Matching & Merging of Parton Showers and Matrix Elements

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Outline

introduction: why event generators?

and why is precision an issue?

Physics at the LHC & the need for event generators

- proton-proton collisions at the LHC:
  - processes with the highest energies ever at accelerator experiments
  - characteristically, signals and their backgrounds from hard interactions, with many particles in the final state

    (due to strong interaction and huge phase space)
  - complex final states in many channels,
    hard to gain detailed quantitative understanding from first principles/analytical work

need simulation $\Longrightarrow$ event generators

The dual role of event generators

- dichotomy in the application/use of event generators by experiments
- "experimental tool":
  - unfolding of the detector,
  - determination of acceptance and corrections, ...
  - grasping corrections due to hadronisation, multiple interactions etc.
  - typically many parameters, allowing for more freedom
  - can be improved by improved parametrisations and tuning

(clearly, an understanding of physics helps in devising successful parametrisations!)

- "theory tool":

(this is the kind of tool SHERPA strives to be)

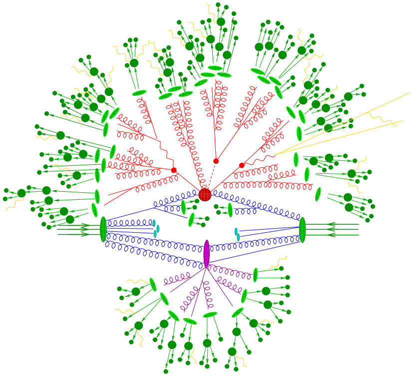
  - accurate description of signal and background,
  - extrapolation from control to signal region,
  - extraction of physics through comparison with data, ...
  - typically few parameters, allowing for less freedom
  - can be improved by improved accuracy of underlying calculations

(this yields improved/reduced errors)

The inner working of event generators . . .

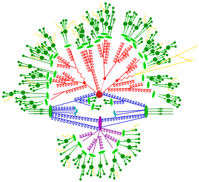
simulation: divide et impera

- hard process:
  fixed order perturbation theory
  traditionally: Born-approximation
- bremsstrahlung:
  resummed perturbation theory
- hadronisation:
  phenomenological models
- hadron decays:
  effective theories, data
- "underlying event":
  phenomenological models



. . . and possible improvements

possible strategies:



- improving the phenomenological models:
  - "tuning" (fitting parameters to data)
  - replacing by better models, based on more physics

    (my hot candidate: "minimum bias" and "underlying event" simulation)

- improving the perturbative description:
  - inclusion of higher order exact matrix elements and correct connection to resummation in the parton shower:

    "NLO-Matching" & "Multijet-Merging"
  - systematic improvement of the parton shower: next-to leading (or higher) logs & colours

Reminder: parton-level in perturbation theory

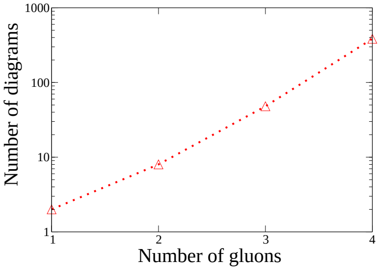
Cross sections at the LHC: Born approximation

$$d\sigma_{ab\to N} = \int_0^1 dx_a dx_b f_a(x_a, \mu_F) f_b(x_a, \mu_F) \int_{\text{cuts}} d\Phi_N \frac{1}{2\hat{s}} \left|\mathcal{M}_{p_a p_b \to N}(\Phi_N; \mu_F, \mu_R)\right|^2$$

- parton densities $f_a(x, \mu_F)$ (PDFs)
- phase space Φ_N for N-particle final states
- incoming current $1/(2\hat{s})$
- squared matrix element $\mathcal{M}_{p_a p_b \to N}$ (summed/averaged over polarisations)
- renormalisation and factorisation scales μ_R and μ_F
- complexity demands numerical methods for large N

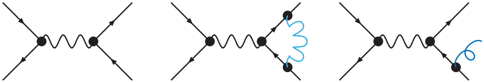
Complexity: factorial growth in $e^+e^- \to q\bar{q} + ng$

n	$\#_{\text{diags}}$
0	1
1	2
2	8
3	48
4	384



Higher orders: some general thoughts

- obtained from adding diagrams with additional:
  loops (virtual corrections) or legs (real corrections)



- effect: reducing the dependence on μ_R & μ_F

  (NLO first order allowing for meaningful estimate of uncertainties)

- additional difficulties when going NLO:
  ultraviolet divergences in virtual correction
  infrared divergences in real and virtual correction

  enforce
  UV regularisation & renormalisation
  IR regularisation & cancellation

  (Kinoshita–Lee–Nauenberg–Theorem)

Structure of an NLO calculation

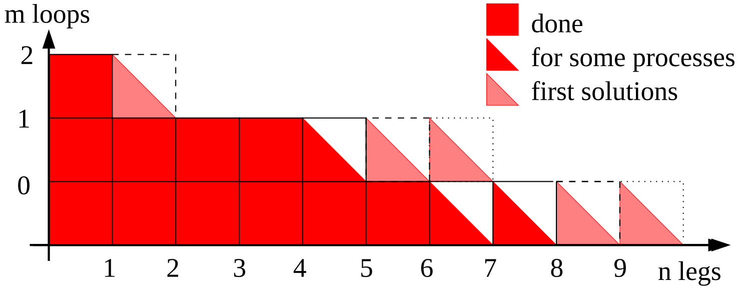
- sketch of cross section calculation

$$d\sigma_N^{(\mathrm{NLO})} = \underbrace{d\Phi_N \mathcal{B}_N}_{\substack{\text{Born}\\ \text{approximation}}} + \underbrace{d\Phi_N \mathcal{V}_N}_{\substack{\text{renormalised}\\ \text{virtual correction}\\ \text{IR-divergent}}} + \underbrace{d\Phi_{N+1} \mathcal{R}_N}_{\substack{\text{real correction}\\ \text{IR-divergent}}}$$

$$= d\Phi_N \left[\mathcal{B}_N + \mathcal{V}_N + \mathcal{B}_N \otimes \mathcal{S}\right] + d\Phi_{N+1}\left[\mathcal{R}_N - \mathcal{B}_N \otimes d\mathcal{S}\right]$$

- subtraction terms $\mathcal{S}$ (integrated) and $d\mathcal{S}$:
  exactly cancel IR divergence in $\mathcal{R}$ – process-independent structures
- result: terms in both brackets separately infrared finite

Availability of exact calculations for hadron colliders



Parton showers

Probabilistic treatment of emissions

- Sudakov form factor (**no-decay** probability)

$$\Delta^{(\mathcal{K})}_{ij,k}(t,t_0) = \exp\left[-\int\limits_{t_0}^{t}\frac{\mathrm{d}t}{t}\frac{\alpha_S}{2\pi}\int \mathrm{d}z\frac{\mathrm{d}\phi}{2\pi}\quad\underbrace{\mathcal{K}_{ij,k}(t,z,\phi)}_{\substack{\text{splitting kernel for}\\ (ij)\to ij\ (\text{spectator } k)}}\quad\right]$$

- evolution parameter t defined by kinematics

generalised angle (HERWIG++) or transverse momentum (PYTHIA, SHERPA)

