Deep Inelastic Scattering
Part 2

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Jefferson Lab

CTEQ 2015 Summer School

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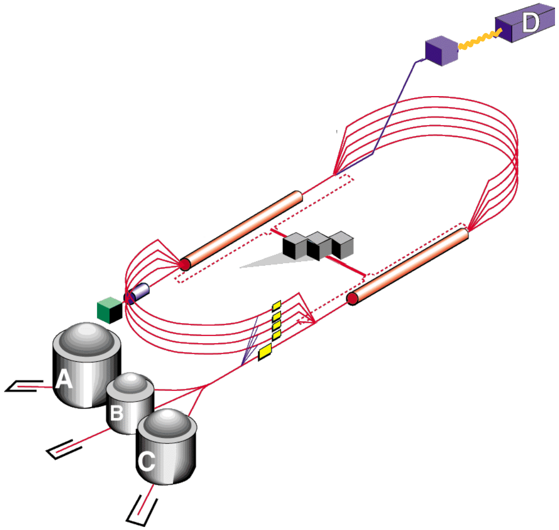


Outline

Day 1:

1. Introduction to DIS and the Quark Parton Model
2. Formalism
   →Unpolarized DIS
   →Polarized DIS
3. Results and examples

Day 2:

1. Nuclear Effects in DIS
2. Beyond inclusive scattering
   → Semi-inclusive reactions (SIDIS)

Deep Inelastic Scattering

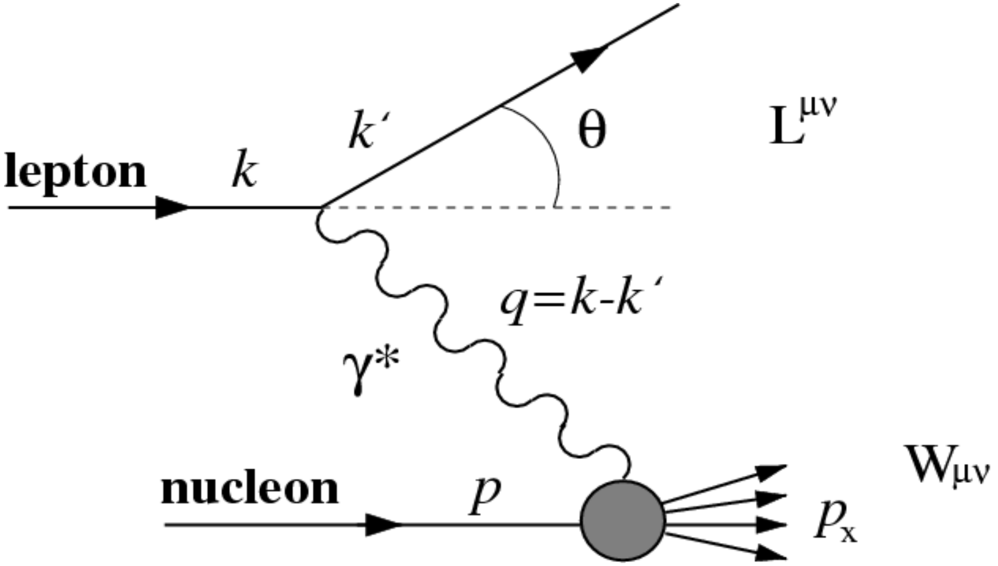
$$\frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2 (E')^2}{Q^4}\left[W_2(\nu,Q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,Q^2)\sin^2\frac{\theta}{2}\right]$$

Large Q^2

$$MW_1(\nu,Q^2) \rightarrow F_1(x)$$

$$\nu W_2(\nu,Q^2) \rightarrow F_2(x)$$

$$x = \frac{Q^2}{2M\nu}$$



F_2 interpreted in the **quark-parton model** as the charge-weighted sum over quark distributions:

$$F_2(x) = \sum_i e_i^2 x q_i(x)$$

Nuclear Effects in DIS

Typical nuclear binding energies
→ MeV while DIS scales → GeV

(super) Naïve expectation:

$$F_2^A(x) = ZF_2^p(x) + (A-Z)F_2^n(x)$$

More sophisticated approach includes effects from Fermi motion

$$F_2^A(x) = \sum_i \int_x^{M_A/m_N} dy f_i(y) F_2^N(x/y)$$

Quark distributions in nuclei were not expected to be significantly different (below x=0.6)

