_H^2)^2 + m_H^2\Gamma_H^2} \,\mathfrak{Im}\left\{2\widetilde{\mathcal{A}}_{\rm Higgs}\mathcal{A}^*_{\rm box}\right\}$$

- For our light Higgs the second term is negligible.

- If the full *s*-dependence of the first term can be represented by factor from the propagator, it should vanish on integration (odd about the Higgs mass).
  - but *s*-dependence is more complicated because the box diagrams favour large invariant masses (W pairs).



- Long destructive tail required by unitarity; integrated contribution significant, (negative) 10-15%.

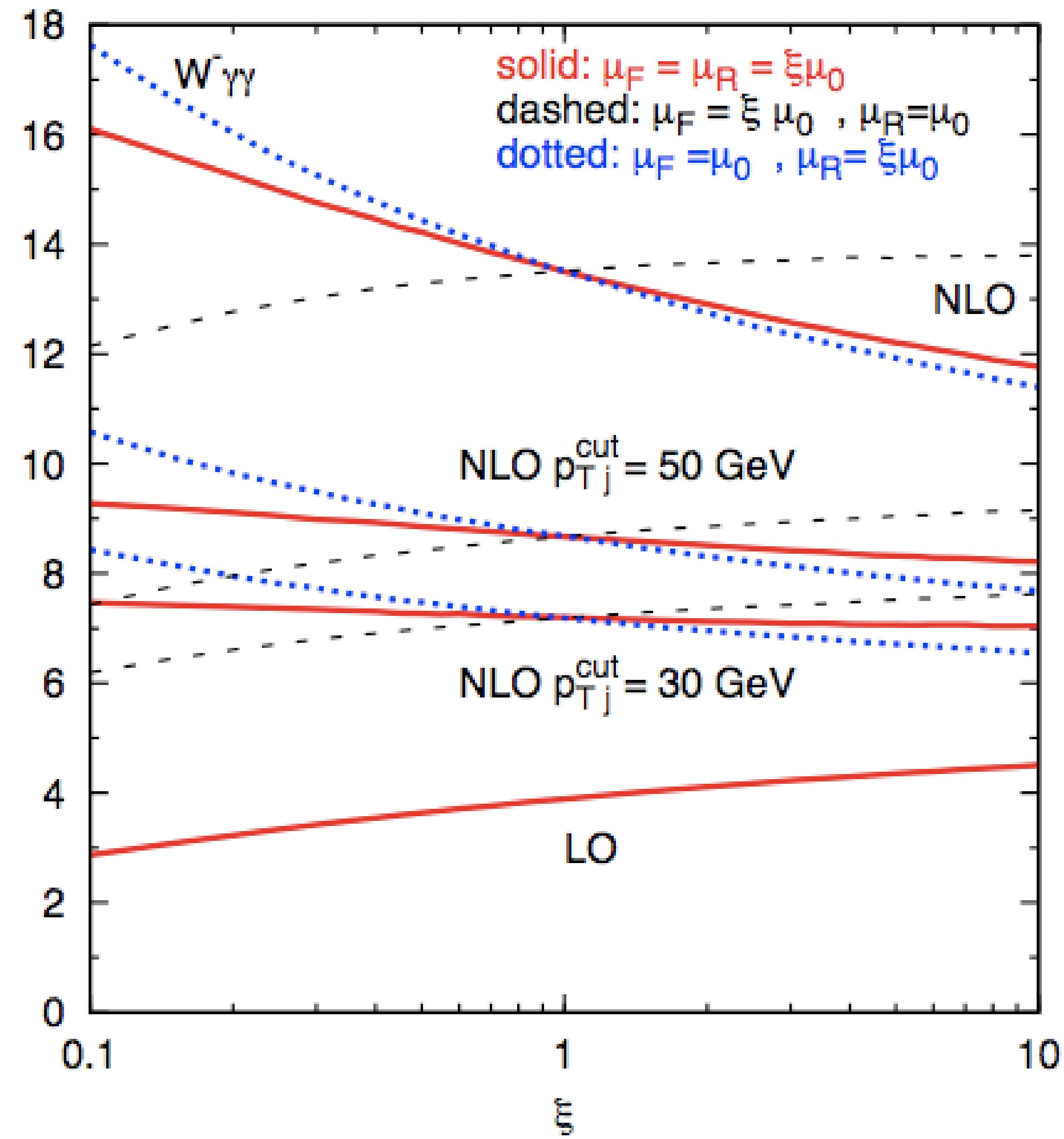
Tri-boson production

- Cross sections very small: after including decays and cuts, cross sections are in the region of tens of femtobarns (at most).
- All modes available in VBFNLO, Zγγ in MCFM.
- Example: Wγγ scale dependence (VBFNLO).