- will replace $\frac{\mathrm{d}t}{t}\mathrm{d}z\frac{\mathrm{d}\phi}{2\pi} \longrightarrow \mathrm{d}\Phi_1$
- scale choice for strong coupling: $\alpha_S(k_\perp^2)$

resums classes of higher logarithms

- regularisation through cut-off t_0

Emissions off a Born matrix element

- "compound" splitting kernels $\mathcal{K}_n$ and Sudakov form factors $\Delta_n^{(\mathcal{K})}$ for emission off n-particle final state:

$$\mathcal{K}_n(\Phi_1) = \frac{\alpha_S}{2\pi} \sum_{\text{all }\{ij,k\}} \mathcal{K}_{ij,k}(\Phi_{ij,k}) \,, \quad \Delta_n^{(\mathcal{K})}(t, t_0) = \exp\left[-\int\limits_{t_0}^{t} \mathrm{d}\Phi_1\, \mathcal{K}_n(\Phi_1) \right]$$

- consider first emission only off Born configuration

$$\begin{aligned} \mathrm{d}\sigma_B &= \mathrm{d}\Phi_N\, \mathcal{B}_N(\Phi_N) \\ &\cdot \underbrace{\left\{ \Delta_N^{(\mathcal{K})}(\mu_N^2, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \left[\mathcal{K}_N(\Phi_1) \Delta_N^{(\mathcal{K})}(\mu_N^2, t(\Phi_1)) \right] \right\}}_{\text{integrates to unity} \longrightarrow \text{“unitarity” of parton shower}} \end{aligned}$$

- further emissions by recursion with $Q^2 = t$ of previous emission

Aside: connection to resummation formalism

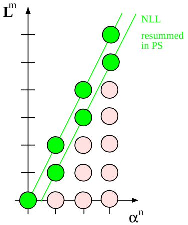
- consider standard Collins-Soper-Sterman formalism (CSS):

$$\frac{\mathrm{d}\sigma_{AB\to X}}{\mathrm{d}y\,\mathrm{d}Q_\perp^2} = \mathrm{d}\Phi_X\,\mathcal{B}_{ij}(\Phi_X)\cdot\underbrace{\int\frac{\mathrm{d}^2b_\perp}{(2\pi)^2}\exp(i\vec{b}_\perp\cdot\vec{Q}_\perp)}_{\text{guarantee 4-mom conservation}}\underbrace{\tilde{W}_{ij}(b;\,\Phi_X)}_{\text{higher orders}}$$

with

$$\tilde{W}_{ij}(b;\,\Phi_X) = \overbrace{C_i(b;\,\Phi_X,\,\alpha_S)C_j(b;\,\Phi_X,\,\alpha_S)}^{\text{collinear bits}}\overbrace{H_{ij}(\alpha_S)}^{\text{loops}}$$
$$\underbrace{\exp\left[-\int\limits_{1/b_\perp^2}^{Q_X^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\left(A(\alpha_S(k_\perp^2))\log\frac{Q_X^2}{k_\perp^2}+B(\alpha_S(k_\perp^2))\right)\right]}_{\text{Sudakov form factor, } A,\ B \text{ expanded in powers of } \alpha_S}$$

Connection to resummation formalism: log accuracy



- analyse structure of emissions above
- logarithmic accuracy in $\log\frac{\mu_N}{k_\perp}$ (a la CSS)
  possibly up to next-to leading log,
  - if evolution parameter $\sim$ transverse momentum,
  - if argument in α_S is $\propto$ $k_\perp$ of splitting,
  - if $K_{ij,k} \to$ terms $A_{1,2}$ and B_1 upon integration

  (okay, if soft gluon correction is included, and if $K_{ij,k}$ $\to$ AP splitting kernels)
- in CSS $k_\perp$ typically is the transverse momentum of produced system, in parton shower of course related to the cumulative effect of explicit multiple emissions
- resummation scale $\mu_N \approx \mu_F$ given by (Born) kinematics –
  simple for cases like $q\bar{q}' \to V$, $gg \to H$, ...
  tricky for more complicated cases

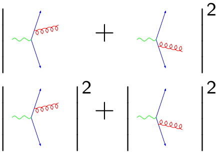
A simple improvement: matrix element corrections

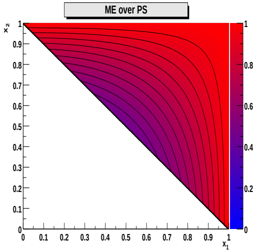
(M. Seymour, Comp. Phys. Comm. 90 (1995) 95 & E. Norrbin & T. Sjostrand, Nucl. Phys. B603 (2001) 297)

- parton shower ignores interferences typically present in matrix elements
- pictorially

ME :

PS :





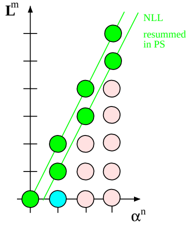
- form many processes $\mathcal{R}_N < \mathcal{B}_N \times \mathcal{K}_N$
- typical processes: $q\bar{q}' \to V$, $e^-e^+ \to q\bar{q}$, $t \to bW$
- practical implementation: shower with usual algorithm, but reject first/hardest emissions with probability $\mathcal{P} = \mathcal{R}_N/(\mathcal{B}_N \times \mathcal{K}_N)$

- analyse **first** emission, given by

$$\mathrm{d}\sigma_B = \mathrm{d}\Phi_N\,\mathcal{B}_N(\Phi_N)$$

$$\cdot\underbrace{\left\{\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0) + \int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\left[\frac{\mathcal{R}_N(\Phi_N\times\Phi_1)}{\mathcal{B}_N(\Phi_N)}\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t(\Phi_1))\right]\right\}}_{\text{once more: integrates to unity}\ \longrightarrow\ \text{“unitarity” of parton shower}}$$

- radiation given by $\mathcal{R}_N$ (correct at $\mathcal{O}(\alpha_S)$)
  (but modified by logs of higher order in α_S from $\Delta_N^{(\mathcal{R}/\mathcal{B})}$)
- emission phase space constrained by μ_N
- also known as “soft ME correction”
  hard ME correction fills missing phase space
- used for “power shower”:
  $\mu_N \to E_{pp}$ and apply ME correction



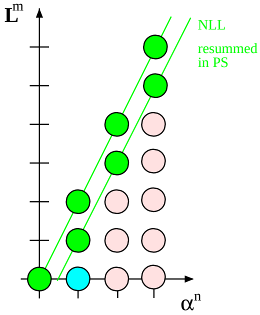
NLO improvements: Matching

NLO matching: Basic idea

- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- adjust ("match") terms:
  - cross section at NLO accuracy &
    correct hardest emission in PS to exactly
    reproduce ME at order α_S
    ($\mathcal{R}$-part of the NLO calculation)

    (this is relatively trivial)
  - maintain (N)LL-accuracy of parton shower

    (this is not so simple to see)



The POWHEG-trick: modifying the Sudakov form factor

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

- reminder: $\mathcal{K}_{ij,k}$ reproduces process-independent behaviour of $\mathcal{R}_N/\mathcal{B}_N$ in soft/collinear regions of phase space

$$\mathrm{d}\Phi_1\,\frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)} \xrightarrow{\text{IR}} \mathrm{d}\Phi_1\,\frac{\alpha_S}{2\pi}\,\mathcal{K}_{ij,k}(\Phi_1)$$

- define modified Sudakov form factor (as in ME correction)

$$\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,t_0) = \exp\left[-\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)}\right],$$

- assumes factorisation of phase space: $\Phi_{N+1} = \Phi_N \otimes \Phi_1$
- typically will adjust scale of α_S to parton shower scale

