Introduction to neutrino theory

Pilar Coloma

Fermilab

CTEQ school

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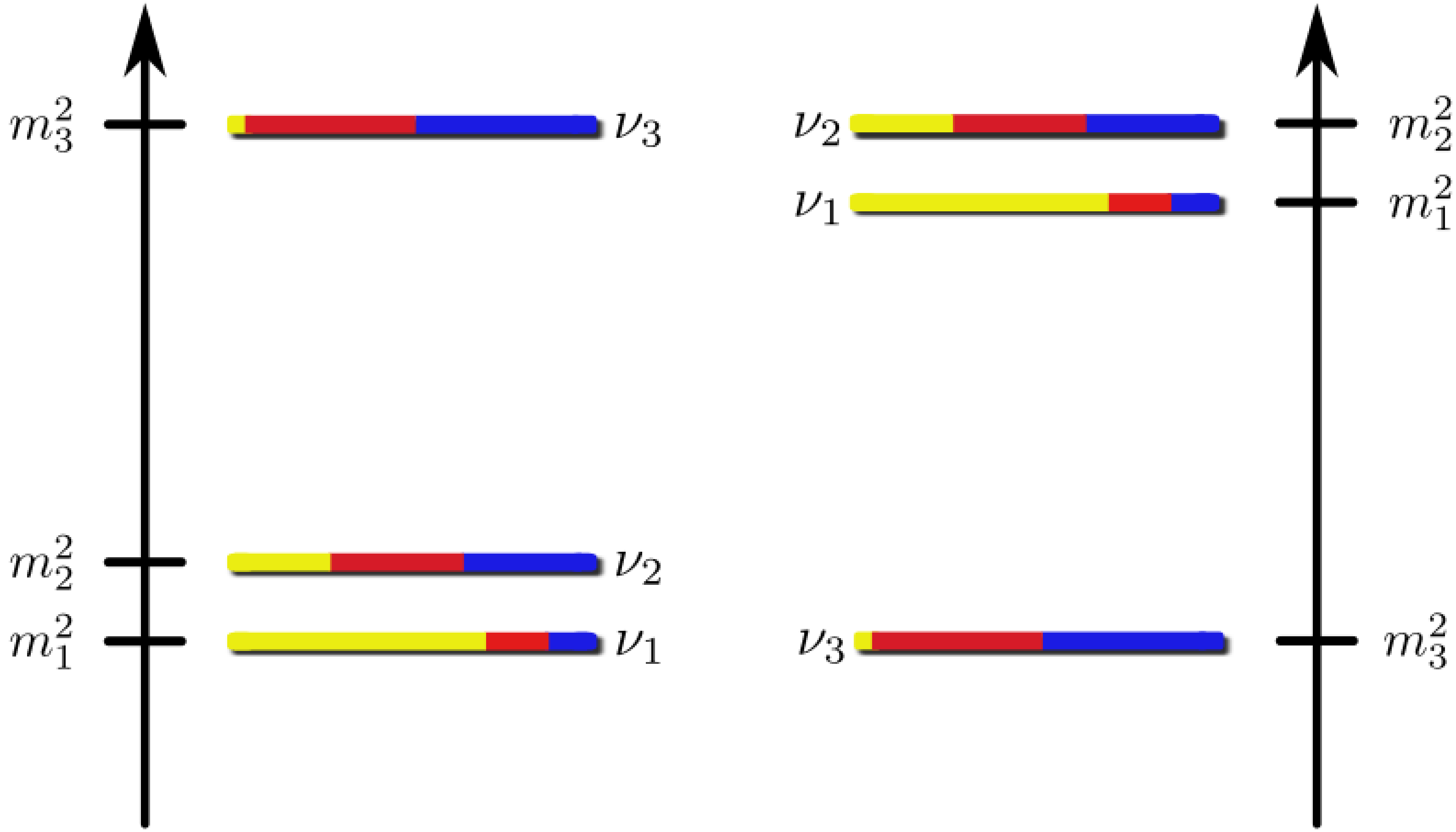
Leptonic mixing



Neutrino oscillation probabilities

$$\mathcal{L}^\nu = \mathcal{L}^\nu_{CC} + \mathcal{L}^\nu_{NC} + \mathcal{L}^\nu_{k} + \underbrace{\mathcal{L}^\nu_{m}}_{\nu_i}$$

CC interactions mix charged leptons and neutrinos:

$$\mathcal{L}^\nu_{CC} \sim U^*_{i\alpha} \left(\bar{l}_\alpha \, \gamma^\mu_L \, \nu_i W^+_\mu + h.c.\right)$$

Neutrinos in flavor space are superposition of mass eigenstates. In propagation, each wave packet evolves separately.

$$|\nu_\alpha\rangle = \sum_j U^*_{\alpha j} |\nu_j\rangle$$

The leptonic mixing matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric — Reactor/Interference — Solar

$$\times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}$$

Pontecorvo, 1957

Maki, Nakagawa, Sakata, 1962

Neutrinos in flavor space are superposition of mass eigenstates. In propagation, each wave packet evolves separately.

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m_{21}^2}{2E} & 0 \\ 0 & 0 & \frac{\Delta m_{31}^2}{2E} \end{pmatrix} U^\dagger + V \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

Outline

- Part I: Adding neutrino masses to the SM
  - Dirac/Majorana masses
  - See Saw models
  - Neutrinoless double beta decay
- Part II: Neutrino Oscillations in three families
  - Oscillation probabilities in vacuum
  - Matter effects
  - The degeneracy problem

Outline

- Part III: New physics in neutrino oscillations
  - Non-Standard Interactions
  - Sterile neutrinos

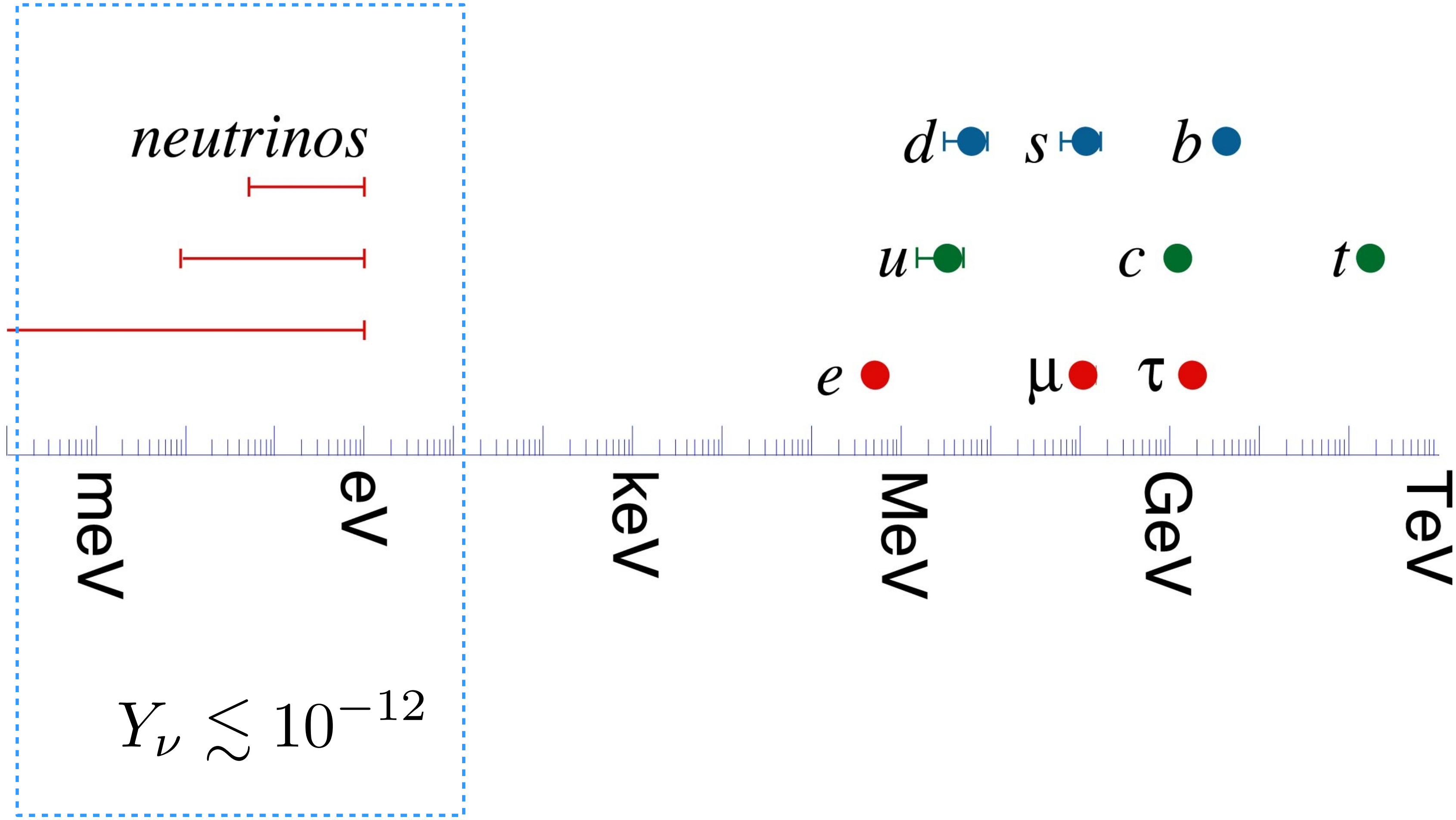
Part I: Adding neutrino masses to the SM

Are ν masses different?