Large enhancement due to gluon flux

Even after strong jet veto, still significant enhancement (partially due to RAZ)

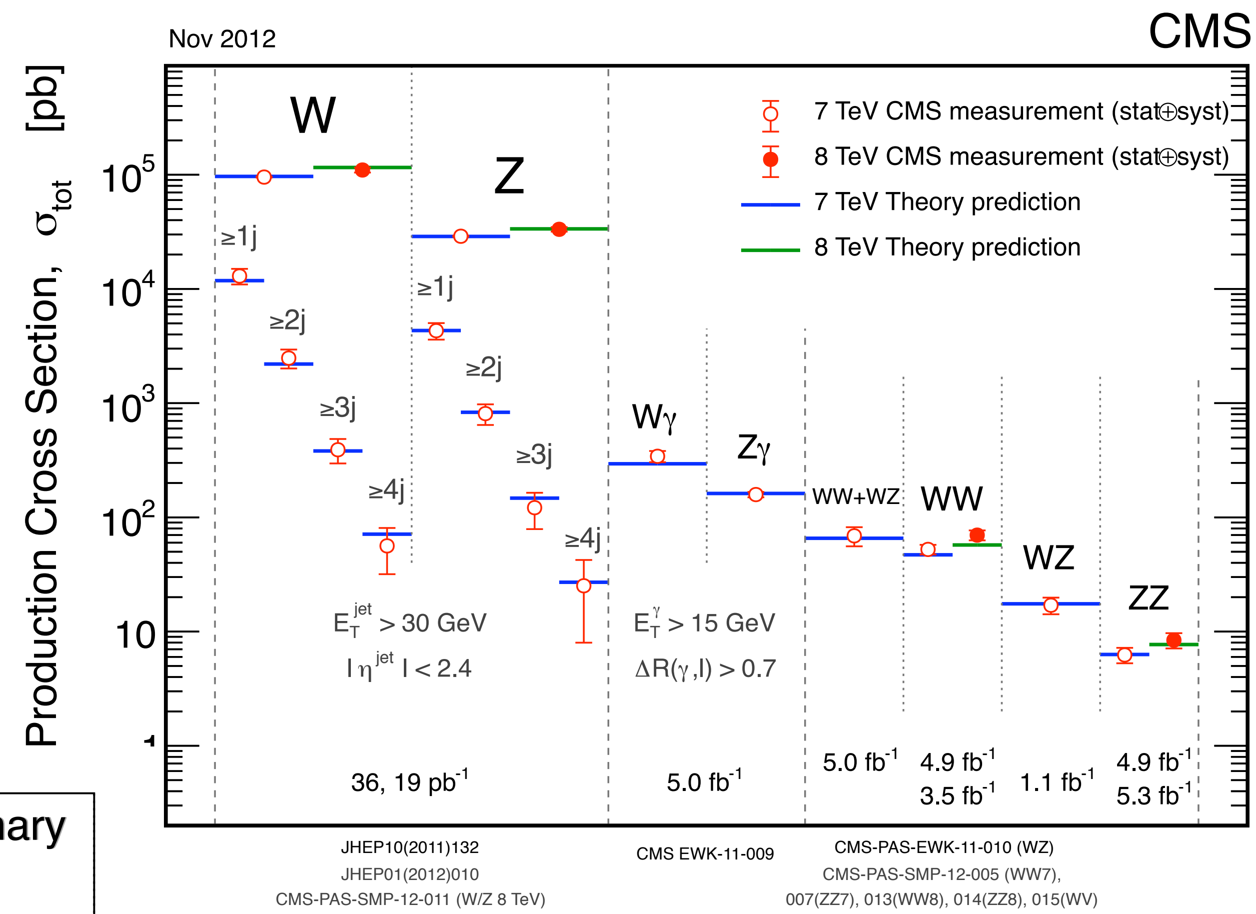
Bozzi et al (2011)