Local K-factors

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

- start from Born configuration Φ_N with NLO weight:

("local K-factor")

$$\begin{aligned} d\sigma_N^{(\mathrm{NLO})} &= d\Phi_N\, \bar{\mathcal{B}}(\Phi_N) \\ &= d\Phi_N \left\{ \mathcal{B}_N(\Phi_N) + \underbrace{\mathcal{V}_N(\Phi_N) + \mathcal{B}_N(\Phi_N) \otimes \mathcal{S}}_{\tilde{\mathcal{V}}_N(\Phi_N)} \right. \\ &\qquad \left. + \int d\Phi_1 \left[\mathcal{R}_N(\Phi_N \otimes \Phi_1) - \mathcal{B}_N(\Phi_N) \otimes d\mathcal{S}(\Phi_1) \right] \right\} \end{aligned}$$

- by construction: exactly reproduce cross section at NLO accuracy
- note: second term vanishes if $\mathcal{R}_N \equiv \mathcal{B}_N \otimes d\mathcal{S}$

(relevant for MC@NLO)

NLO accuracy in radiation pattern

(P. Nason, JHEP 0411 (2004) 040 & S. Frixione, P. Nason & C. Oleari, JHEP 0711 (2007) 070)

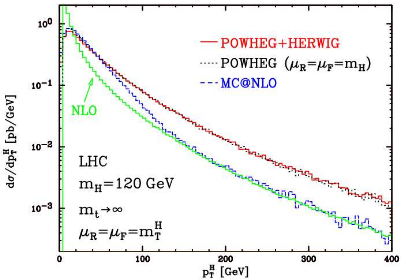
- generate emissions with $\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0)$:

$$\mathrm{d}\sigma_N^{(\mathrm{NLO})} = \mathrm{d}\Phi_N\,\bar{\mathcal{B}}(\Phi_N)$$
$$\times \underbrace{\left\{\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \frac{\mathcal{R}_N(\Phi_N\otimes\Phi_1)}{\mathcal{B}_N(\Phi_N)}\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, k_\perp^2(\Phi_1))\right\}}_{\text{integrating to yield 1 - “unitarity of parton shower”}}$$

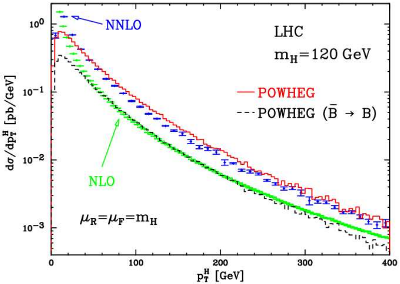
- radiation pattern like in ME correction
- pitfall, again: choice of upper scale μ_N^2 (this is vanilla POWHEG!)
- apart from logs: which configurations enhanced by local K-factor

(K-factor for inclusive production of X adequate for $X+$ jet at large $p_\perp$?)

POWHEG features



S. Alioli, P. Nason, C. Oleari, & E. Re, JHEP 0904 (2009) 002



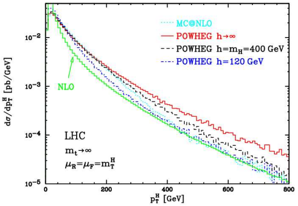
- large enhancement at high $p_{T,h}$
- can be traced back to large NLO correction
- fortunately, NNLO correction is also large $\to \sim$ agreement

Improved POWHEG

S. Alioli, P. Nason, C. Oleari, & E. Re, JHEP 0904 (2009) 002

- split real-emission ME as

$$\mathcal{R} = \mathcal{R}\left(\underbrace{\frac{h^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(S)}} + \underbrace{\frac{p_\perp^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(F)}}\right)$$



- can "tune" h to mimick NNLO - or maybe resummation result
- differential event rate up to first emission

$$\mathrm{d}\sigma = \mathrm{d}\Phi_B \bar{\mathcal{B}}^{(\mathrm{R}^{(S)})}\left[\Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, t_0) + \int\limits_{t_0}^{s} \mathrm{d}\Phi_1 \frac{\mathcal{R}^{(S)}}{\mathcal{B}}\, \Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, k_\perp^2)\right]$$
$$+\mathrm{d}\Phi_R\, \mathcal{R}^{(F)}(\Phi_R)$$

Resummation in MC@NLO

- divide $\mathcal{R}_N$ in soft ("S") and hard ("H") part:

$$\mathcal{R}_N = \mathcal{R}_N^{(S)} + \mathcal{R}_N^{(H)} = \mathcal{B}_N \otimes \mathrm{d}\mathcal{S}_1 + \mathcal{H}_N$$

- identify subtraction terms and shower kernels $\mathrm{d}\mathcal{S}_1 \equiv \sum_{\{ij,k\}} \mathcal{K}_{ij,k}$

(modify $\mathcal{K}$ in 1st emission to account for colour)

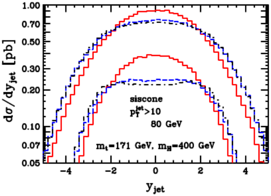
$$\begin{aligned}\mathrm{d}\sigma_N \quad = \quad & \mathrm{d}\Phi_N \underbrace{\tilde{\mathcal{B}}_N(\Phi_N)}_{\mathcal{B}+\tilde{\mathcal{V}}} \left[\Delta_N^{(\mathcal{K})}(\mu_N^2,\, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1\, \mathcal{K}_{ij,k}(\Phi_1)\, \Delta_N^{(\mathcal{K})}(\mu_N^2,\, k_\perp^2)\right] \\ & + \mathrm{d}\Phi_{N+1}\, \mathcal{H}_N\end{aligned}$$

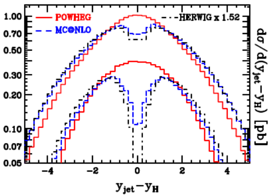
- effect: only resummed parts modified with local K-factor

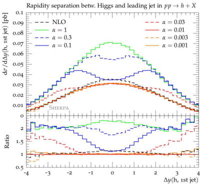
Aside: phase space/K-factor effects

(S. Alioli, P. Nason, C. Oleari, & E. Re, JHEP 0904 (2009) 002 &

S. Hoeche, F. Krauss, M. Schoenherr, & F. Siegert, JHEP 1209 (2012) 049)





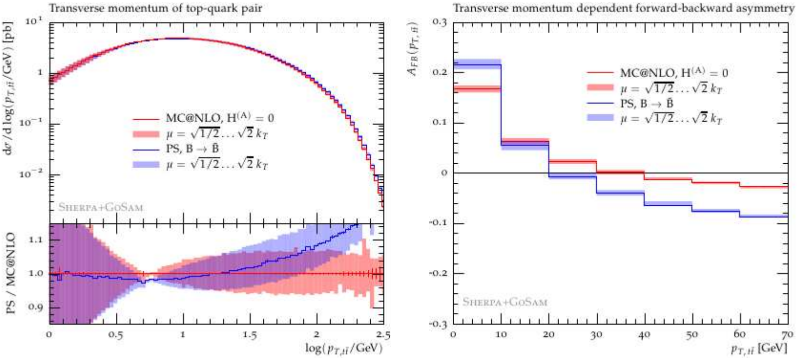


- problem: impact of subtraction terms on local K-factor (filling of phase space by parton shower)
- studied in case of $gg \to H$ above
- proper filling of available phase space by parton shower paramount