When the SM was formulated, neutrino masses had not been observed yet. The simplest way to give them a mass is:

$$Y \overline{L}_L \widetilde{\phi} \nu_R + h.c. \xrightarrow{\text{EWSB}} m_{dirac} \propto Y v$$

Smallness of neutrino masses



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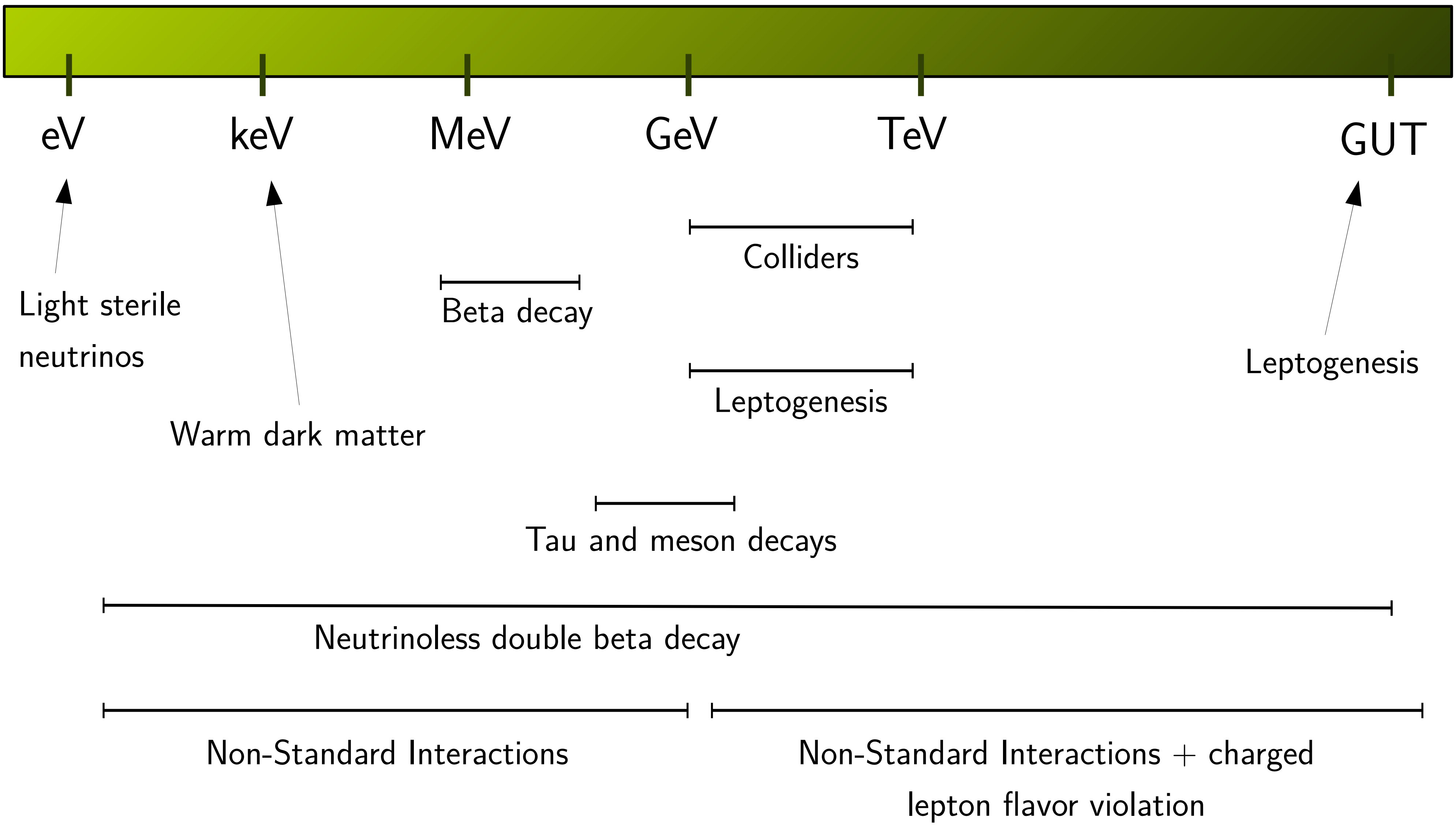
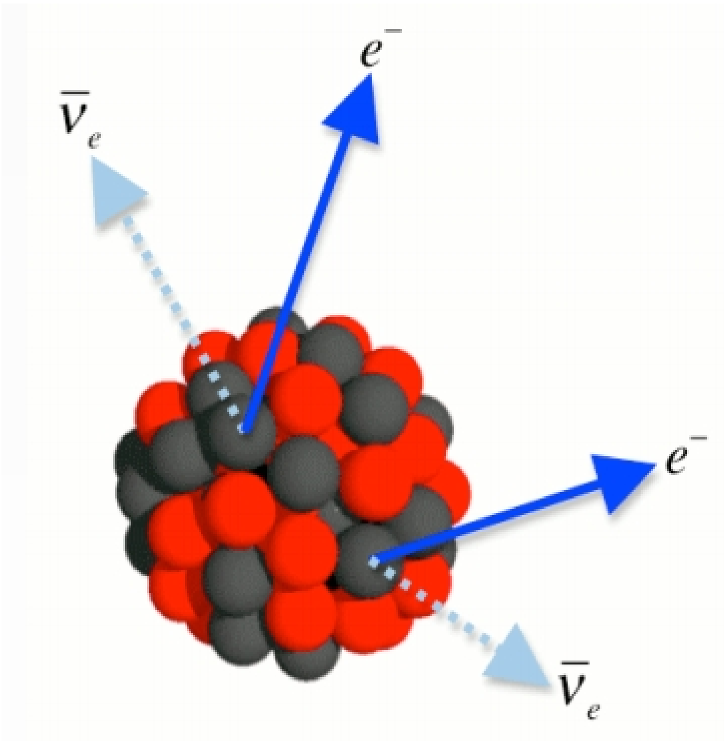
Right-handed neutrinos are SM singlets, though...

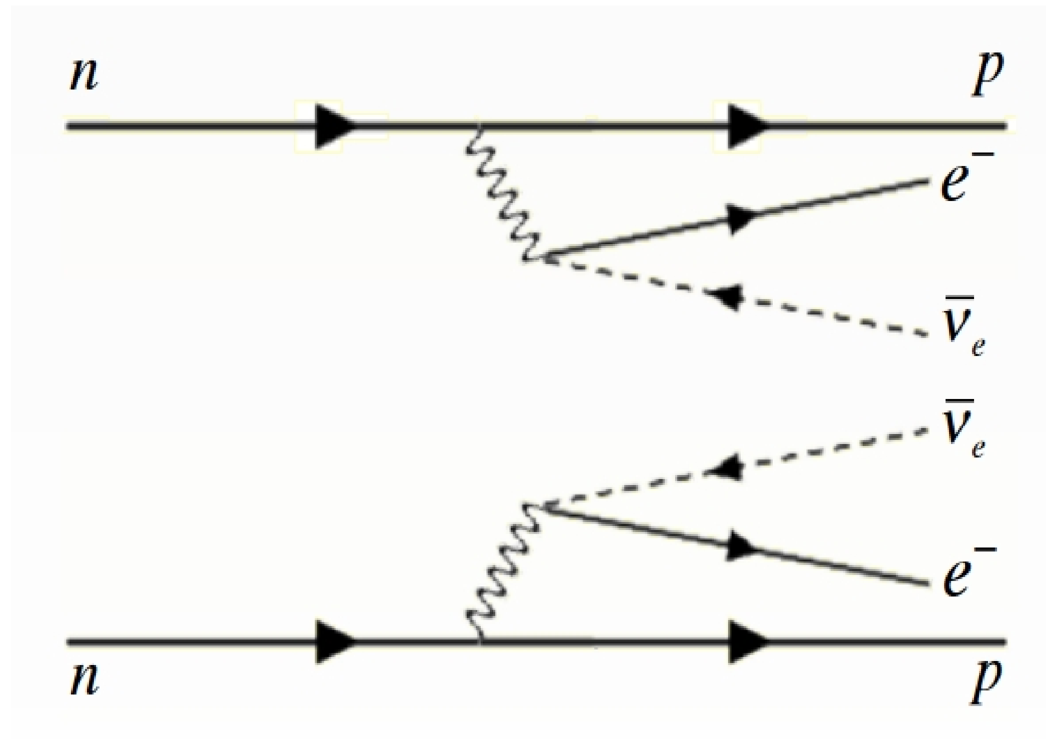
$$Y \overline{L}_L \widetilde{\phi} \nu_R + \frac{1}{2} M \overline{\nu}^c_R \nu_R + h.c. \qquad (\not{L})$$