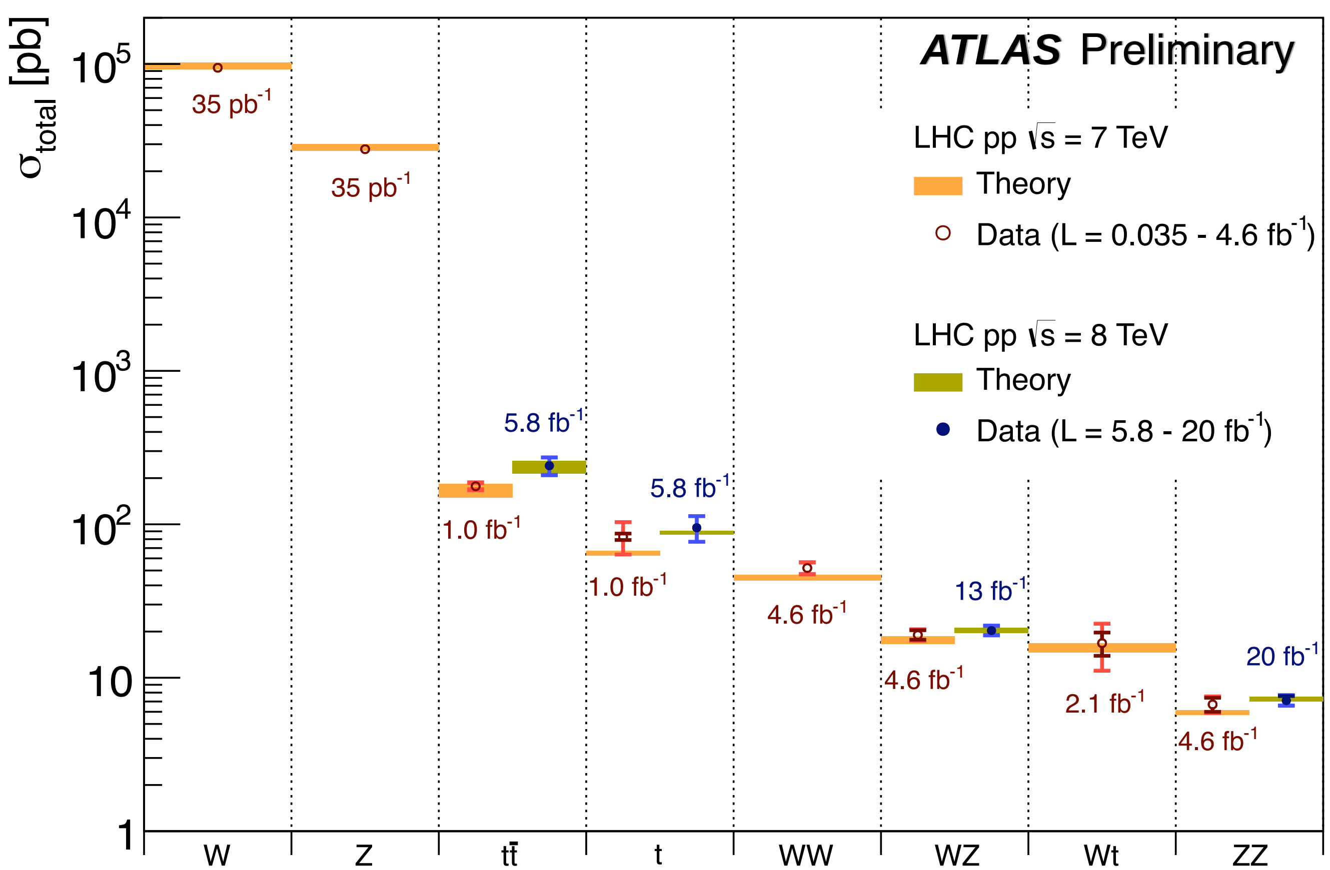


- Era of tri-boson measurements just beginning at LHC.

