Heavy Quark Theory

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What is special about heavy quarks?

$[m_c(\sim 1.3\,\mathrm{GeV}),\ m_b(\sim 4.4\,\mathrm{GeV}),\ m_t(\sim 170\,\mathrm{GeV})]$

- $\alpha_s(M)$ is in perturbative region (defining property)
  - Enables some kinds of system perturbative calculation ($\Longrightarrow$ predictions) not possible for light quarks and gluons
- They can be usefully non-relativistic, with lack of significant pair production
- Decoupling of quarks of mass much heavier than scale Q of process.
  [Leads to simplifications in regions with 3, 4, 5, . . . active quark flavors.]

This list gives insights/ideas/motivations for detailed technical work.

Techniques for heavy quarks

- HQET (heavy quark effective theory):
  - E.g., treat B meson ($b\bar{u}$ etc) as single non-relativistic heavy quark ($m_b(\sim 4.4\,\mathrm{GeV})$) plus few hundred MeV of non-perturbative light quark stuff.
  - Show how to factorize the non-perturbative part.
  - . . .
  - Apply to decays, e.g., with $b \to c$.

- Similar methods for hadronization of heavy quarks, etc.

- . . .

- *(My focus today)* Decoupling theorem and related physics.

  Leads to 3-, 4-, 5-, . . . flavor versions of α_s and pdfs, and their uses, etc

What are 3-, 4-, 5-, . . . flavor versions of α_s and pdfs?

Methods we'll need to answer this question:

- Insight/intuition:

  A process at invariant-mass scale Q and distance scale $1/Q$ is expected to be insensitive to much higher mass particles and details of the theory at much shorter distances.

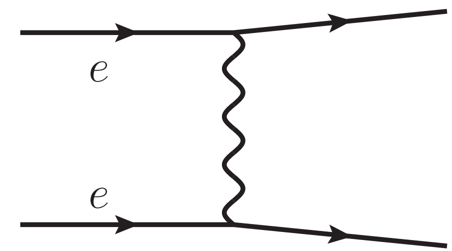
- One formalization is in terms of EFT (effective field theory). E.g.,:

  QCD$_{u,d,s,c,b,t}$ at Q of couple of GeV reduces to QCD$_{u,d,s}$, by dropping inactive quarks, *ETC . . .*

- A useful different approach is by CWZ/ACOT—later. These keep heavy quarks in the theory, but change calculational methods.

Example of low-energy process to motivate ideas of EFT & decoupling

ee scattering with γ and Z exchange:



$$\text{EM: } \frac{e^2}{q^2} \qquad \text{WI: } \frac{e^2 \times \text{few}}{q^2 - m_Z^2}$$

When $|q^2| \ll m_Z^2$

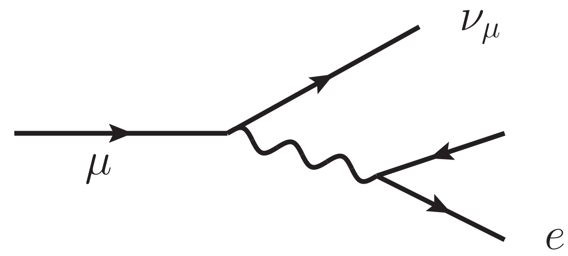
$$\left|\frac{\text{WI}}{\text{EM}}\right| \sim \frac{|q^2|}{m_Z^2}$$

Basic general phenomenon:

- To leading power: One can drop the heavy field,
- since particles of mass $\gg Q$ give power-suppressed contribution

(E.g., in 1975 QCD$_{u,d,s,c}$ was a good enough theory. One could make *valid* predictions without knowing about b and t quarks.)

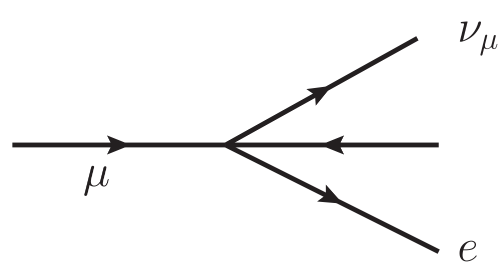
Example where power-suppressed term is all there is

μ decay: 