Scale of new physics



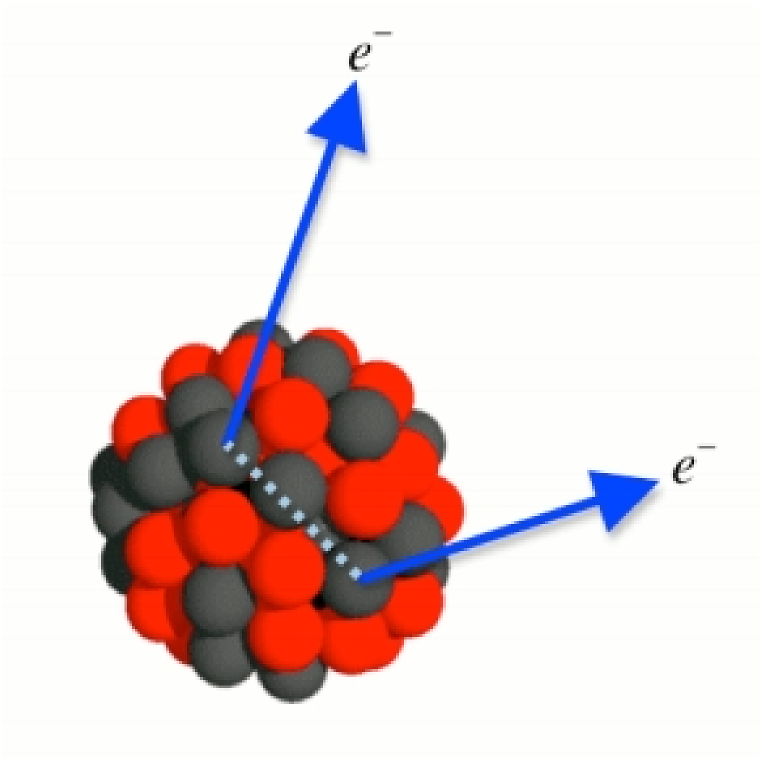
Neutrinoless double beta decay

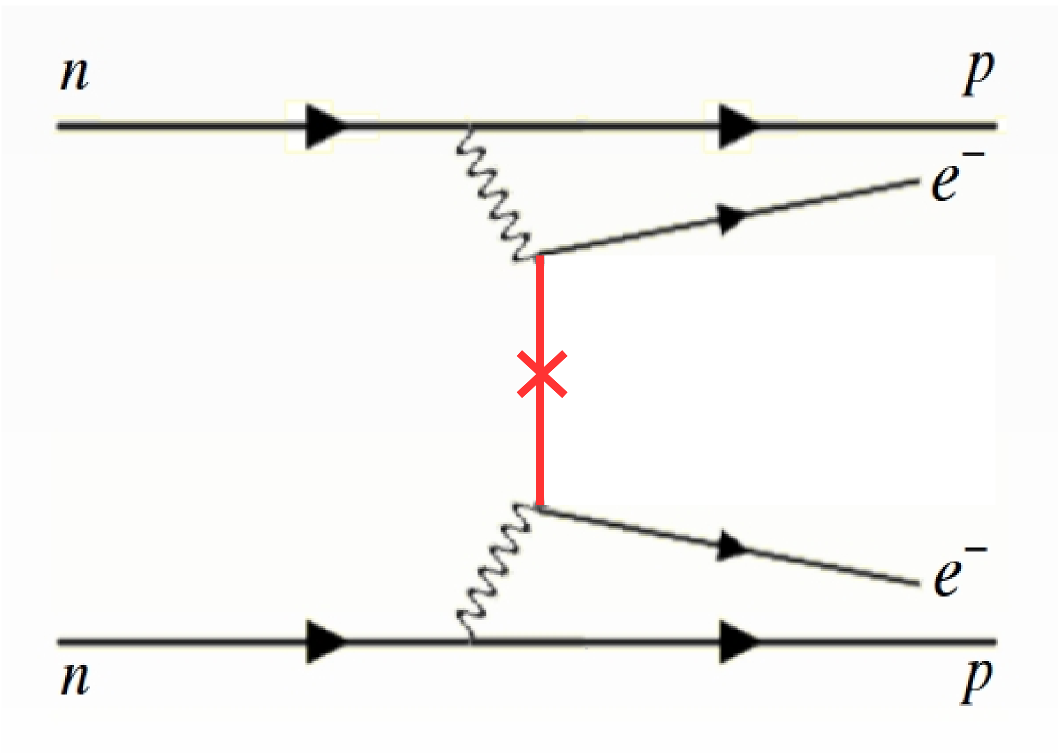




Figures from R. Saakyan's talk at NuPhys2014

Neutrinoless double beta decay





Figures from R. Saakyan's talk at NuPhys2014

Neutrinoless double beta decay

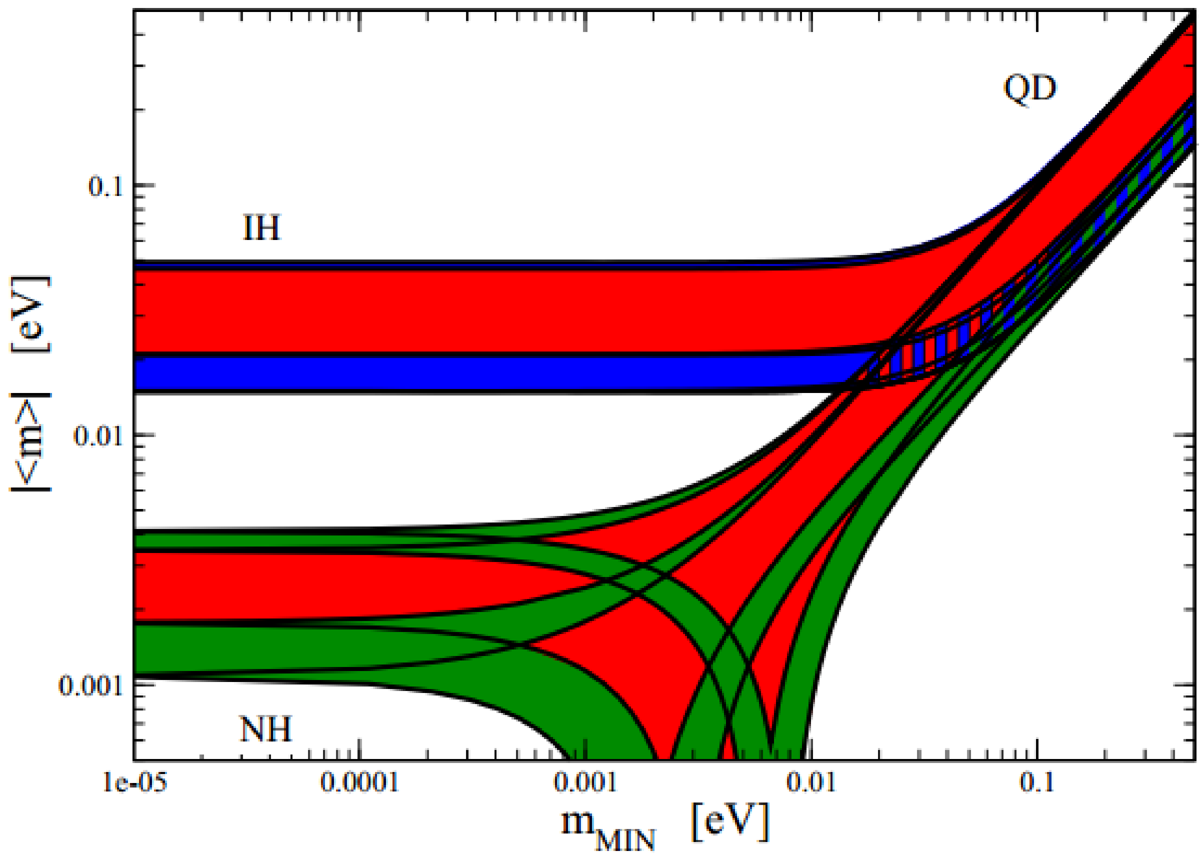


Figure from PDG

See also Pascoli and Petcov, 0711.4993 and hep-ph/0205022, and Bilenky, Pascoli and Petcov, hep-ph/0102265, among others

Neutrinoless double beta decay

$$\mathcal{A} \propto \sum_i \bar{e}\, U_{ei}(\gamma_\mu P_L)^\dagger C \frac{\not{p} + m_i}{p^2 - m_i^2} U_{ei}\gamma_\nu P_L\, e$$

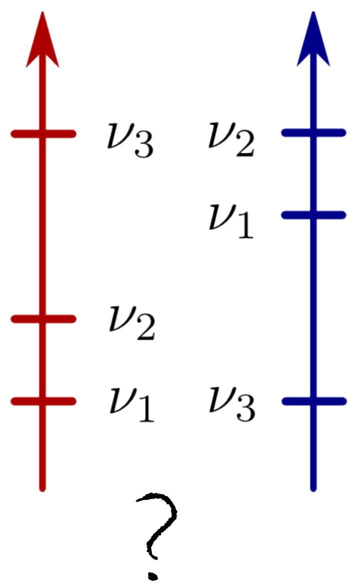
$$= \sum_i \bar{e}^c\, U_{ei}\gamma_\mu P_R \frac{m_i}{p^2 - m_i^2} P_R \gamma_\nu U_{ei}\, e$$

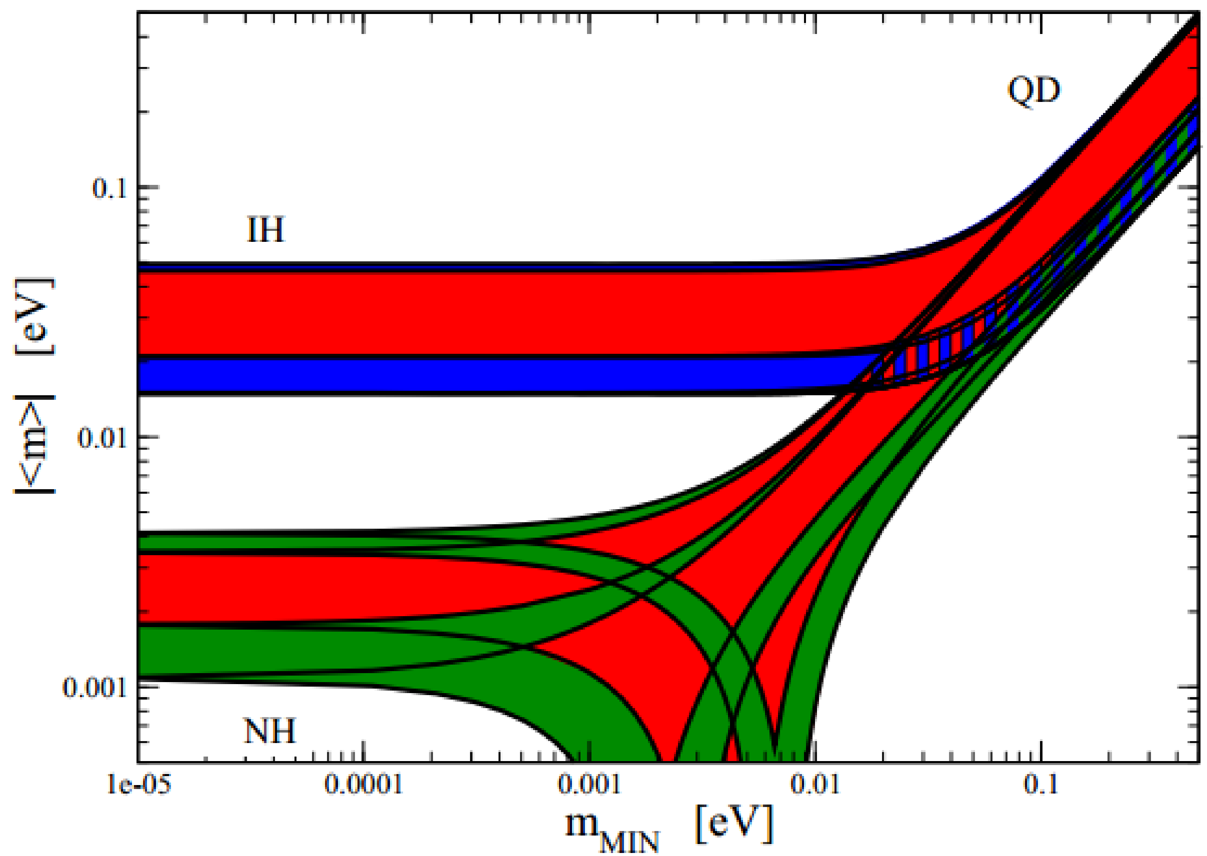
In the case where the Majorana masses are heavy, only the light neutrinos contribute, and we get:

$$\mathcal{A} \propto \langle m_{0\nu\beta\beta} \rangle = \sum_{i=1}^{3} m_i U_{ei}^2$$