Vector bosons: experimental summary

Good consistency with expectations of NNLO (W/Z) and NLO (di-bosons) for all processes in both experiments.



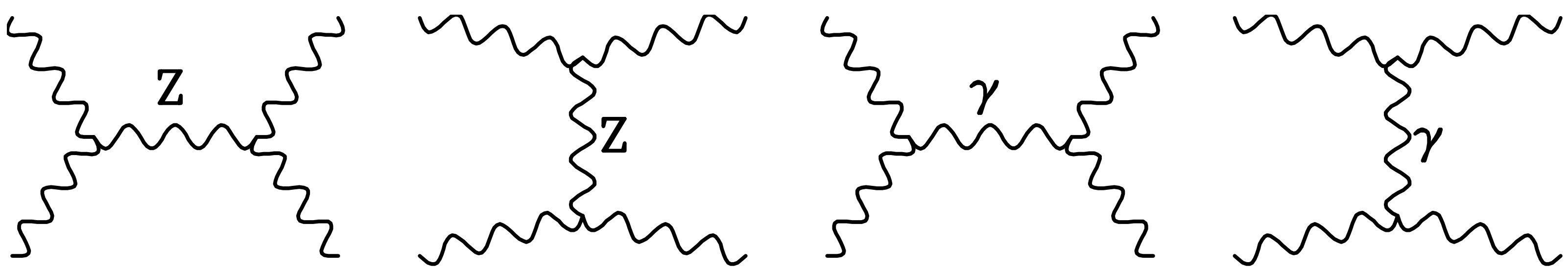


Slight exception: WW has a small error and looks high throughout.

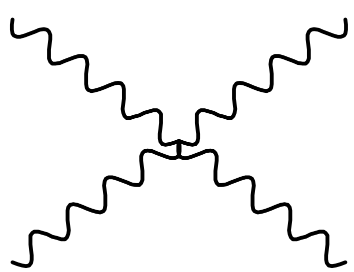
Vector boson scattering

- One way of probing the electroweak sector further is through vector boson scattering.
- Simplest to consider the amplitudes not a hadron collider but in the pure scattering process:

$$W^+(p_+) + W^-(p_-) \rightarrow W^+(q_+) + W^-(q_-)$$



(a) (b) (c) (d)



(e)

Five diagrams all involving self-couplings of the vector bosons

High-energy limit (again)

- Once again consider the high-energy behaviour, concentrating on leading behaviour given by the scattering of longitudinal W bosons.
- Incoming W's along the *z*-axis:

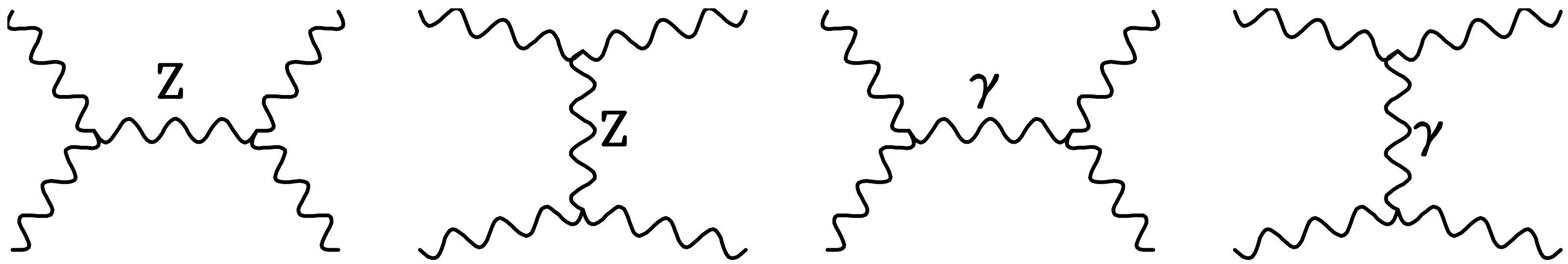
$$
\begin{aligned}
p_\pm &= (E, 0, 0, \pm p)\,, \\
q_\pm &= (E, 0, \pm p \sin\theta, \pm p \cos\theta)
\end{aligned}
$$

  and longitudinal polarizations a slight generalization of previous form:

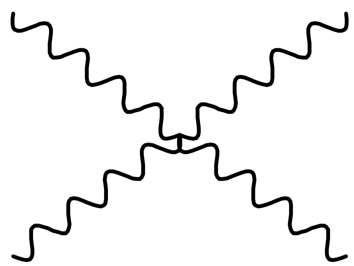
$$
\begin{aligned}
\varepsilon_L(p_\pm) &= \left(\frac{p}{M_W}, 0, 0, \pm\frac{E}{M_W}\right)\,, \\
\varepsilon_L(q_\pm) &= \left(\frac{p}{M_W}, 0, \pm\frac{E}{M_W}\sin\theta, \pm\frac{E}{M_W}\cos\theta\right)
\end{aligned}
$$

- Use these to calculate the form of the diagrams in the high-energy limit, i.e. dropping terms without factor of p^2/m_W^2.

Result



(a) (b) (c) (d)



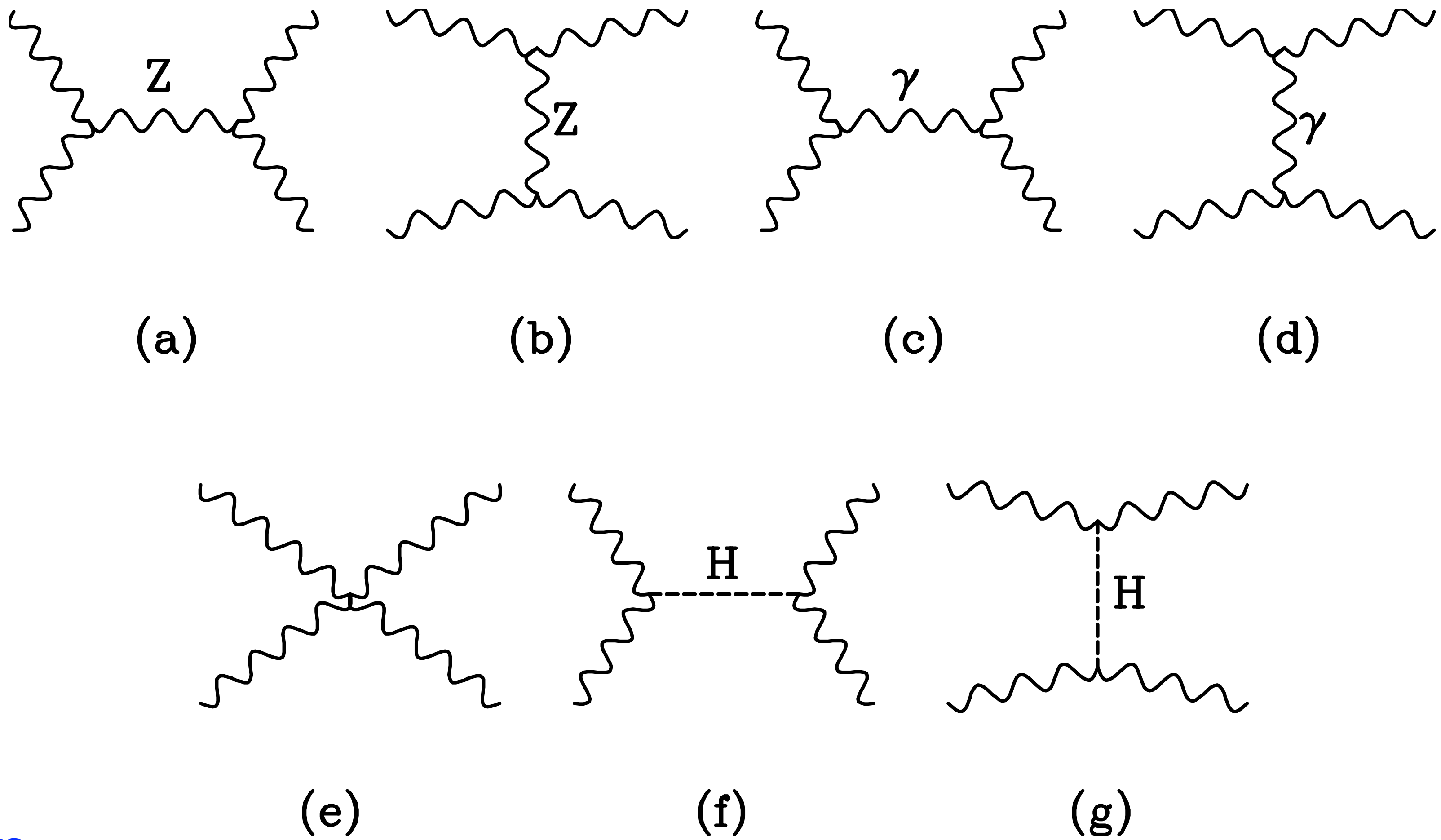
(e)