$$F_2^{Fe} / \left(ZF_2^p + (A-Z)F_2^n \right)$$

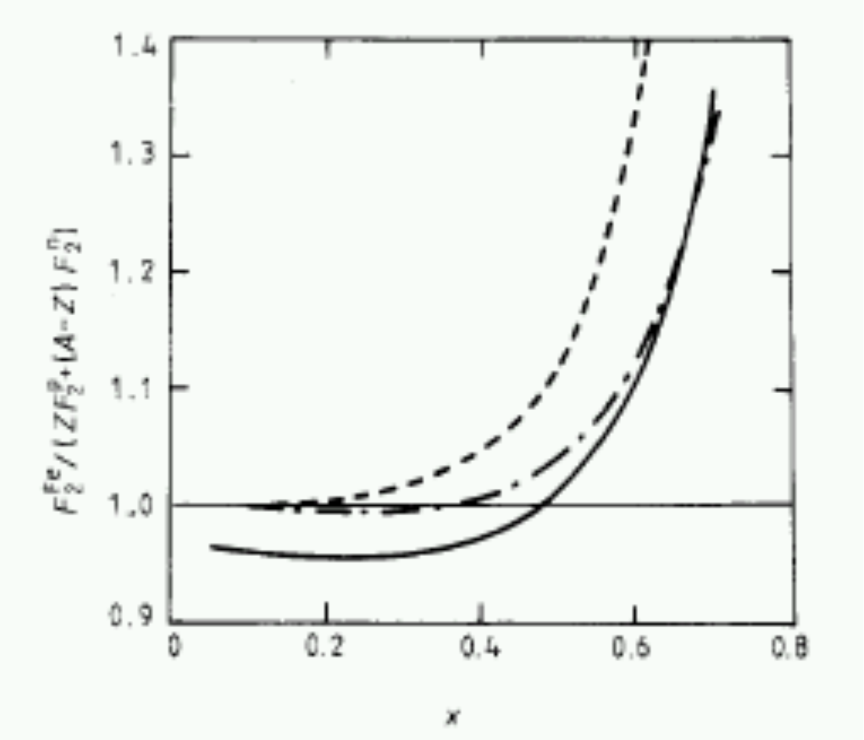
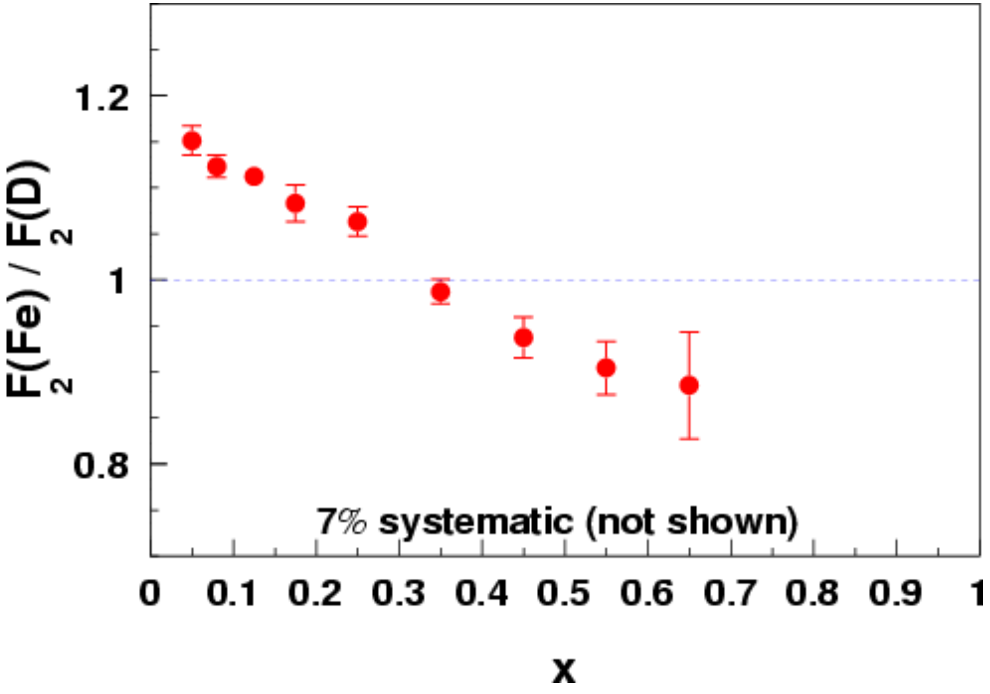