The basic low energy EFT is QED + QCD, all of whose interactions exactly preserve lepton (and quark) flavor. So decay amplitude with WI is

$$0 + \text{factor} \times \frac{e^2}{m_W^2}$$

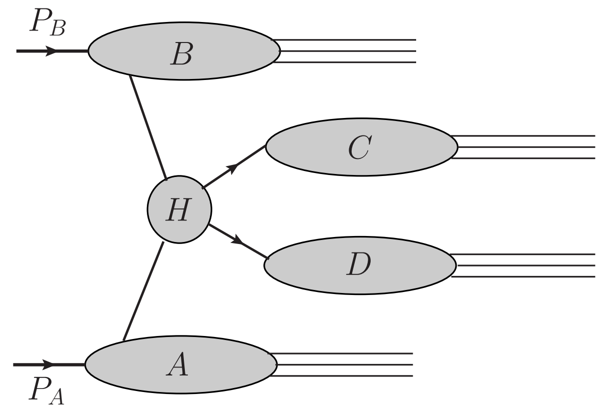
Result approximated by non-renormalizable point-like 4-fermion interaction



Quark masses and perturbation theory in factorization in QCD

(E.g.: Production of high-p_T jets, DIS, Drell-Yan)

Factorization structure, with hard scale $Q \sim p_T$), e.g.,



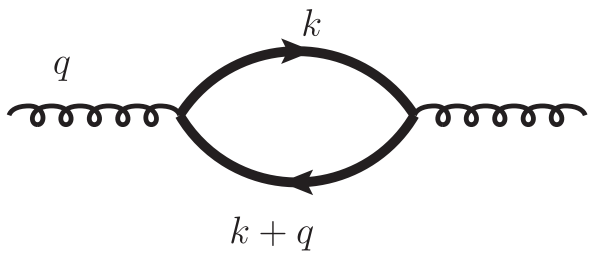
$$\mathrm{d}\sigma_{\mathrm{had}} = \sum \int \text{(pdf(s)) (ff(s))}\ \mathrm{d}\sigma_{\text{partonic, hard}}\, \mathrm{d}(\text{partonic variables})$$

quark mass	in hard sc.	in evol.	in non-pert. factors
$m \ll Q$	$m \to 0$ useful	No m	Preserve m
$M \sim Q$	Preserve M	(. . .)	Decouples (. . .)
$M \gg Q$	Decouples (. . .)	Decouples (. . .)	Decouples (. . .)

Is it really true that . . .

- EFT $\text{QCD}_{n \text{ active flavors}}$ is obtained simply by dropping the $6 - n$ inactive flavors?
- there is just one characteristic scale?

Unsuppressed effects when $M^2 \gg Q^2$ ($\overline{\mathrm{MS}}$ renormalization)



$$\propto g^2 \int \frac{\mathrm{tr}\,\gamma^\mu(\not{k}+M)\gamma^\nu(\not{q}+\not{k}+M)}{(k^2-M^2)\,\left[(k+q)^2-M^2\right]} \frac{\mathrm{d}^4 k}{(2\pi)^4} + \text{c.t.}$$

$$\propto (q^2 g^{\mu\nu} - q^\mu q^\nu) \frac{\alpha_s}{\pi} \int_0^1 x(1-x) \ln \frac{M^2 - q^2 x(1-x)}{\mu^2} \, \mathrm{d}x$$

$$= (q^2 g^{\mu\nu} - q^\mu q^\nu)\, \frac{\alpha_s}{6\pi} \ln \frac{M^2}{\mu^2} + \text{power-suppressed}$$

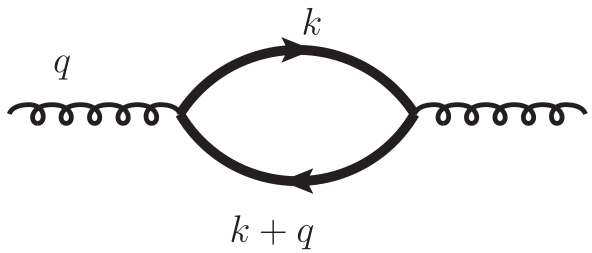
when $|q^2| \ll M^2$. This is *not* suppressed when $M^2 \gg |q^2|$.

Add in light-quark graph. Mass m, with $m^2 \ll |q^2|$:

$$(q^2 g^{\mu\nu} - q^\mu q^\nu)\, \frac{\alpha_s}{6\pi} \left[\ln \frac{q^2}{\mu^2} + \text{constant} \right]$$

So no single choice of $\overline{\mathrm{MS}}$ μ eliminates large logarithms for sum of both heavy and light-quark graphs when $m^2 \ll |q^2| \ll M^2$.