Leading term (p^4) cancels due to gauge structure, but p^2 remains:

$$T^{e} = g_w^2 \left\{ \frac{p^4}{M_W^4} \left[-3 + 6\cos\theta + \cos^2\theta \right] + \frac{p^2}{M_W^2} \left[-4 + 6\cos\theta + 2\cos^2\theta \right] \right\}$$

$$T^{a-d} = g_w^2 \left\{ \frac{p^4}{M_W^4} \left[3 - 6\cos\theta - \cos^2\theta \right] + \frac{p^2}{M_W^2} \left[\frac{9}{2} - \frac{11}{2}\cos\theta - 2\cos^2\theta \right] \right\}$$

Result (continued)



Cancellation now complete

$$T^{f-g} = g_w^2 \left\{ \frac{p^2}{M_W^2} \left[-\frac{1}{2} - \frac{1}{2}\cos\theta \right] - \frac{M_H^2}{4M_W^2} \left[\frac{s}{s-M_H^2} + \frac{t}{t-M_H^2} \right] \right\}$$

$$T^{e} = g_w^2 \left\{ \frac{p^4}{M_W^4} \left[-3 + 6\cos\theta + \cos^2\theta \right] + \frac{p^2}{M_W^2} \left[-4 + 6\cos\theta + 2\cos^2\theta \right] \right\}$$

$$T^{a-d} = g_w^2 \left\{ \frac{p^4}{M_W^4} \left[3 - 6\cos\theta - \cos^2\theta \right] + \frac{p^2}{M_W^2} \left[\frac{9}{2} - \frac{11}{2}\cos\theta - 2\cos^2\theta \right] \right\}$$

Discussion

- As a result, WW scattering amplitude does not diverge at high-energy.
  - however, it may still be too large for perturbative unitarity to hold
- Can examine using a partial-wave analysis.

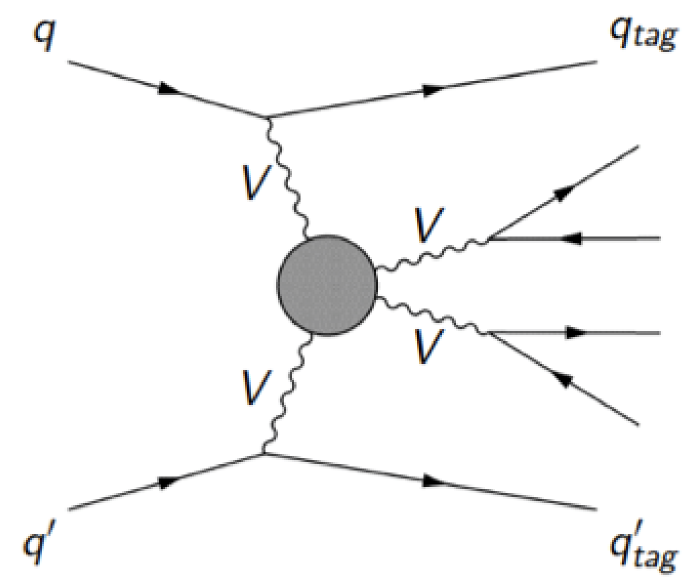
  Lee, Quigg and Thacker (1977)
- Looking at all channels of vector boson scattering and requiring unitarity results in a constraint on the Higgs boson mass:

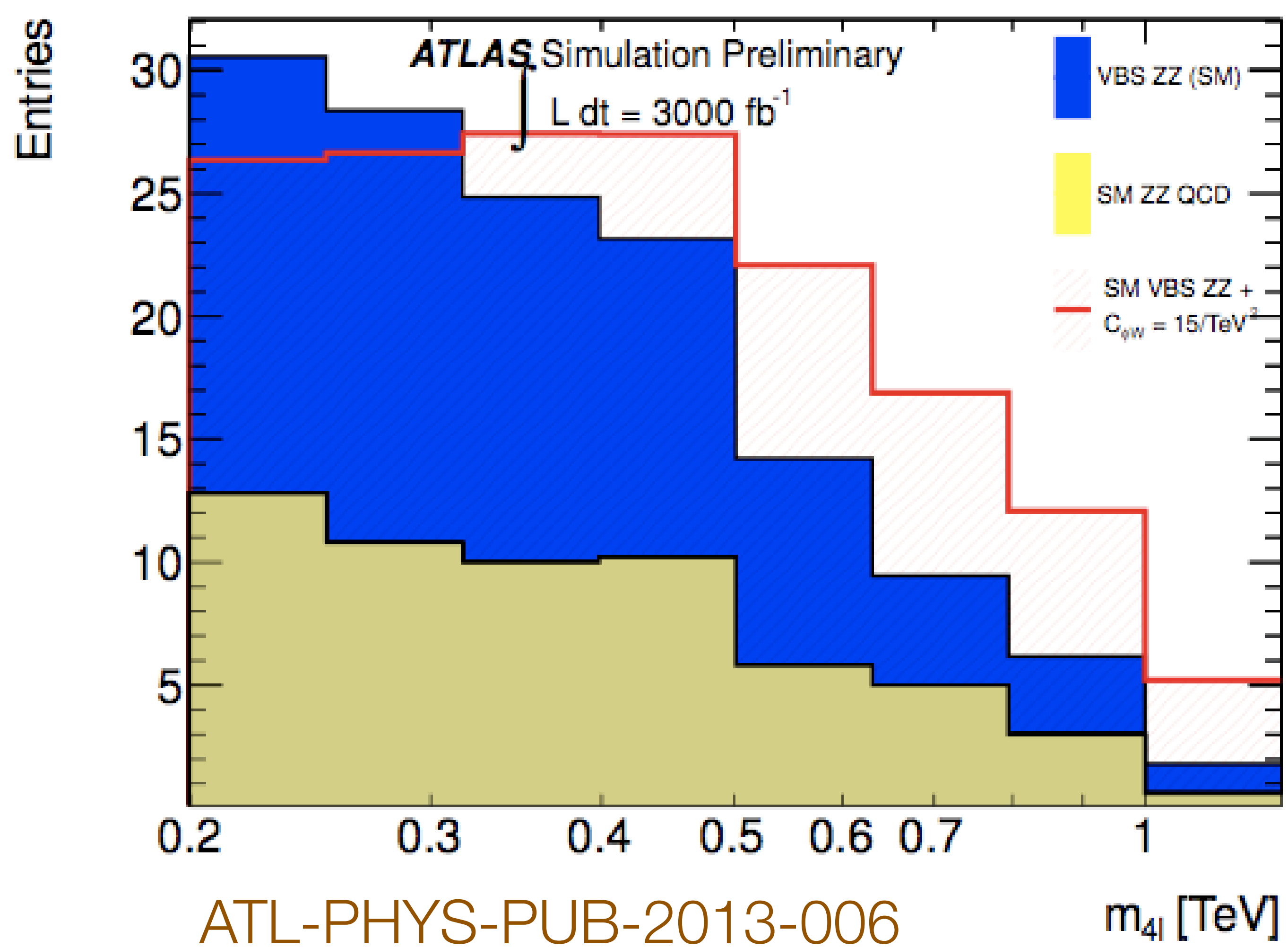
$$M_H < \left(\frac{8\sqrt{2}\pi}{3G_F}\right)^{\frac{1}{2}} \approx 1 \text{ TeV}$$

- Observation of a Higgs boson violating this bound would have meant strong interactions of W,Z bosons that could not be described perturbatively.
- Even with a light Higgs, it is possible that it is not entirely responsible for the unitarization at high energies
  - essential to probe vector boson scattering to look for anomalous couplings/ hints of new particles.