Neutrinoless double beta decay

$$\langle m_{0\nu\beta\beta}\rangle = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i2\alpha_{21}} + m_3 s_{13}^2 e^{i2\alpha_{31}}$$





See Saw models

After EWSB, the mass lagrangian for neutrinos with Majorana masses can be written as:

$$-\mathcal{L}^{\nu}_{mass} = \frac{1}{2}\bar{n}^c_L \mathcal{M}^* n_L + \text{h.c.}$$

$$\mathcal{M} = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} Y_\nu \\ \frac{v}{\sqrt{2}} Y^\dagger_\nu & M \end{pmatrix} \qquad n_L = \begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix}$$

See Saw models

In the limit M » v, the diagonalization of the mass matrix gives:

$$m_{light} \sim \frac{v^2}{2} Y_\nu \frac{1}{M} Y^t$$

$$m_{heavy} \sim M$$

Type I See Saw:
only a right handed singlet is added to the SM particles

See Saw models

Another way to obtain the same result is to start from an effective operator approach:

$$\mathcal{L}^{eff} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\delta\mathcal{L}^{d=5} + \frac{1}{\Lambda^2}\delta\mathcal{L}^{d=6} + \ldots$$

See Saw models

Another way to obtain the same result is to start from an effective operator approach:

$$\mathcal{L}^{eff} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\delta\mathcal{L}^{d=5} + \frac{1}{\Lambda^2}\delta\mathcal{L}^{d=6} + \ldots$$

The only d=5 operator which can be built within the SM particle content is

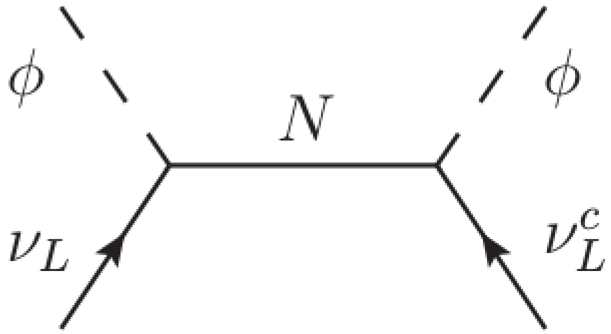
$$\mathcal{L}^{(5)} = \frac{c_5}{\Lambda_{NP}}(\overline{L}_L\widetilde{\phi})(\widetilde{\phi}^t L_L^c) \longrightarrow m_\nu \propto c_5\frac{v^2}{\Lambda}$$

Weinberg, 1979

If neutrino masses are generated through this operator, we should expect additional effects coming from higher dimension operators too...

See Saw models

Type I

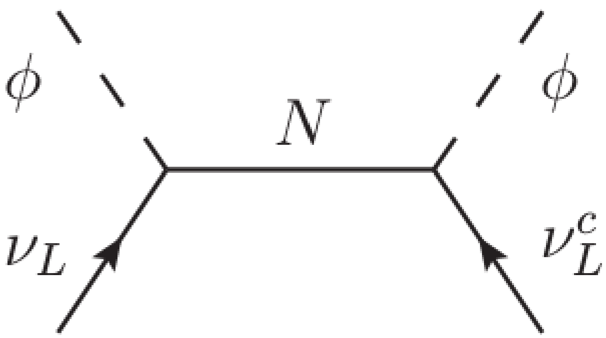


SM + fermion singlet

$$c_5 \sim Y_\nu \frac{1}{M} Y^t$$

See Saw models

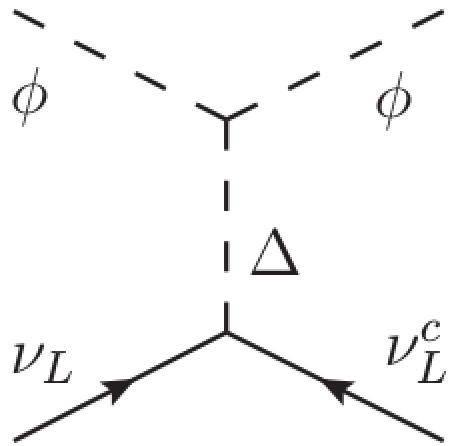
Type I



SM + fermion singlet

$$c_5 \sim Y_\nu \frac{1}{M} Y^t$$

Type II

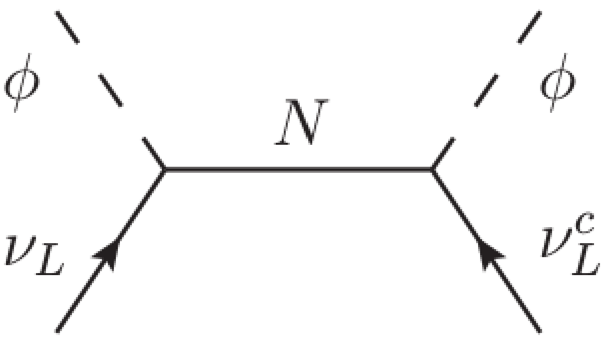


SM + scalar triplet

$$c_5 \sim Y_\nu \frac{\mu_\Delta}{M_\Delta^2}$$

See Saw models

Type I



SM + fermion singlet

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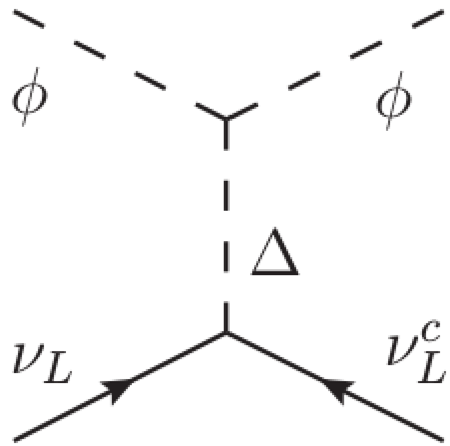
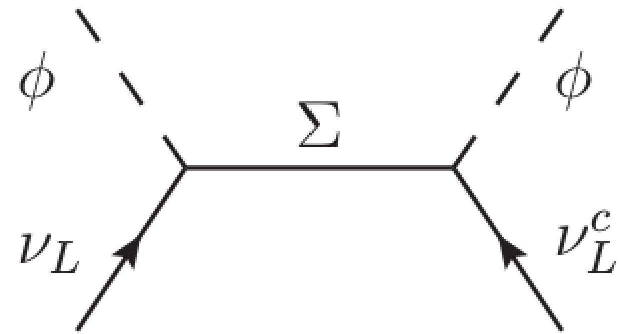


SM + scalar triplet

$$c_5 \sim Y_\nu \frac{\mu_\Delta}{M_\Delta^2}$$

Type III



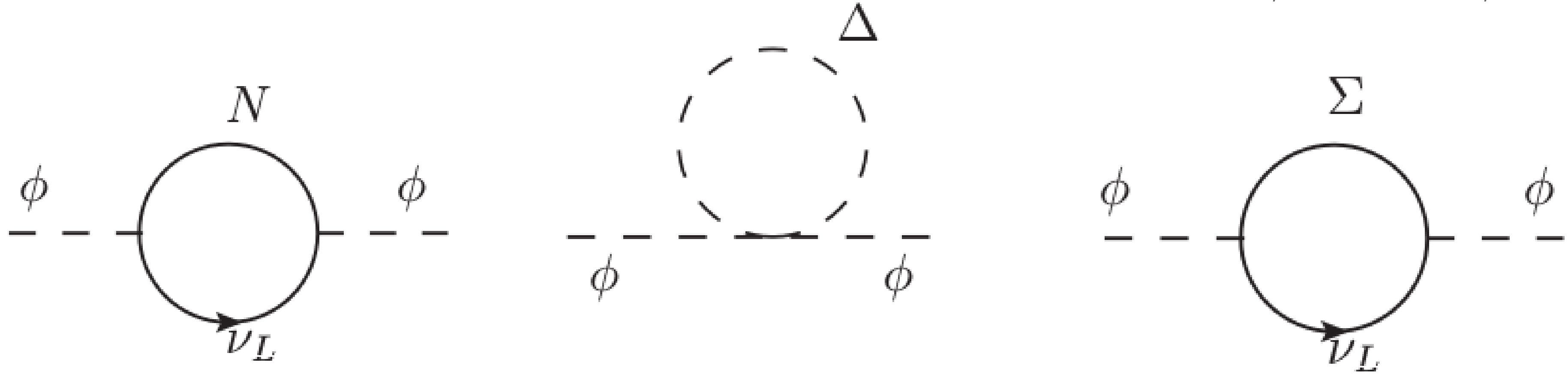
SM + fermion triplet

$$c_5 \sim Y_\nu \frac{1}{M} Y^t$$

See Saw models

See Saw models with very large Majorana masses contribute to the naturalness problem for the Higgs mass:

Vissani, ´98, Casas et al, ´04, Abada et al, ´07



E.g., for type I seesaw:

$$\delta m_H^2 \sim -\frac{Y_N^\dagger Y_N}{16\pi^2}\left[2\Lambda^2 + 2M_N^2 \log\frac{M_N^2}{\Lambda^2}\right]$$

Part II: neutrino oscillations in the standard picture

Current status in neutrino oscillations

$$
\begin{array}{rrcl}
\theta_{12}: & 31.3^\circ & \to & 35.9^\circ \\
\theta_{13}: & 7.8^\circ & \to & 9.1^\circ \\
\theta_{23}: & 38.3^\circ & \to & 53.3^\circ \\
\delta: & 0 & \to & 360^\circ
\end{array}
$$

$$
\Delta m^2_{21}: \quad (7.02 \quad \to \quad 8.09) \quad \times 10^{-5}\ \mathrm{eV}^2
$$

$$
\Delta m^2_{atm}: \left[\begin{array}{ccc} 2.3 & \to & 2.6 \\ -2.6 & \to & -2.3 \end{array} \right] \times 10^{-3}\mathrm{eV}^2
$$

Gonzalez-Garcia, Maltoni, Schwetz, 1409.5439
(see also 1312.2878 and 1405.7540)

Neutrino oscillations in vacuum

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \left[U_{2\times 2}\begin{pmatrix} 0 & 0 \\ 0 & \frac{\Delta m^2}{2E} \end{pmatrix} U^\dagger_{2\times 2}\right]\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$U_{2\times 2} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$|\nu_\alpha(t)\rangle = \sum_j U_{\alpha j} e^{-iE_j t}|\nu_j\rangle$$

Neutrino oscillations in vacuum

In propagation, each mass eigenstate acquires a different phase. This produces the oscillation:

$$|\nu_\alpha(t)\rangle = \sum_j U_{\alpha j} e^{-iE_j t} |\nu_j\rangle$$

$$\mathrm{A}_{\alpha\beta}(t) = \langle \nu_\beta | \nu_\alpha(t) \rangle = \sum_j U^*_{\beta j} U_{\alpha j} e^{-iE_j t}$$

$$P_{\alpha\beta}(t) = |\mathcal{A}_{\alpha\beta}(t)|^2$$

The two family approximation

Due to the very different oscillation amplitudes, the two-family approximation works very well in most oscillation experiments $\rightarrow$ one oscillation frequency can usually be neglected

Oscillation probabilities in this approximation are rather simple:

$$P_{app} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$P_{dis} = 1 - P_{app}$$

Types of oscillation experiments

Disappearance

If $\alpha = \beta$

Very common in current/past experiments

Appearance

If $\alpha \neq \beta$

CP violation is observable, but we need 3+ families

Note! the neutrino energy needs to be sufficient to create a charged lepton in the final state.