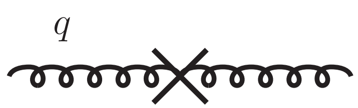
Unsuppressed effects when $M \gg Q$ equivalent to change of parameters



$$\propto (q^2 g^{\mu\nu} - q^\mu q^\nu)\ \frac{\alpha_s}{6\pi} \ln \frac{M^2}{\mu^2} + \text{power-suppressed}$$

when $q^2 \ll M^2$.

Renormalization was implemented by adding to basic graph a counterterm graph:



$$= (q^2 g^{\mu\nu} - q^\mu q^\nu) \times (q\text{-independent coefficient})$$

So non-suppressed $M \gg Q$ contribution is equivalent to changing the counterterm, i.e., to a change in parameters of the theory.

Decoupling theorem

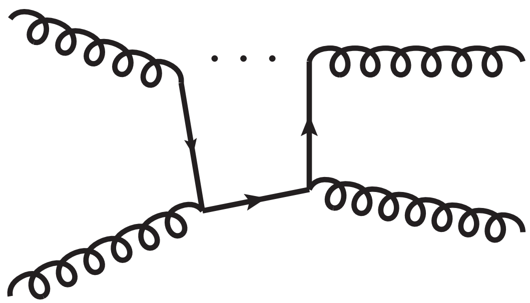
Let Q be the maximum external momentum scale of the processes considered, and let the full theory have a field/particle of much larger mass M. Then to leading power in M/Q, equivalent results are obtained from an EFT obtained by

- Deleting the large mass fields.
- Adjusting the parameters of the theory. ("Matching")

[Power corrections implemented similarly by adding non-renormalizable local interactions in EFT.]

Sketch of general rationale for decoupling theorem

Generalizes from one-loop example:



$$\propto \int \frac{\text{num}}{(k+q_1)^2 - M^2} \, \frac{\text{num}}{(k+q_1+q_2)^2 - M^2} \dots \mathrm{d}^4 k$$

with N external gluons.

Region	Power-counting	$N > 4$	$N \leq 4$
$k \to \infty$	$\lvert k \rvert^{4-N}$	Convergent	Divergent, needs c.t.
$\lvert k \rvert \sim M$	M^{4-N}	Suppressed	Non-suppressed, like c.t.
		Expand in powers of qs	
$\lvert k \rvert \ll M$	Suppressed.		

Matching conditions: theory with and without quark of mass M

- Compute graphs needed for renormalization in full theory and effective theory
- Adjust parameters to give agreement at low scales.
- Use $\mu \sim M$ to avoid logarithms of M/μ
- Renormalization theorem: *Counterterms* don't have logarithms of small scales.
- So we have matching calculation without large logarithms; Useful expansion in powers of small coupling $\alpha_s(M)$
- Evolve to other scales by RG, etc.

Series of effective QCD theories

		Best accuracy is power of:
$\mathrm{QCD}_{u,d,s}$	3 flavors	$\dfrac{\Lambda}{m_c} \sim \dfrac{1}{7}$
$\mathrm{QCD}_{u,d,s,c}$	4 flavors	$\dfrac{m_c}{m_b} \sim \dfrac{1}{3}$
$\mathrm{QCD}_{u,d,s,c,b,t}$	5 flavors	$\dfrac{m_b}{m_t} \sim \dfrac{1}{40}$
$\mathrm{QCD}_{u,d,s,c,b,t}$	6 flavors	Unknown territory

Use one of these where:

- the retained flavors have $m \lesssim Q$
- the omitted flavors have $M \gg Q$

Further issues with simplest EFT view

Simple method:

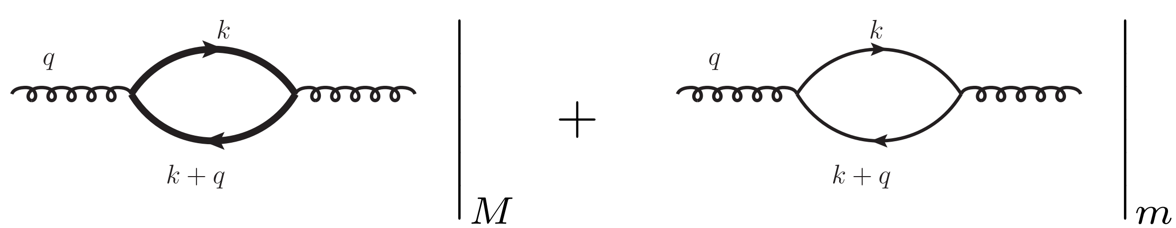
> Going up in mass scale, successively use 3-, 4-, . . . flavor versions of QCD, as appropriate for the quantity calculated (single scale assumed).