Aside': impact of full colour

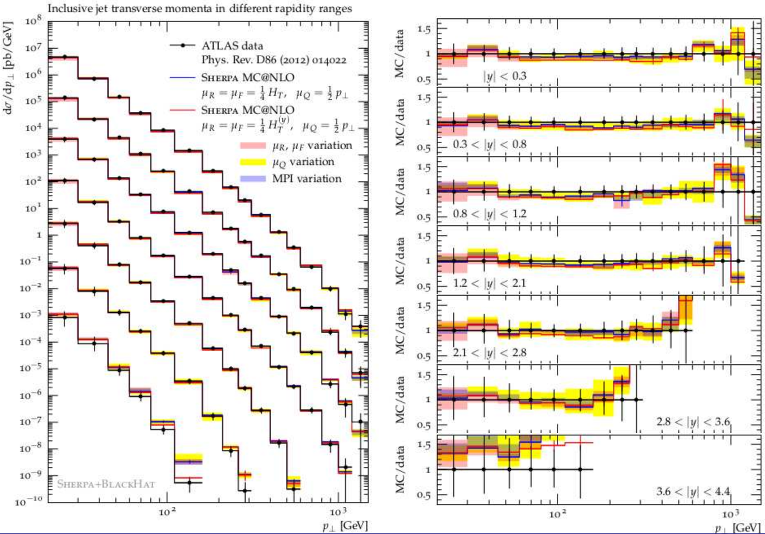
(S. Hoeche, J. Huang, G. Luisoni, M. Schoenherr, & J. Winter, arXiv:1306.2703 [hep-ph])

- evaluate effect of full colour treatment, MC@NLO without **H**-part vs. parton shower with $\mathcal{B} \longrightarrow \tilde{\mathcal{B}}$
- take $t\bar{t}$ production



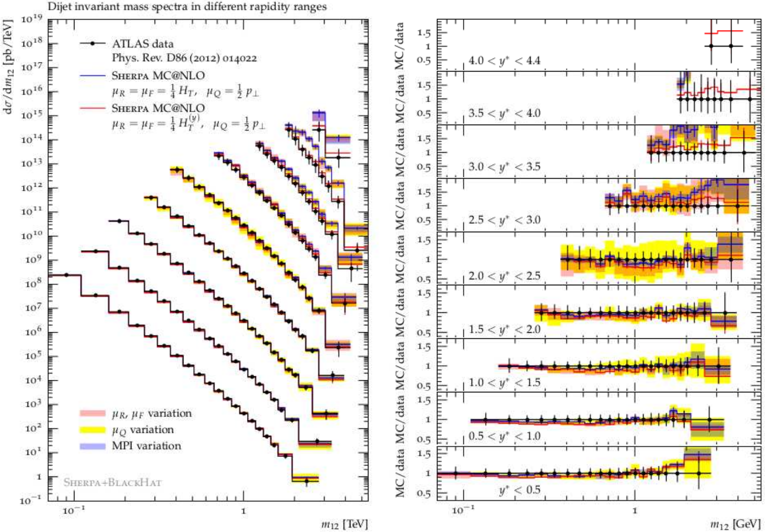
MC@NLO for light jets: jet-$p_\perp$

(S. Hoeche & M. Schoenherr, Phys. Rev. D86 (2012) 094042)



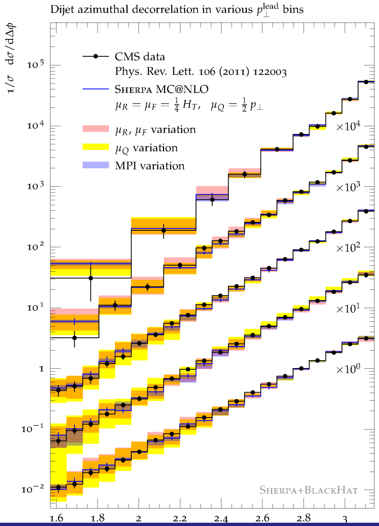
MC@NLO for light jets: dijet mass

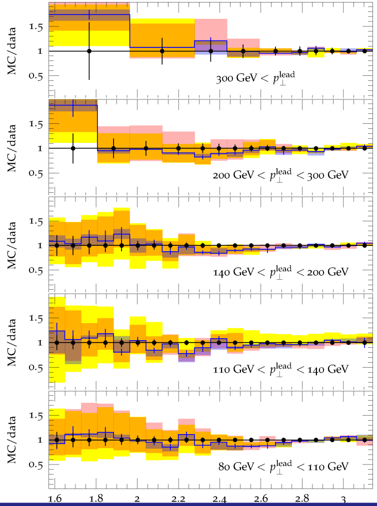
(S. Hoeche & M. Schoenherr, Phys. Rev. D86 (2012) 094042)



MC@NLO for light jets: azimuthal decorrelations

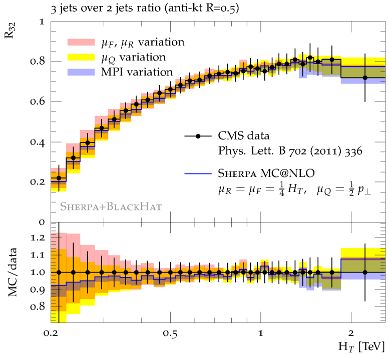
(S. Hoeche & M. Schoenherr, Phys. Rev. D86 (2012) 094042)

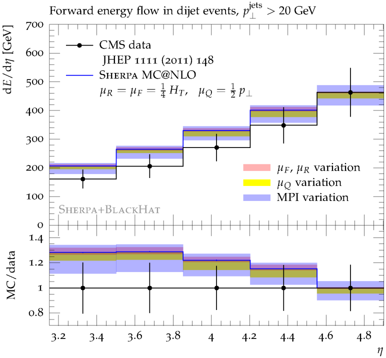




MC@NLO for light jets: R_{32} & forward energy flow

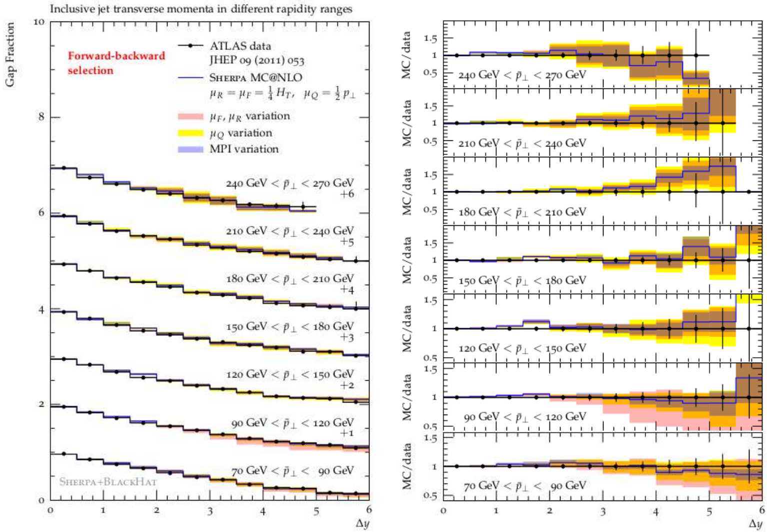
(S. Hoeche & M. Schoenherr, Phys. Rev. D86 (2012) 094042)





MC@NLO for light jets: jet vetoes

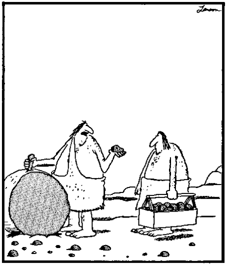
(S. Hoeche & M. Schoenherr, Phys. Rev. D86 (2012) 094042)



Summary of 1^{st} lecture

Summary of 1^{st} lecture

- reviewed higher order calculations and parton shower, with emphasis on accuracy
- these things are not blackboxes and can be understood
- systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:
  - ME corrections
  - NLO matching (“MC@NLO”, “POWHEG”)



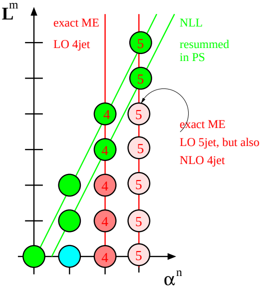
“So what’s this? I asked for a *hammer*! A hammer! *This* is a crescent wrench! ... Well, maybe it’s a hammer. ... Damn these stone tools.”