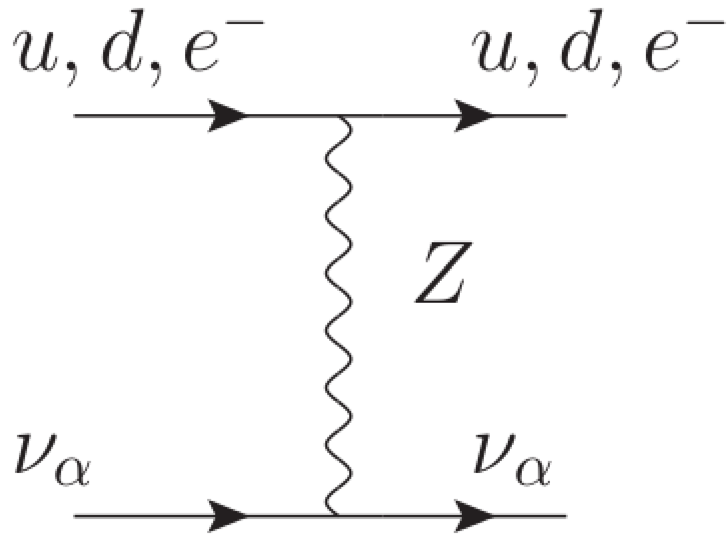
(this is typically an issue with nutau appearance experiments)

Neutrino oscillations in vacuum

Assuming the matrix to be unitary, the probability can be written down, after some algebra, as:

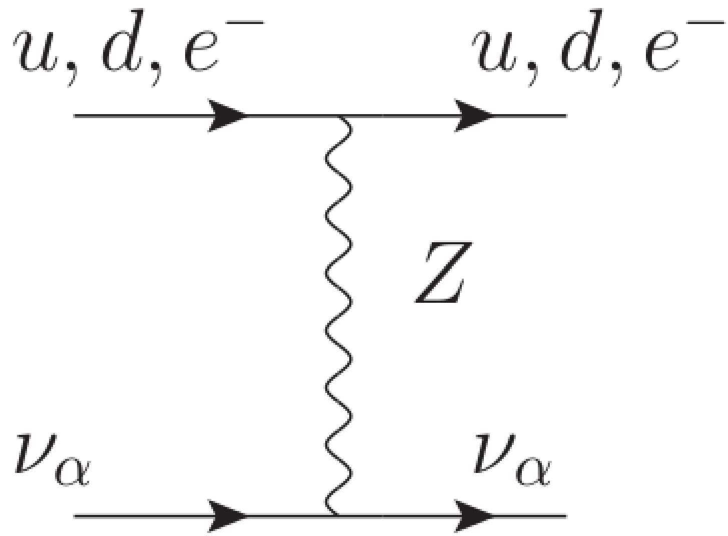
$$
\begin{aligned}
\mathcal{P}_{\alpha\beta}(L) &= \delta_{\alpha\beta} - \\
&- 4 \sum_{k,j>k} \mathrm{Re}(U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k}) \sin^2\left(\frac{\Delta m^2_{jk} L}{4E}\right) + \\
&+ \boxed{2 \sum_{k,j>k} \mathrm{Im}(U_{\alpha j} U^*_{\beta j} U^*_{\alpha k} U_{\beta k}) \sin\left(\frac{\Delta m^2_{jk} L}{2E}\right)}
\end{aligned}
$$

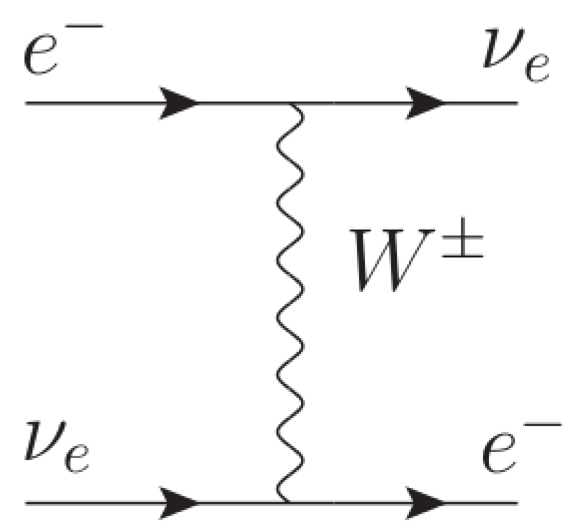
Matter effects on Earth



$$V_{NC} = -\sqrt{2} G_F \frac{N_n}{2}$$

Matter effects on Earth





$$V_{NC} = -\sqrt{2} G_F \frac{N_n}{2} \qquad V_{CC} = \pm\sqrt{2} G_F N_e$$

On Earth, N_e can be considered as a constant.

Otherwise (in the Sun, for instance), things can be more complicated

Matter effects on Earth

In two-families:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \left[U_{2x2}\begin{pmatrix}0 & 0\\ 0 & \frac{\Delta m^2}{2E}\end{pmatrix}U^\dagger_{2x2} + V\right]\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

$$U_{2x2} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$$

$$V = \begin{pmatrix}V_{CC}+V_{NC} & 0\\ 0 & V_{NC}\end{pmatrix}$$

Wolfenstein, 1978

Barger, 1980

Mikheev and Smirnov, 1985

Matter effects on Earth

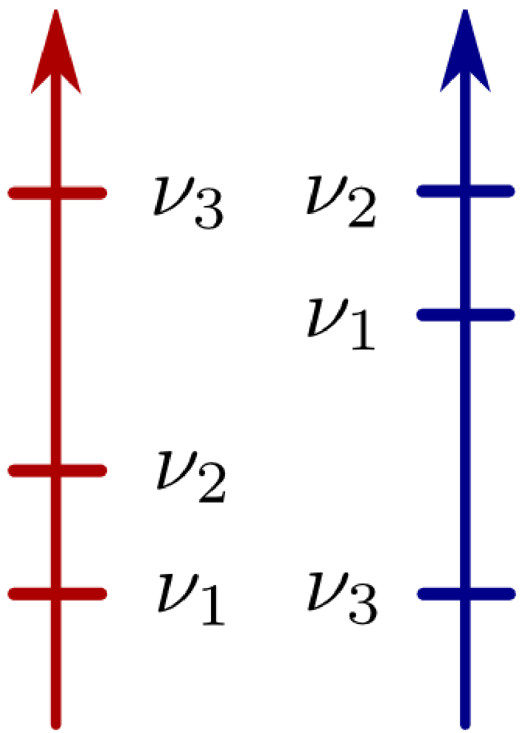
In two-families:

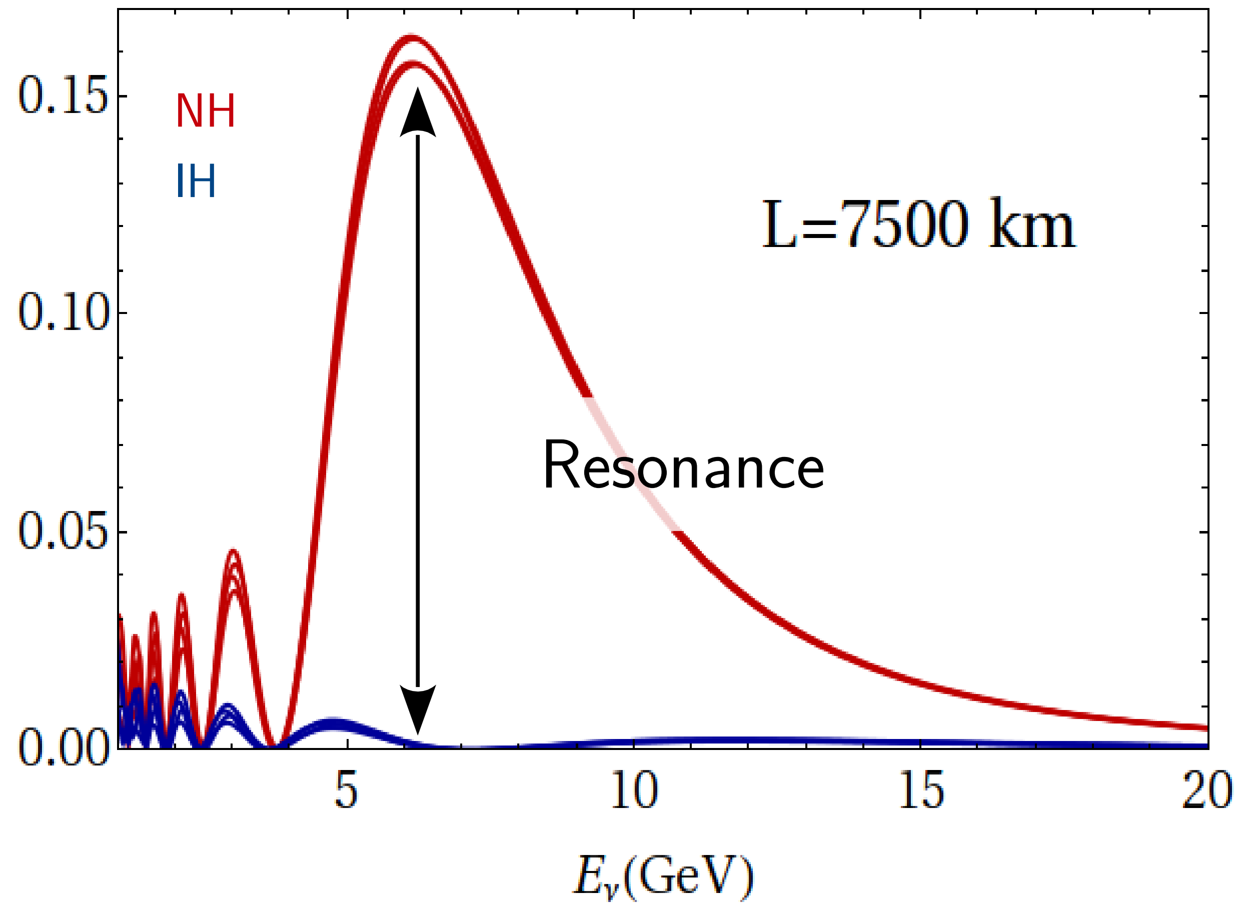
$$P_{app} = \sin^2 2\theta_M \sin^2\left(\frac{\Delta m^2_M L}{4E}\right)$$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \; ; \; A = \frac{2EV}{\Delta m^2}$$

Even for small angles, the effective mixing angle in matter gets enhanced if the resonance condition is satisfied

The matter resonance





Wolfenstein, 1978

Barger, 1980

Mikheev and Smirnov, 1985

Why CP violation searches?