But

- The ratios of successive masses aren't always large.
- Typical contributions to an amplitude/cross section have multiple scales.

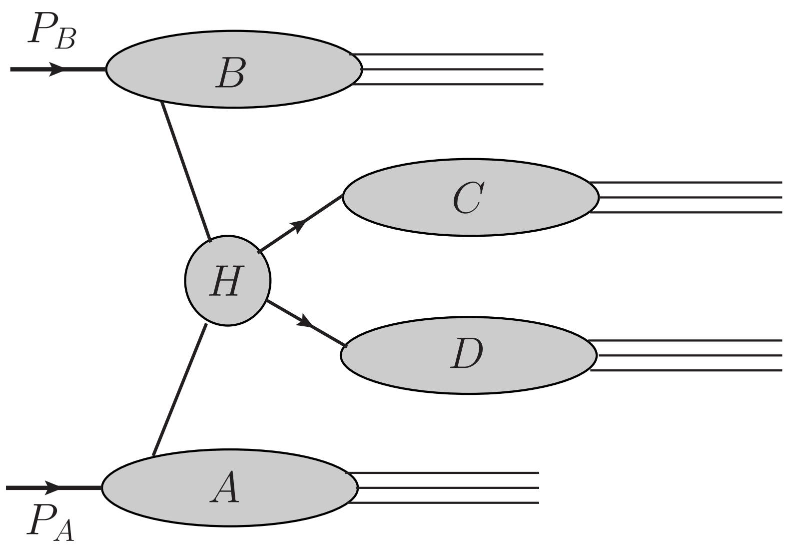
Q: If we have know we have six quarks (u, d, s, c, b, t) why not always use the full theory?

A: (First pass) If we use $\overline{\mathrm{MS}}$, we can't get rid of all logarithms in sum of graphs with heavy and light quarks:



Scales when there is a hard scattering

E.g., jet production at $p_T =$ many $100\,\mathrm{GeV}$ involves factors like

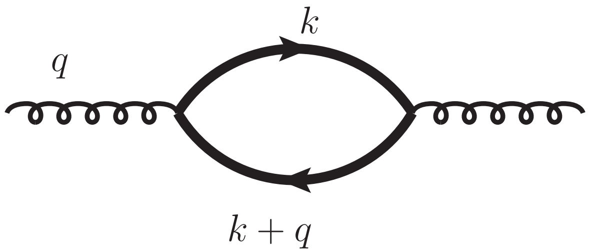


Important scales:

- In hard scattering H: $O(p_T)$.
- In beam and hadronization parts: *Everything* between about Λ_{QCD} and p_T.

How to stay in full theory: CWZ idea

For "inactive" quarks, use zero-momentum subtraction:



$$+ \text{c.t.} \propto (q^2 g^{\mu\nu} - q^\mu q^\nu) \frac{\alpha_s}{\pi} \int_0^1 x(1-x) \ln \frac{M^2 - q^2 x(1-x)}{M^2} \, \mathrm{d}x$$

$$= (q^2 g^{\mu\nu} - q^\mu q^\nu) \, \frac{\alpha_s}{6\pi} \, O\left(\frac{q^2}{M^2} \right)$$

when $|q^2| \ll M^2$.

Use $\overline{\mathrm{MS}}$ for everything else.

Key properties:

- "Manifest decoupling"
- Automatically preserves gauge-invariance of QCD
- RG and DGLAP equations are same (mass-independent) as in the EFT approach.