Multijet merging @ leading order

Multijet merging: basic idea

(S. Catani, F. Krauss, R. Kuhn, B. Webber, JHEP 0111 (2001) 063,
L. Lonnblad, JHEP 0205 (2002) 046, & F. Krauss, JHEP 0208 (2002) 015)

- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- combine ("merge") both:
  result: "towers" of MEs with increasing number of jets evolved with PS
  - multijet cross sections at Born accuracy
  - maintain (N)LL accuracy of parton shower

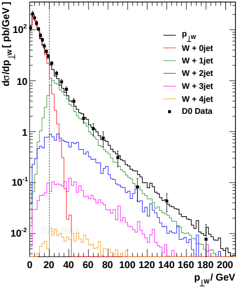


Separating jet evolution and jet production

- separate regions of jet production and jet evolution with jet measure Q_J

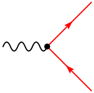
  ("truncated showering" if not identical with evolution parameter)

- matrix elements populate hard regime
- parton showers populate soft domain

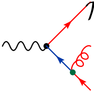


Why it works: jet rates with the parton shower

- consider jet production in $e^+e^- \to$ hadrons
  Durham jet definition: relative transverse momentum $k_\perp > Q_J$
- fixed order: one factor α_S and up to $\log^2 \frac{E_{\rm c.m.}}{Q_J}$ per jet
- use Sudakov form factor for resummation &
  replace approximate fixed order by exact expression:



$$\mathcal{R}_2(Q_J) = \left[\Delta_q(E^2_{\rm c.m.},\, Q_J^2)\right]^2$$



$$\mathcal{R}_3(Q_J) = 2\Delta_q(E^2_{\rm c.m.},\, Q_J^2) \int\limits_{Q_J^2}^{E^2_{\rm c.m.}} \frac{{\rm d}k_\perp^2}{k_\perp^2} \left[\frac{\alpha_S(k_\perp^2)}{2\pi}{\rm d}z\mathcal{K}_q(k_\perp^2, z) \right.$$
$$\left. \times\Delta_q(E^2_{\rm c.m.},\, k_\perp^2)\Delta_q(k_\perp^2,\, Q_J^2)\Delta_g(k_\perp^2,\, Q_J^2)\right]$$

First emission(s), again

(S. Hoeche, F. Krauss, S. Schumann, F. Siegert, JHEP 0905 (2009) 053)

$$\begin{aligned} \mathrm{d}\sigma \;=\;& \mathrm{d}\Phi_N\,\mathcal{B}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,\,t_0)+\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,\,t_{N+1})\Theta(Q_J-Q_{N+1})\right]\\ &+\mathrm{d}\Phi_{N+1}\,\mathcal{B}_{N+1}\,\Delta_N^{(\mathcal{K})}(\mu_{N+1}^2,\,t_{N+1})\Theta(Q_{N+1}-Q_J)\end{aligned}$$

- note: $N+1$-contribution includes also $N+2$, $N+3$, ...

(no Sudakov suppression below t_{n+1}, see further slides for iterated expression)

- potential occurence of different shower start scales: $\mu_{N,N+1,\dots}$
- "unitarity violation" in square bracket: $\mathcal{B}_N\mathcal{K}_N \longrightarrow \mathcal{B}_{N+1}$

(cured with UMEPS formalism, L. Lonnblad & S. Prestel, JHEP 1302 (2013) 094 &

S. Platzer, arXiv:1211.5467 [hep-ph] & arXiv:1307.0774 [hep-ph])

Iterating the emissions

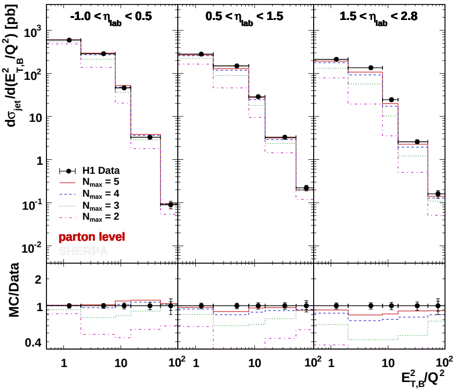
(S. Hoeche, F. Krauss, S. Schumann, F. Siegert, JHEP 0905 (2009) 053)

$$
\begin{aligned}
d\sigma \;=\; & \sum_{n=N}^{n_{max}-1} \Bigg\{ d\Phi_n \, \mathcal{B}_n \overbrace{\left[\prod_{j=N}^{n-1} \Theta(Q_{j+1}-Q_J)\right]}^{(n-N)\text{ extra jets}} \overbrace{\left[\prod_{j=N}^{n-1} \Delta_j^{(\mathcal{K})}(t_j,\, t_{j+1})\right]}^{\text{no emissions off internal lines}} \\
& \times \Bigg[\underbrace{\Delta_n^{(\mathcal{K})}(t_n, t_0)}_{\text{no emission}} + \underbrace{\int_{t_0}^{t_n} d\Phi_1 \, \mathcal{K}_n \Delta_n^{(\mathcal{K})}(t_n, t_{n+1}) \Theta(Q_J - Q_{n+1})}_{\text{next emission no jet \& below last ME emission}} \Bigg] \\
& + d\Phi_{n_{max}} \, \mathcal{B}_{n_{max}} \left[\prod_{j=N}^{n_{max}-1} \Theta(Q_{j+1}-Q_J)\right] \left[\prod_{j=N}^{n_{max}-1} \Delta_j^{(\mathcal{K})}(t_j,\, t_{j+1})\right] \\
& \times \left[\Delta_{n_{max}}^{(\mathcal{K})}(t_{n_{max}}, t_0) + \int_{t_0}^{t_{n_{max}}} d\Phi_1 \, \mathcal{K}_{n_{max}} \Delta_{n_{max}}^{(\mathcal{K})}(t_{n_{max}}, t_{n_{max}+1}) \right]
\end{aligned}
$$

ME & PS results: inclusive jets in DIS

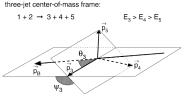
S. Hoeche, T. Gehrmann, T. Carli, arXiv:0912.3715, data from PL **B542** (2002) 193

Variation of maximum matrix-element multiplicity, $N_{\max}$

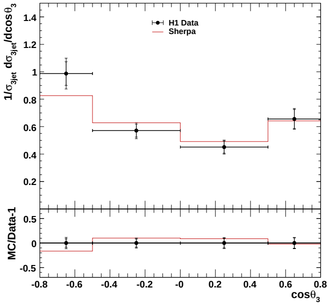


ME & PS results: Inclusive trijets in DIS

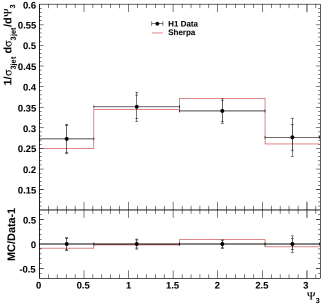
S. Hoeche, T. Gehrmann, T. Carli, arXiv:0912.3715, data from PL **B515** (2001) 17