Recent study

- Like vector-boson fusion: induce scattering in association with two forward jets.
- Sensitivity to operators not probed in di-boson production ($C_{\Phi W}$ here).
- σ_{SM} ~ 0.5 pb (w/o decays), need very high luminosity.





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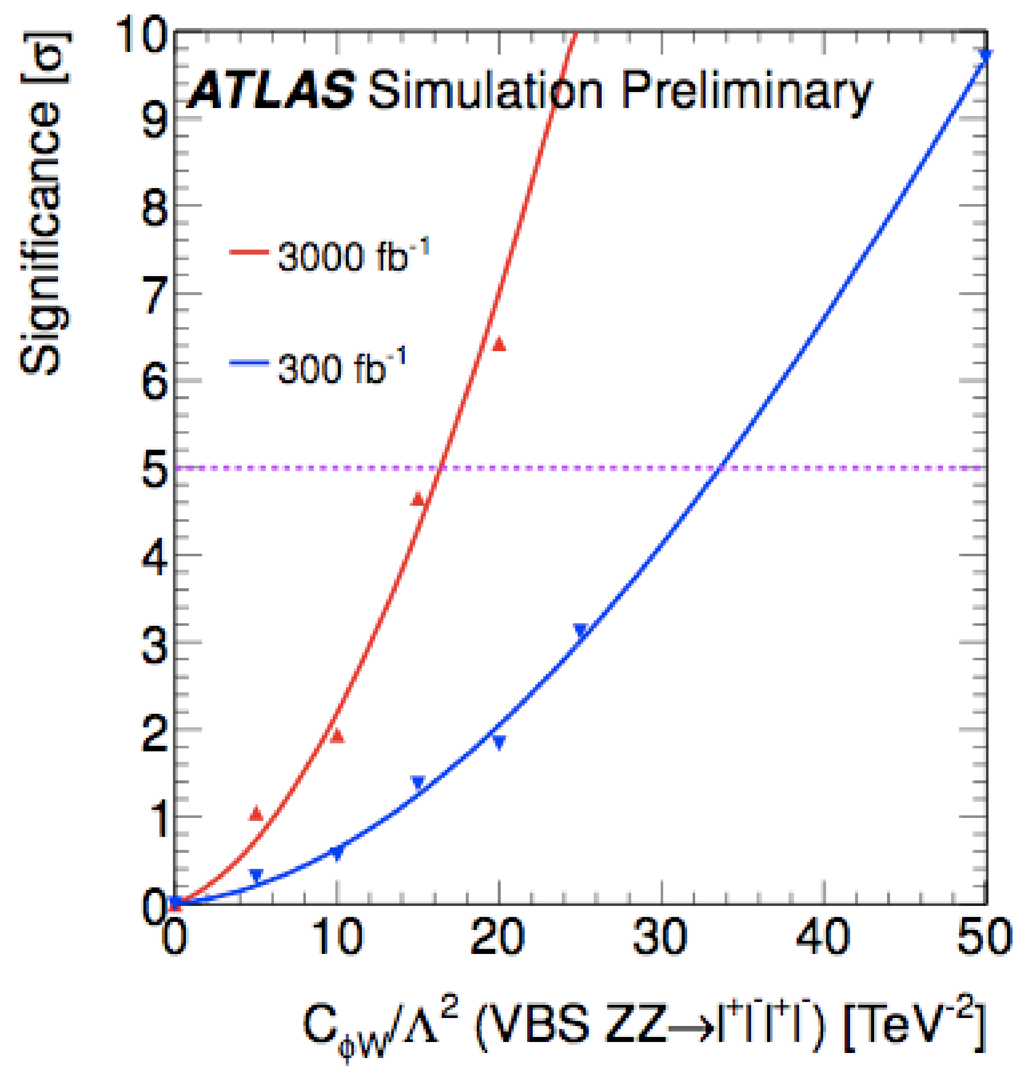

Summary

- The importance of multi-boson production.
  - role of self-interactions (gauge structure) in taming high-energy behaviour
- Review of selected di-boson phenomenology.
  - radiation amplitude zero, jet-binning, aTGCs, interference
- Beyond inclusive di-boson measurements.
  - the importance of vector boson scattering