Statement of CWZ

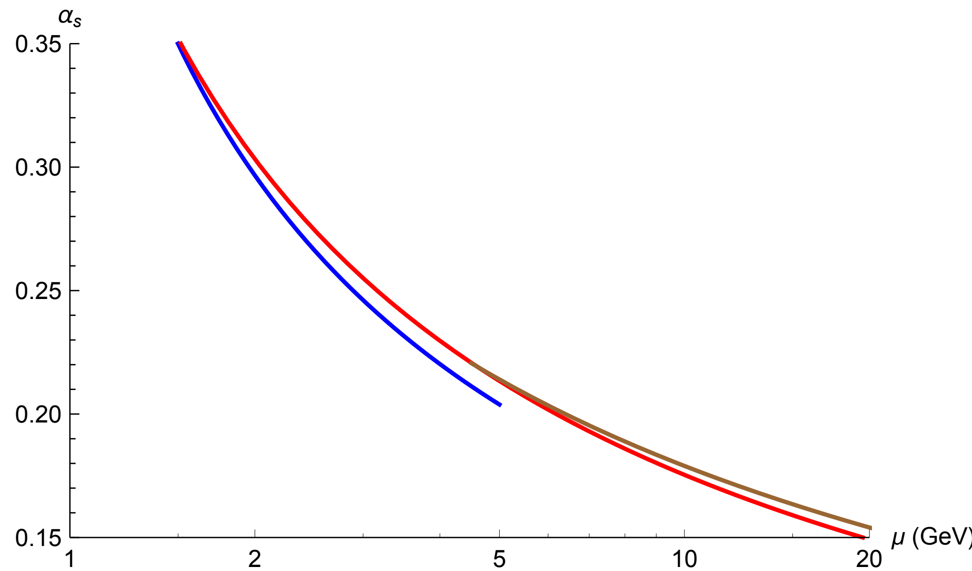
Definition:

- Keep all (known or relevant) quarks in theory
- Define a sequence of (renormalization) subschemes with 3, 4, 5, etc "active" flavors. (u, d, s, u, d, s, c, etc)
- Use $\overline{\mathrm{MS}}$ for active flavors, zero-momentum subtraction for graphs with inactive flavors
- Obtain relations of coupling, etc between subschemes by matching

Adjust choice of $\#$ of active flavors by the following principles:

- At scale Q, quarks with $m \ll Q$ are active
- Quarks with $M \gg Q$ are inactive
- Overlapping ranges of usefulness for $m \sim Q$.
- Manifest decoupling applies; it gives relation to EFT method.

Running coupling with variable numbers of active flavors



RGE:

$$\frac{\mathrm{d}\alpha_s/(4\pi)}{\mathrm{d}\ln\mu^2} = \beta\Big(\frac{\alpha_s}{4\pi}, n_{\mathrm{act}}\Big) = -\left(11 - \frac{2}{3}n_{\mathrm{act}}\right)\Big(\frac{\alpha_s}{4\pi}\Big)^2 - \dots$$

Matching, from calculation of relevant graphs:

$$\alpha_s(\mu,3) = \alpha_s(\mu,4) + \alpha_s(\mu,4)^2\left(\text{coeff.}\ln\frac{m_c^2}{\mu^2} + 0\right) + \alpha_s(\mu,4)^3(\dots) + \dots$$

ACOT idea

Apply CWZ idea to pdfs and factorization, etc

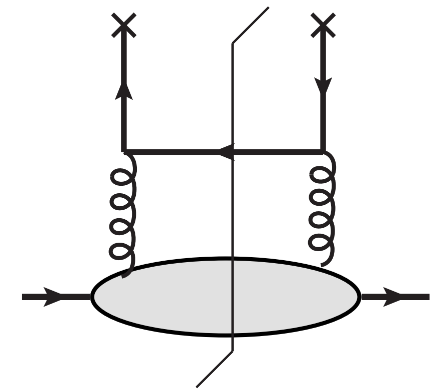
Pdfs:

3-flavor	Evolution: u, d, s only Usual 3-flavor DGLAP	c pdf suppressed by $(\Lambda/m_c)^p$ ($Pace$ Brodsky & intrinsic charm)
4-flavor	Evolution: u, d, s, c Usual 4-flavor DGLAP Start c at $\mu \simeq m_c$ from calculated matching	ETC

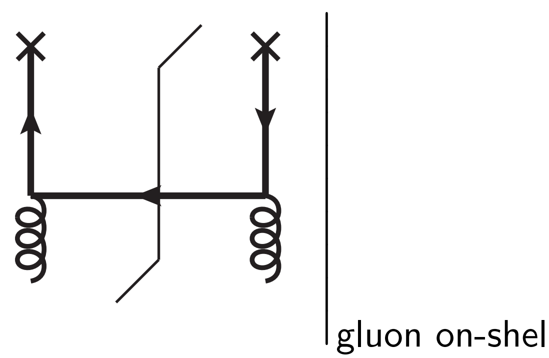
ETC.