$\cos\theta_3$ $Q^2 > 150$ GeV^2

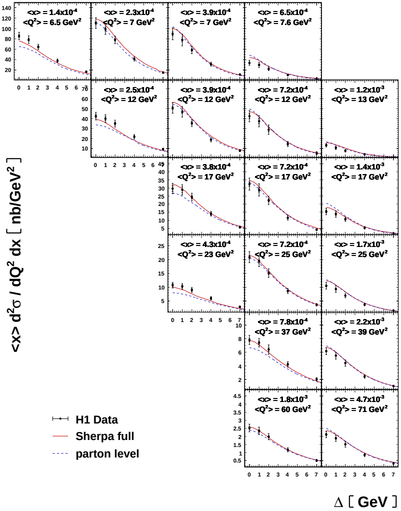


Ψ_3 $Q^2 > 150$ GeV^2



ME & PS results: Low-x dijets in DIS

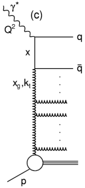
S. Hoeche, T. Gehrmann, T. Carli, arXiv:0912.3715, data from EPJ **C33** (2004) 477



Δ in bins of $\langle x\rangle$ and $\langle Q^2\rangle$

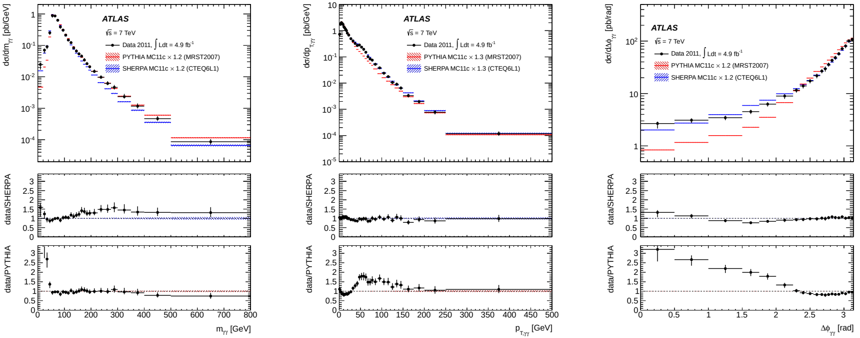
Δ defined as $E^*_{T,\mathrm{max}} > E^*_{T\,\mathrm{cut}} + \Delta$

$E^*_{T\,\mathrm{cut}} \to$ minimum jet transverse energy
$E^*_{T\,\mathrm{max}} \to$ transverse energy of hardest jet



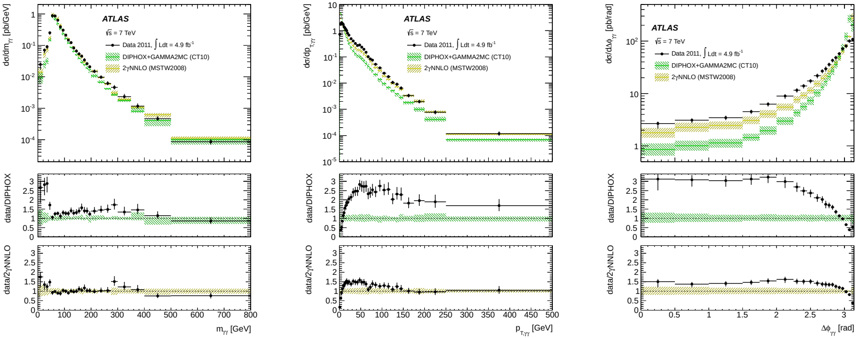
Di-photons @ ATLAS: $m_{\gamma\gamma}$, $p_{\perp,\gamma\gamma}$, and $\Delta\phi_{\gamma\gamma}$ in showers

(arXiv:1211.1913 [hep-ex])



Aside: Comparison with higher order calculations

(arXiv:1211.1913 [hep-ex])



A step towards multijet-merging at NLO: MENLOPS

(K. Hamilton & P. Nason, JHEP 1006 (2010) 039 &

S. Hoeche, F. Krauss, M. Schoenherr, & F. Siegert, JHEP 1108 (2011) 123)

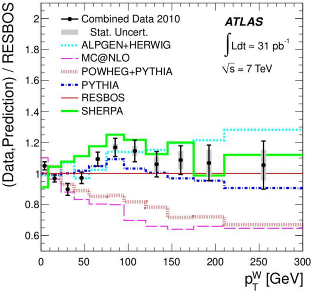
- combine matching for lowest multiplicity with multijet merging
- interpolating local K-factor for reweighting hard emissions

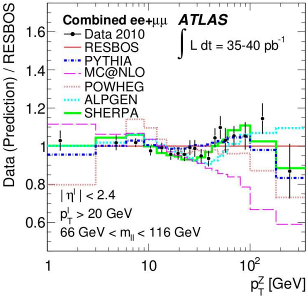
$$k_N(\Phi_{N+1}) = \frac{\tilde{\mathcal{B}}_N}{\mathcal{B}_N}\left(1-\frac{\mathcal{H}_N}{\mathcal{B}_{N+1}}\right)+\frac{\mathcal{H}_N}{\mathcal{B}_{N+1}} \longrightarrow \begin{cases} \tilde{\mathcal{B}}_N/\mathcal{B}_N & \text{for soft emission} \\ 1 & \text{for hard emission} \end{cases}$$

$$\begin{aligned} \mathrm{d}\sigma \;=\;& \mathrm{d}\Phi_N\,\tilde{\mathcal{B}}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,t_0)+\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\right] \\ & +\mathrm{d}\Phi_{N+1}\,\mathcal{H}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1}) \\ & +\mathrm{d}\Phi_{N+1}\,k_N\,\mathcal{B}_{N+1}\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_{N+1}-Q_J) \end{aligned}$$