- In the SM (extended with neutrino masses), there are three possible sources of CP violation:
  - Quark mixing $\rightarrow$ large
  - Strong CP problem $\rightarrow$ tiny!! (if any)
  - Lepton mixing $\rightarrow$??
- The amount of CP violation in the quark sector of the SM is not large enough to explain the matter-antimatter asymmetry of the Universe.
- Leptogenesis?

Yanagida, 1979; Ramond, Gell-Mann, Slansky, 1979

Fukugita, Yanagida, 1986

CP violation searches

Three-family ν_μ appearance oscillation probability, in matter:

$$P_{\nu_e \nu_\mu (\bar{\nu}_e \bar{\nu}_\mu)} = s_{23}^2 \sin^2 2\theta_{13} \left(\frac{\Delta_{13}}{\tilde{B}_\mp} \right)^2 \sin^2 \left(\frac{\tilde{B}_\mp L}{2} \right) +$$

atm

$$+ c_{23}^2 \sin^2 2\theta_{12} \left(\frac{\Delta_{12}}{V} \right)^2 \sin^2 \left(\frac{V L}{2} \right)$$

sol

interf.

$$+ \tilde{J} \frac{\Delta_{12}}{V} \frac{\Delta_{13}}{\tilde{B}_\mp} \sin \left(\frac{VL}{2} \right) \sin \left(\frac{\tilde{B}_\mp L}{2} \right) \cos \left(\pm\delta - \frac{\Delta_{13} L}{2} \right)$$

$$\tilde{J} \equiv \cos\theta_{13} \ \sin 2\theta_{13} \ \sin 2\theta_{23} \ \sin 2\theta_{12}$$

$$\Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{2E}$$

$$\tilde{B}_\mp \equiv |V \mp \Delta_{13}|$$

Cervera et al, hep-ph/0002108
(see also e.g., Freund, hep-ph/0103300, Akhmedov et al, hep-ph/0402175, and Asano, Minakata, 1103.4387)

The degeneracy problem

Since we only have two measurable quantities which depend on the CP phase, degeneracies can arise with the other unknown parameters (θ_{23} and the neutrino mass ordering)

For instance, the value of θ_{23} is usually measured through muon neutrino disappearance:

$$P_{\mu\mu} \simeq 1 - \sin^2 2\theta_{23} \sin^2 \left(\frac{\Delta m^2_{\mu\mu} L}{4E} \right)$$

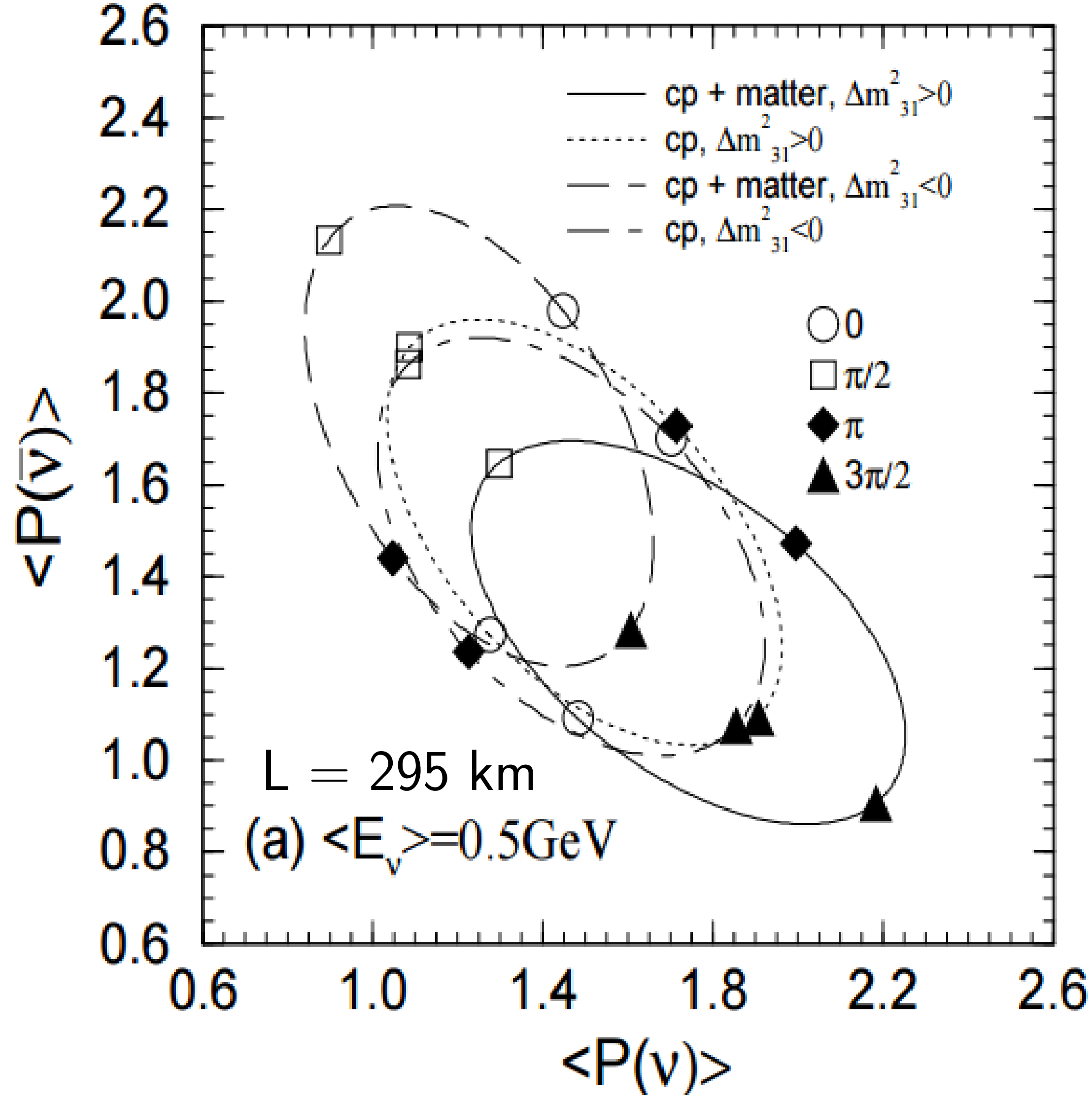
Burguet-Castell *et al.*, hep-ph/0103258