Heavy-quark pdfs are from perturbative short distance effects

Simple Feynman graph for c (etc) pdf in proton:

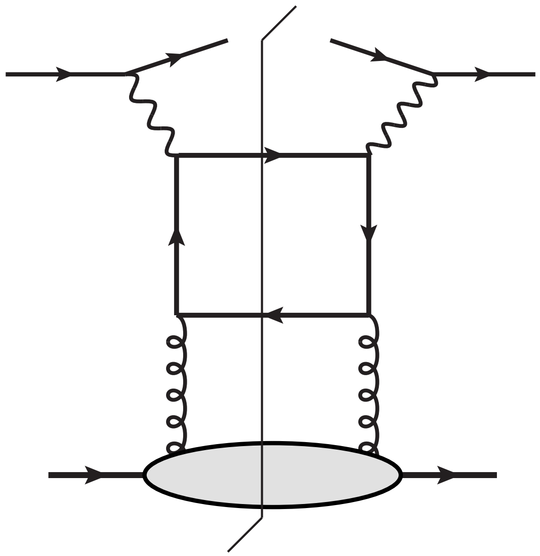


Leading approximation:

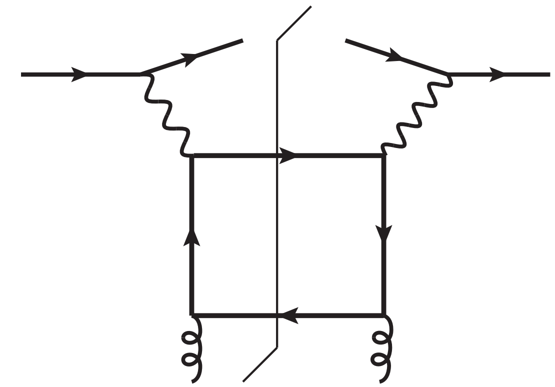
- Gluon of low p_T
- Get $f_c = f_g \otimes$ 

Then there are perturbative leading-power corrections in powers of $\alpha_s(m_c)$

Charm in DIS at $Q =$ few GeV: 3 active flavors



$+ \text{etc} = f_g \otimes \Big[$



$+ \text{etc} \Big]_{\text{gluon on-shell}}$

- Charm generated dynamically in hard scattering only
- No gluon-to-$c\bar{c}$ collinear divergence
- At Q of a few GeV: Not even a collinear region, with associated logarithm
- So, there is no subtraction in hard scattering, unlike light-quark case
- Etc for b quark, etc.

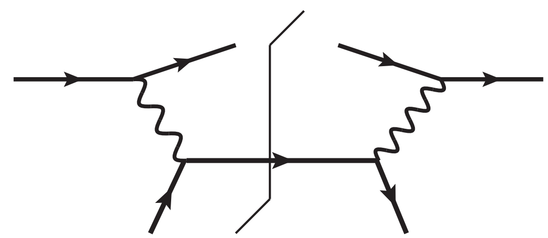
FFNS (fixed-flavor-number scheme): Do this for all Q.

Charm in DIS at $Q \gg$ few GeV: 4 active flavors

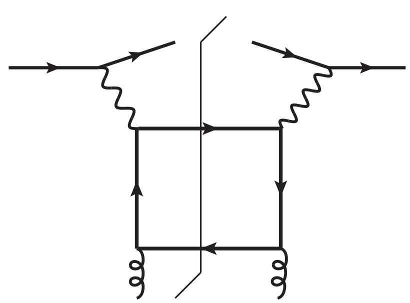
VFNS (variable-flavor-number scheme), ACOT style.

When Q is enough larger than m_c, use 4 active flavors:

- Include c pdf term

$f_c \otimes$  $\Big|_{\text{on-shell quark}}$

- Have collinear region in NLO hard scattering
- Must impose subtraction to avoid double counting (and avoid large logarithm):

$\Big[$  $+$ etc $-$ subtraction for c-in-g pdf $\Big]_{\text{gluon on-shell}}$

- Calculation from definition of pdf
- Can keep m_c in hard scattering

Overall view for factorization of hard process

With n_{act} (= 3, 4, . . .) active flavors:

- The active flavors:
  - are the n_{act} lightest quarks,
  - have masses (well) below Q
  - have pdfs, which evolve normally.
- The inactive flavors
  - are the heavier quarks
  - are only generated in the hard scattering
- Masses can be preserved in hard scattering

Summary

- Heavy quarks, i.e., with masses in perturbative region, allow simplifications, and extra perturbative predictions c.w. light quarks.
- Simplest methods involve decoupling theorem and EFTs
- Fancier methods (CWZ/ACOT) allow keeping heavy quarks in the theory, without penalty of large logarithms
- Get concept of number of "active" partonic quarks
- See the vast literature for a range of views