Transverse momentum of W & Z boson

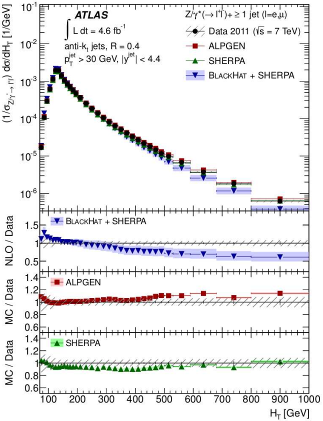
ATLAS, arXiv:1108.6308, arXiv:1107.2381

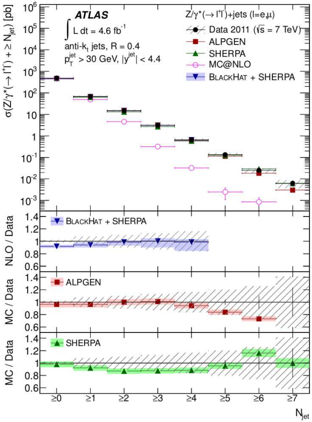




Z+jets: inclusive quantities

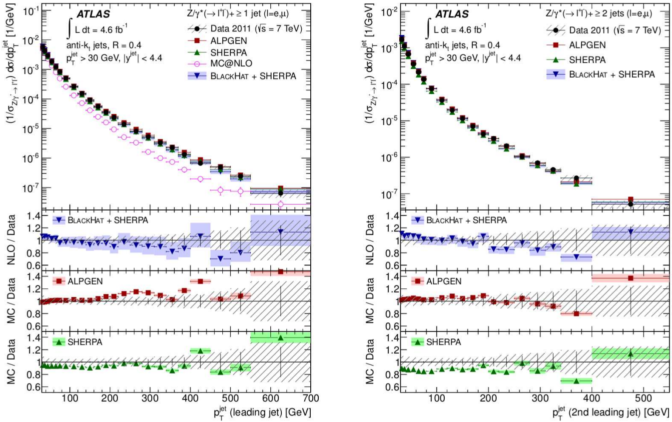
ATLAS, arXiv:1111.2690





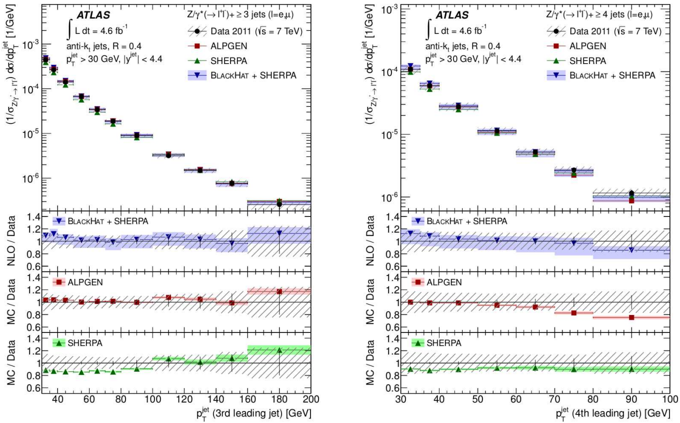
Z+jets: jet transverse momenta

ATLAS, arXiv:1111.2690



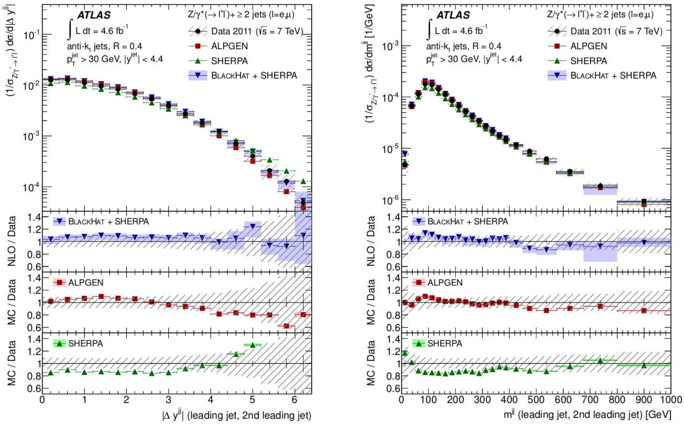
Z+jets: jet transverse momenta

ATLAS, arXiv:1111.2690



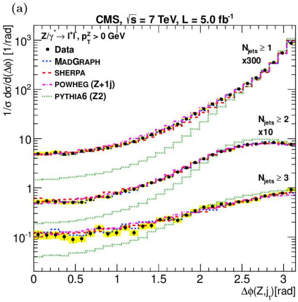
Z+jets: correlation of leading jets

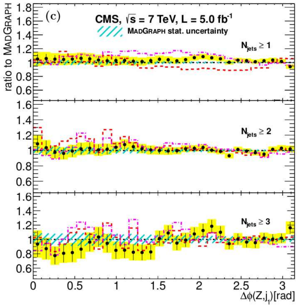
ATLAS, arXiv:1111.2690



Z+jets: $\Delta\phi_{Zj}$ in unboosted sample

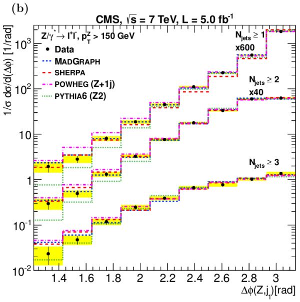
CMS, arXiv:1301.1646

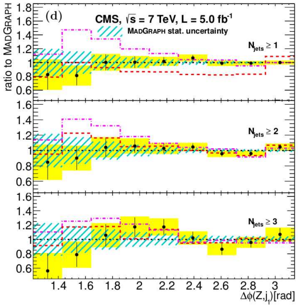




Z+jets: $\Delta\phi_{Zj}$ in boosted sample

CMS, arXiv:1301.1646





Multijet merging @ next-to leading order

Multijet-merging at NLO: MEPS@NLO

- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting

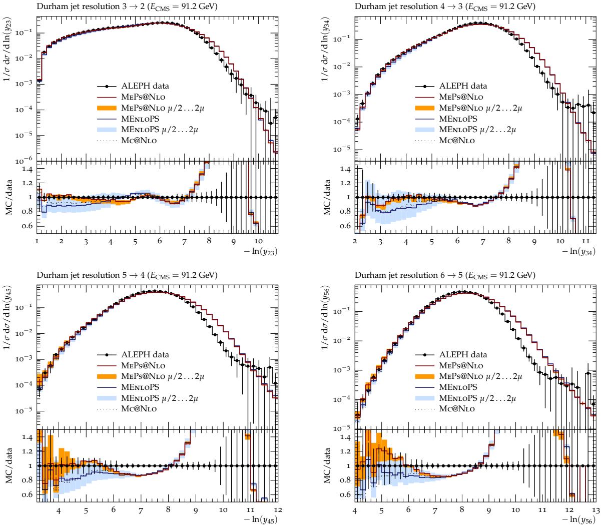
maintain NLO and (N)LL accuracy of ME and PS

- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities

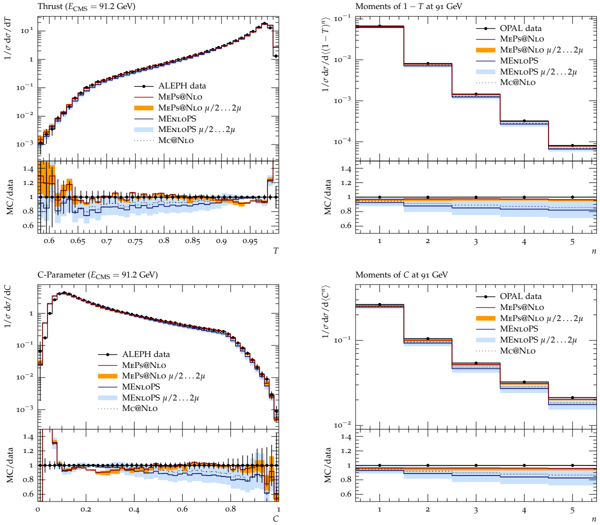
First emission(s), once more

$$
\begin{aligned}
\mathrm{d}\sigma \;=\; & \mathrm{d}\Phi_N\,\tilde{\mathcal{B}}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,t_0)+\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\right]\\
& +\mathrm{d}\Phi_{N+1}\,\mathcal{H}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\\
& +\mathrm{d}\Phi_{N+1}\,\tilde{\mathcal{B}}_{N+1}\left(1+\frac{\mathcal{B}_{N+1}}{\tilde{\mathcal{B}}_{N+1}}\int\limits_{t_{N+1}}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\right)\Theta(Q_{N+1}-Q_J)\\
& \quad\cdot\left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_0)+\int\limits_{t_0}^{t_{N+1}}\mathrm{d}\Phi_1\,\mathcal{K}_{N+1}\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\right]\\
& +\mathrm{d}\Phi_{N+2}\,\mathcal{H}_{N+1}\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\Theta(Q_{N+1}-Q_J)+\ldots
\end{aligned}
$$