Minakata, Nunokawa, hep-ph/0108085

Fogli, Lisi, hep-ph/9604415

Barger, Marfatia, Whisnant, hep-ph/0112119

The degeneracy problem



Minakata, Nunokawa, hep-ph/0108085

The degeneracy problem

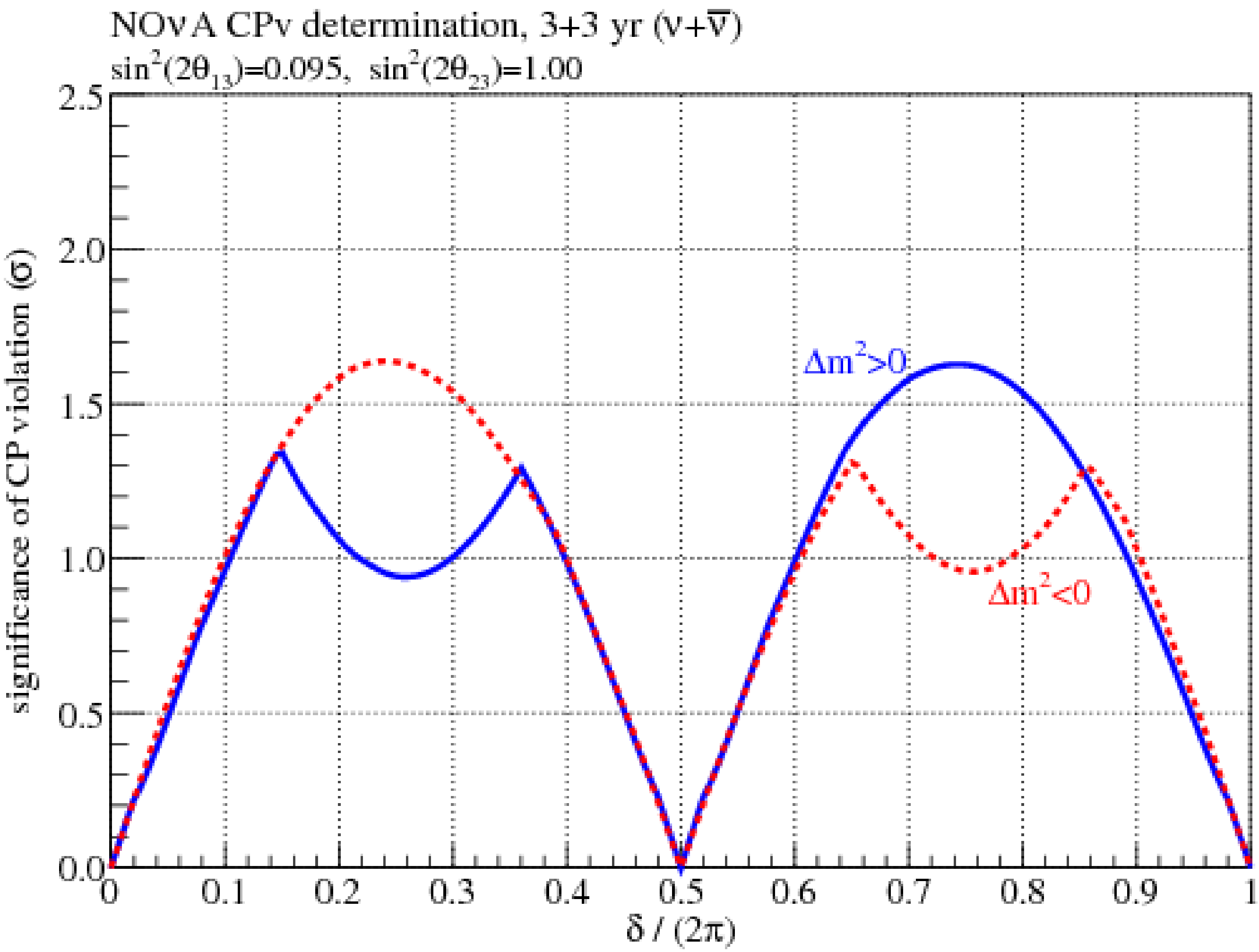


Figure taken from the webpage of the NOvA experiment

Part III: effects of new physics, some examples

Non-Standard Interactions

$$\mathcal{L}^{eff} = \mathcal{L}_{SM} + \frac{1}{\Lambda}\,\delta\mathcal{L}^{d=5} + \frac{1}{\Lambda^2}\,\delta\mathcal{L}^{d=6} + \ldots$$

Non-Standard Interactions

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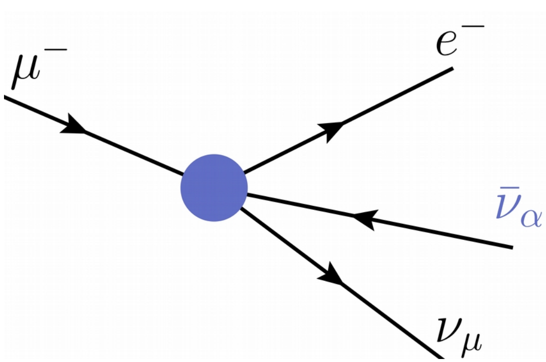
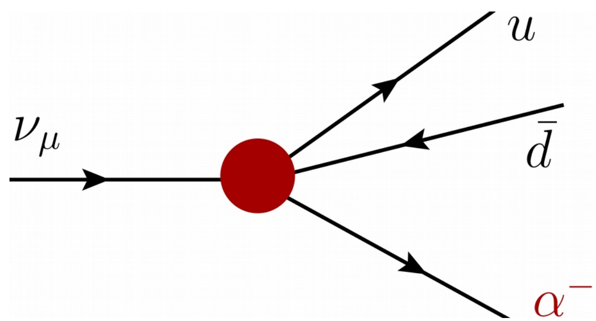
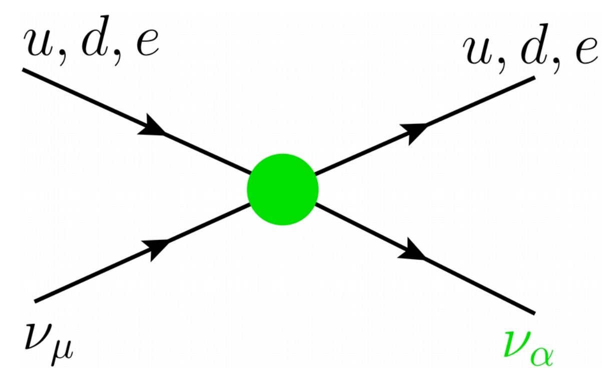
NSI can affect neutrinos in production, detection and propagation processes





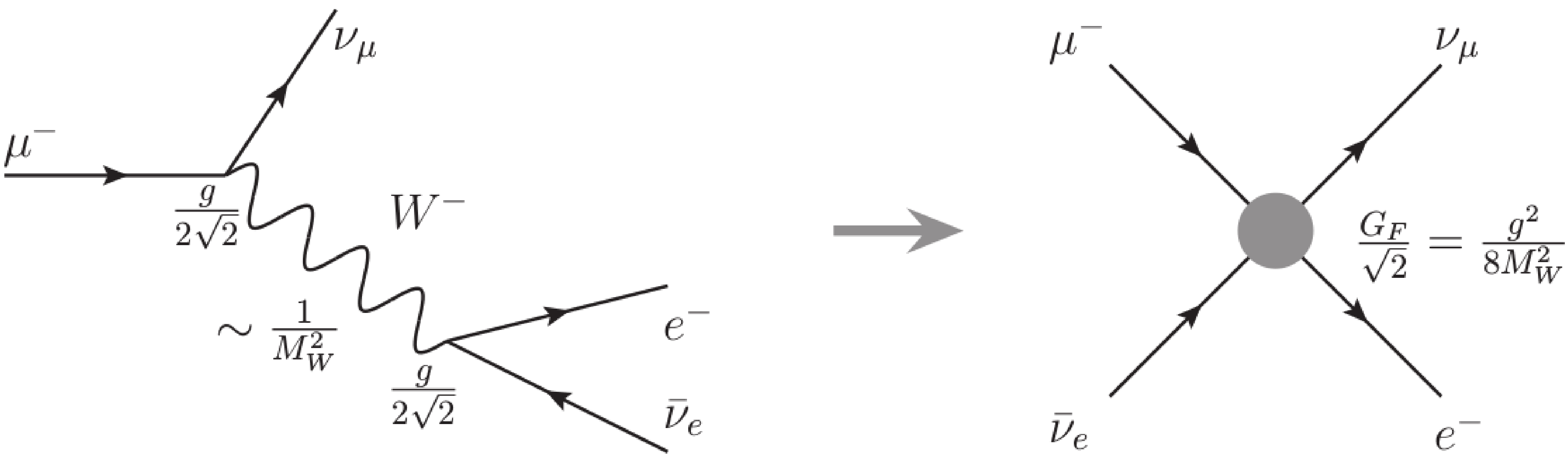


$\varepsilon^{e\mu}_{\mu\alpha}\left(\overline{e}\gamma^\rho\mu\right)\left(\overline{\nu}_\mu\gamma_{\rho,L}\nu_\alpha\right)$ $\varepsilon^{ud}_{\mu\alpha}V_{ud}\left(\overline{d}\gamma^\rho u\right)\left(\overline{\nu}_\mu\gamma_{\rho,L}\alpha\right)$ $\varepsilon^{f}_{\mu\alpha}\left(\overline{f}\gamma^\rho f\right)\left(\overline{\nu}_\mu\gamma_{\rho,L}\nu_\alpha\right)$

Near detectors

Far detectors

Non-Standard Interactions



Non-Standard Interactions

$$|\epsilon^{\oplus}_{\alpha\beta}| < \begin{pmatrix} 4.2 & 0.33 & 3.0 \\ 0.33 & 0.068 & 0.33 \\ 3.0 & 0.33 & 21 \end{pmatrix}$$

Davidson, Pena-Garay, Rius, Santamaria
hep-ph/0302093
Biggio, Blennow, Fernandez-Martinez
0907.0097 [hep-ph]

Model independent bounds are rather weak. However,

- It is expected that any model giving NSI would produce small effects at low energies, since they are (at least) quadratically suppressed with the scale of NP

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Model independent bounds are rather weak. However,

- It is expected that any model giving NSI would produce small effects at low energies, since they are (at least) quadratically suppressed with the scale of NP
- Any model of NP should preserve gauge invariance. This imposes stronger bounds on NSI through charged lepton processes (at least, $\sim 10^{-2}$)

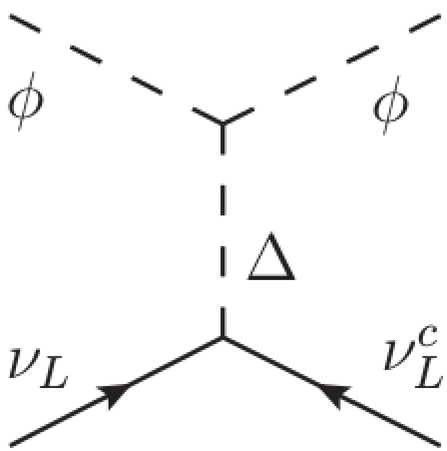
Antusch, Baumann, Fernandez-Martinez, 0807.1003 [hep-ph]
Gavela, Hernandez, Ota, Winter, 0809.3451 [hep-ph]

Non-Standard Interactions

$$\mathcal{L}^{eff} = \mathcal{L}_{SM} + \frac{1}{\Lambda_L}\delta\mathcal{L}^{d=5} + \frac{1}{\Lambda_{Fl}{}^2}\delta\mathcal{L}^{d=6} + \ldots$$

Example:

Type II



SM + scalar triplet

$$c_5 \sim Y_\nu \frac{\mu_\Delta}{M_\Delta^2}$$

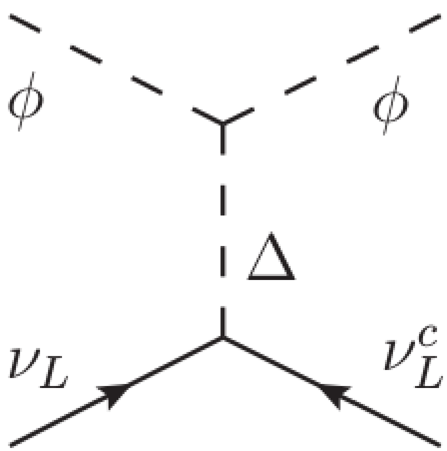
$$c_6 \sim Y_\nu^\dagger \frac{1}{M_\Delta^2} Y_\nu$$

Non-Standard Interactions

$$\mathcal{L}^{eff} = \mathcal{L}_{SM} + \frac{1}{\Lambda_L}\delta\mathcal{L}^{d=5} + \frac{1}{{\Lambda_{Fl}}^2}\delta\mathcal{L}^{d=6} + \ldots$$

Example:

Type II



SM + scalar triplet

$$c_5 \sim Y_\nu \frac{\mu_\Delta}{M_\Delta^2} \qquad \Lambda_L \to \frac{M_\Delta^2}{\mu_\Delta}$$

$$c_6 \sim Y_\nu^\dagger \frac{1}{M_\Delta^2} Y_\nu \qquad \Lambda_{Fl} \to M_\Delta$$