MEPS@NLO: example results for $e^-e^+ \to$ hadrons

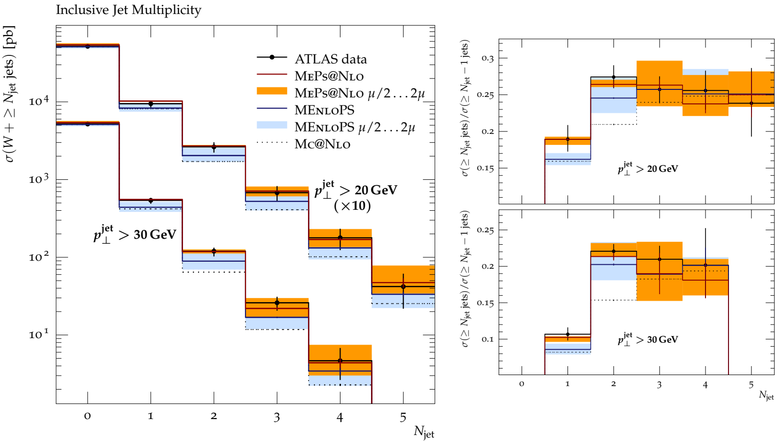


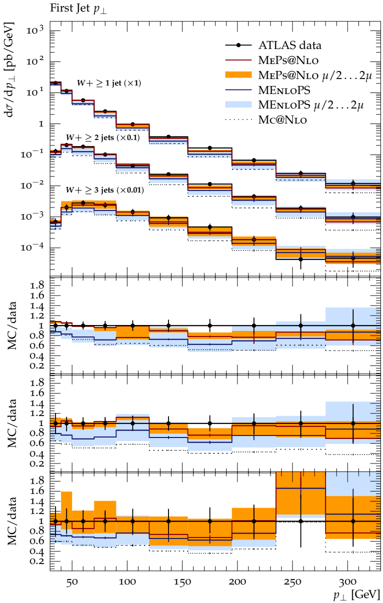
MEPS@NLO: example results for $e^-e^+ \to$ hadrons

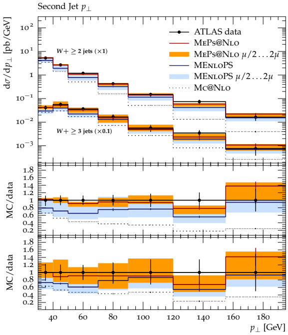


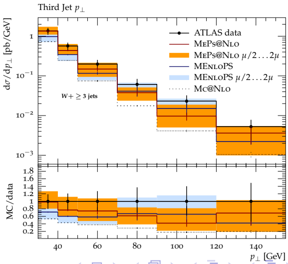
Example: MEPS@NLO for W+jets

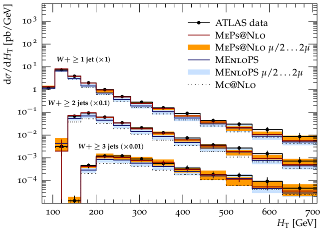
(up to two jets @ NLO, from BlackHat, see arXiv: 1207.5031 [hep-ex])

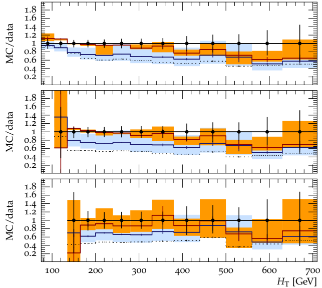



First Jet $p_\perp$
ATLAS data
MEPS@NLO
MEPS@NLO $\mu/2\ldots2\mu$
MENLOPS
MENLOPS $\mu/2\ldots2\mu$
MC@NLO
$W+\geq 1$ jet ($\times 1$)
$W+\geq 2$ jets ($\times 0.1$)
$W+\geq 3$ jets ($\times 0.01$)
$d\sigma/dp_\perp$ [pb/GeV]
MC/data
$p_\perp$ [GeV]

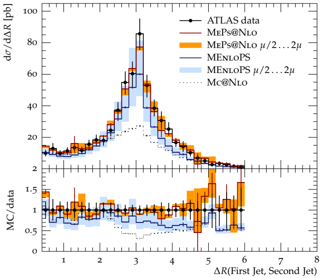

Second Jet $p_\perp$
ATLAS data
MEPS@NLO
MEPS@NLO $\mu/2\ldots2\mu$
MENLOPS
MENLOPS $\mu/2\ldots2\mu$
MC@NLO
$W+\geq 2$ jets ($\times 1$)
$W+\geq 3$ jets ($\times 0.1$)
$d\sigma/dp_\perp$ [pb/GeV]
MC/data
$p_\perp$ [GeV]


Third Jet $p_\perp$
ATLAS data
MEPS@NLO
MEPS@NLO $\mu/2\ldots2\mu$
MENLOPS
MENLOPS $\mu/2\ldots2\mu$
MC@NLO
$W+\geq 3$ jets
$d\sigma/dp_\perp$ [pb/GeV]
MC/data
$p_\perp$ [GeV]

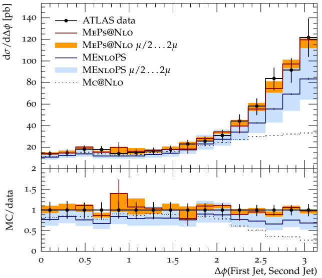




ΔR Distance of Leading Jets

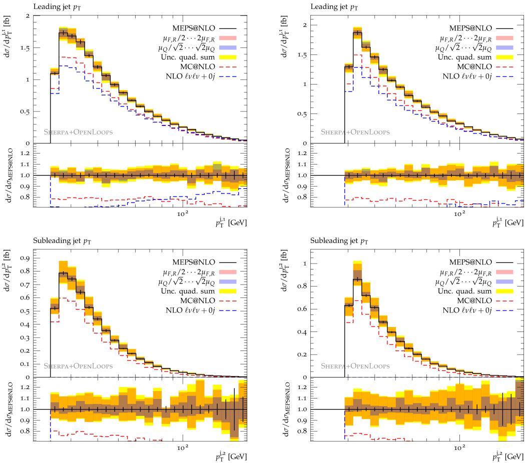


Azimuthal Distance of Leading Jets



Example: MEPS@NLO for W^+W^-+jets

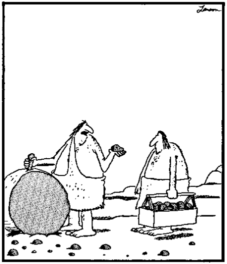
(in prep., up to one jets @ NLO, virtuals from OPENLOOPS, all interferences, no Higgs)



Summary

Summary

- systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:
  - multijet merging ("CKKW", "MLM")
  - NLO matching ("MC@NLO", "POWHEG")
  - MENLOPS – combination of matching and merging
  - multijet merging at NLO (MEPS@NLO, "FxFx")



"So what's this? I asked for a *hammer*! A hammer! *This* is a crescent wrench! ... Well, maybe it's a hammer. ... Damn these stone tools."

(first 3 methods well understood and used in experiments)

- multijet merging at NLO under scrutiny
- complete automation of NLO calculations ≈ done
  time to optimise the impact of this gargantuan task