Sterile neutrinos at the eV scale

- Sterile neutrinos at the eV scale could only be observed via oscillations (sterile!)
- Possible signatures:
  - Disappearance
  - NC event rates
  - Appearance
- Some anomalies observed at the 2-3 sigma CL

Sterile neutrinos at the eV scale

- Furthermore, there is a tension between different data sets

$$P_{app} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$P_{dis} = 1 - P_{app}$$

$$\sin^2 2\theta_{\mu e} \approx \frac{1}{4}\sin^2 2\theta_{ee} \sin^2 2\theta_{\mu\mu}$$

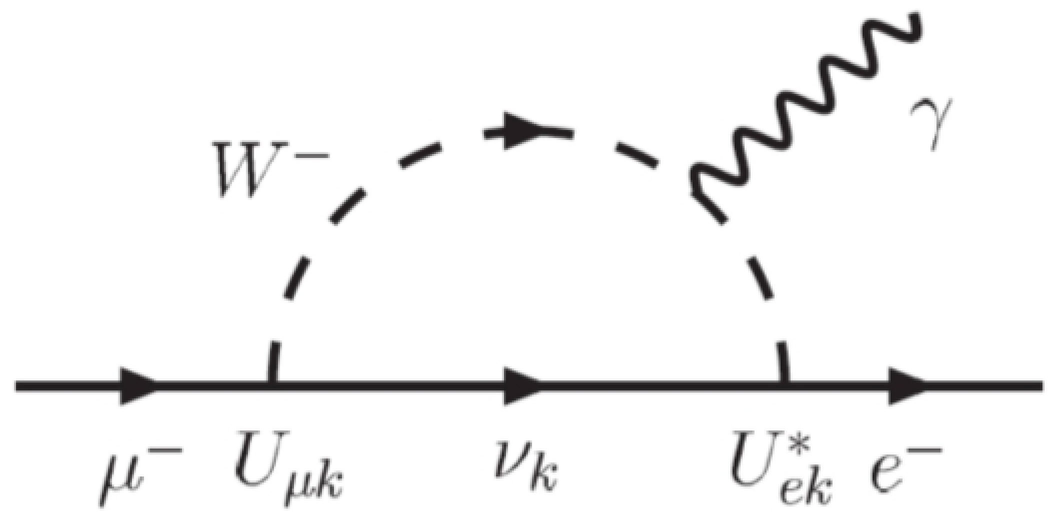
Neutrino tasks for the next 20 years

- is there CP violation in the leptonic sector? What is the value of δ?
- what is the ordering of neutrino masses?
- which flavor of neutrinos dominates the third mass eigenstate?
- why is the mixing in the leptonic sector so different from the mixing in the quark sector? does the flavour of the SM obey a certain pattern?
- are there more than three neutrino species?
- are neutrinos Majorana particles?
- why are neutrinos so light with respect to the charged leptons?
- what is the value of the lightest neutrino mass?
- are there non-standard neutrino interactions?
- ...

Thank you, and hope to
see you tonight at recitation!

Backup

Charged lepton flavor processes

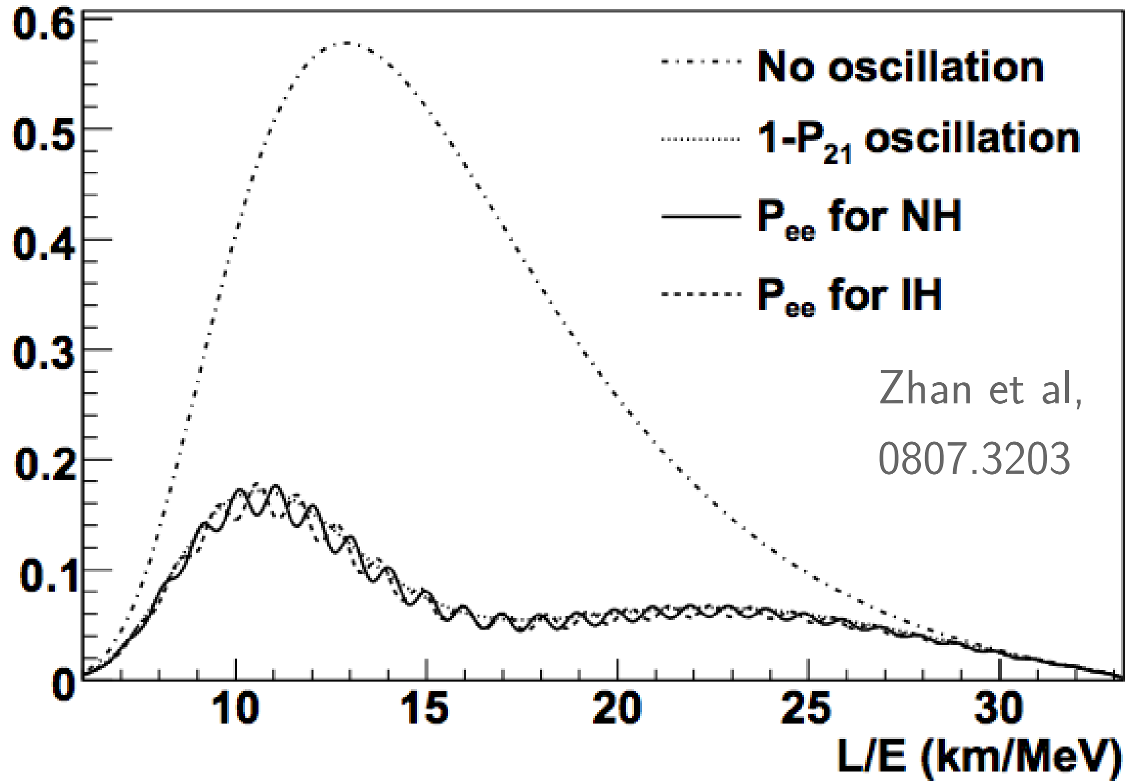


In the SM with neutrino masses, this process exists, but is tiny!

$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W} \right|^2 < 10^{-54}$$

Reactor experiments at medium L

Petcov, Piai, hep-ph/01102074
Choubey, Petcov, Piai,
hep-ph/0306017

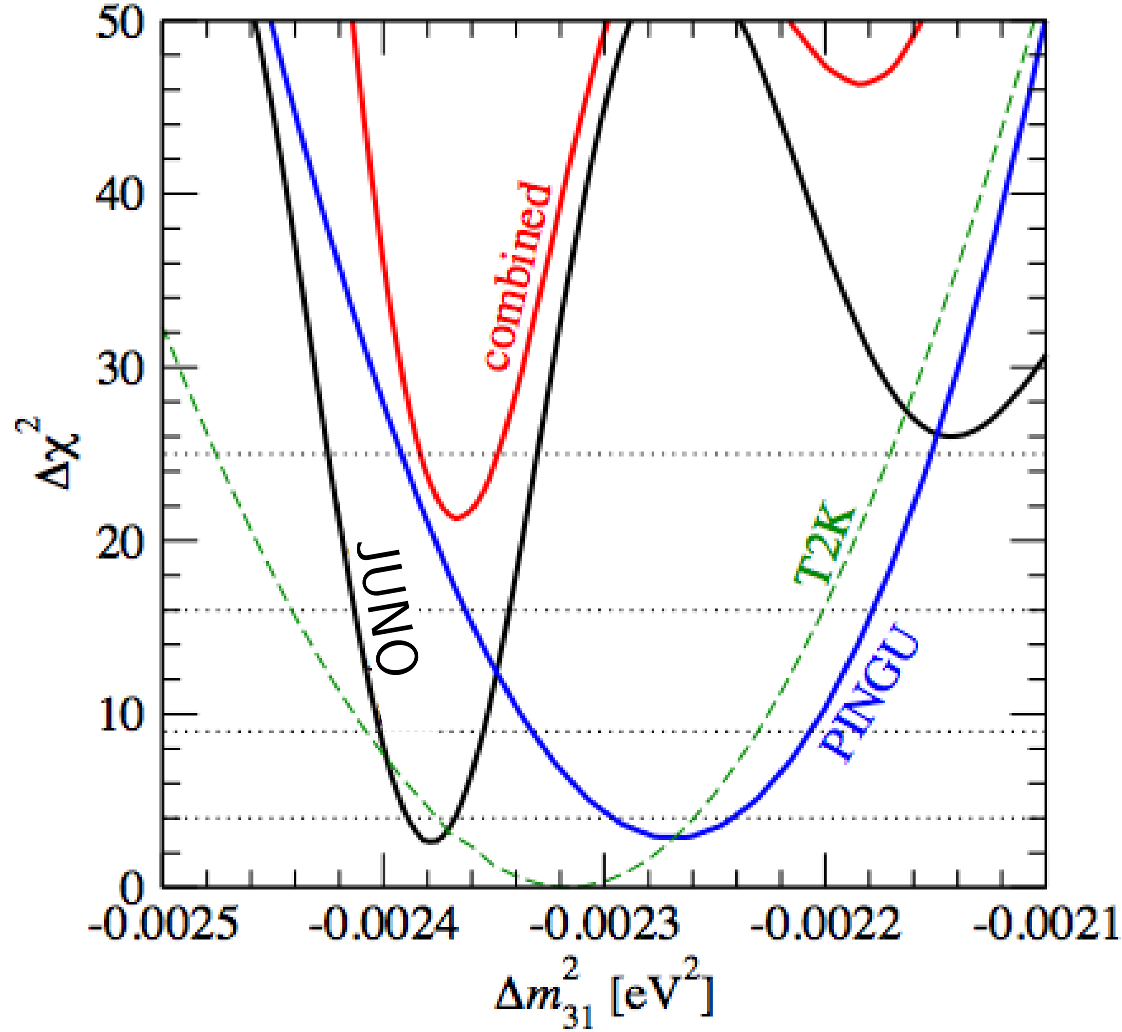


Zhan et al, 0807.3203

$$
\begin{aligned}
P_{ee} &= 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right) \\
&- \sin^2 2\theta_{13}\left[c_{12}^2 \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) + s_{12}^2 \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)\right]
\end{aligned}
$$

Precise measurement of mass splittings

The ordering of neutrino masses may as well come from a global fit to different data



Blennow, Schwetz, 1306.3988 [hep-ph]

(see also Li *et al*, 1303.6733 [hep-ph], for instance)