Figure from Bickerstaff and Thomas, J. Phys. G 15, 1523 (1989)
Calculation: Bodek and Ritchie PRD 23, 1070 (1981)

Discovery of the EMC Effect

- First published measurement of nuclear dependence of F_2 by the European Muon Collaboration in 1983
- Observed 2 mysterious effects
  - Significant enhancement at small *x* → Nuclear Pions! (no)
  - Depletion at large *x* → the "EMC Effect"
- Enhancement at *x<0.1* later went away

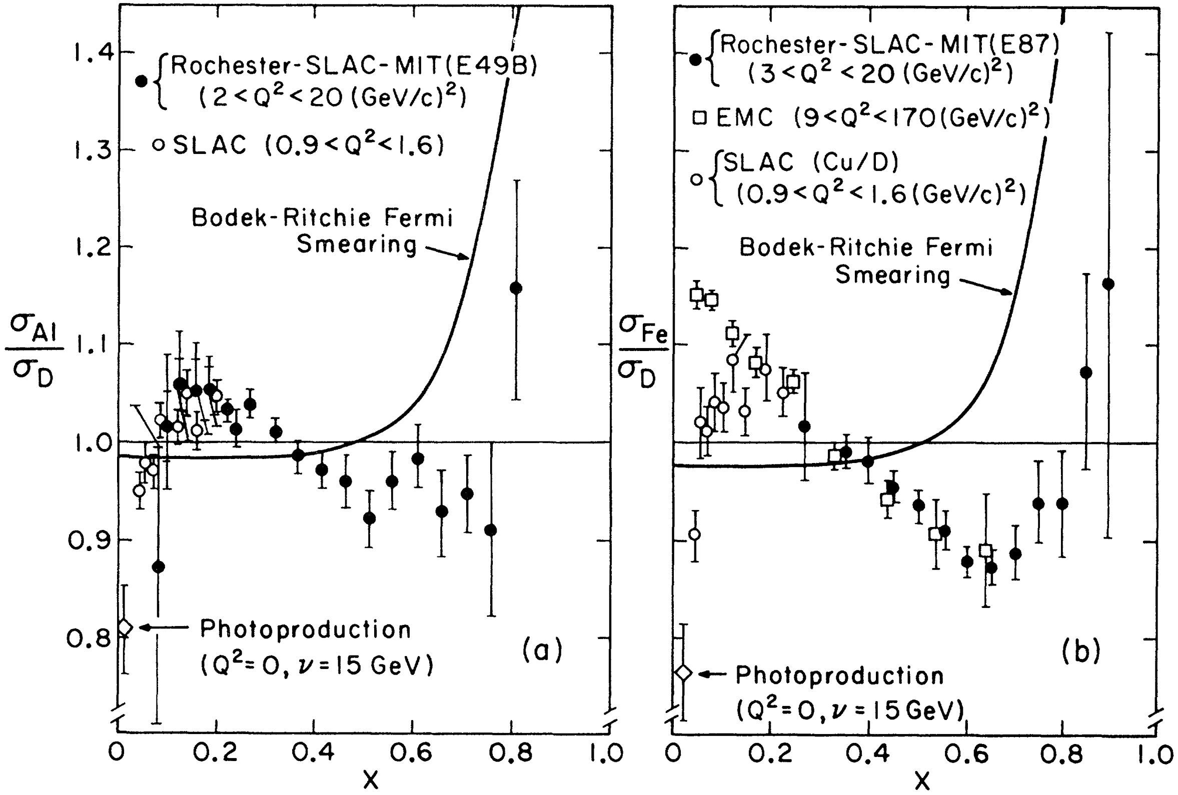


Aubert et al, Phys. Lett. B123, 275 (1983)

Confirmation of the Effect

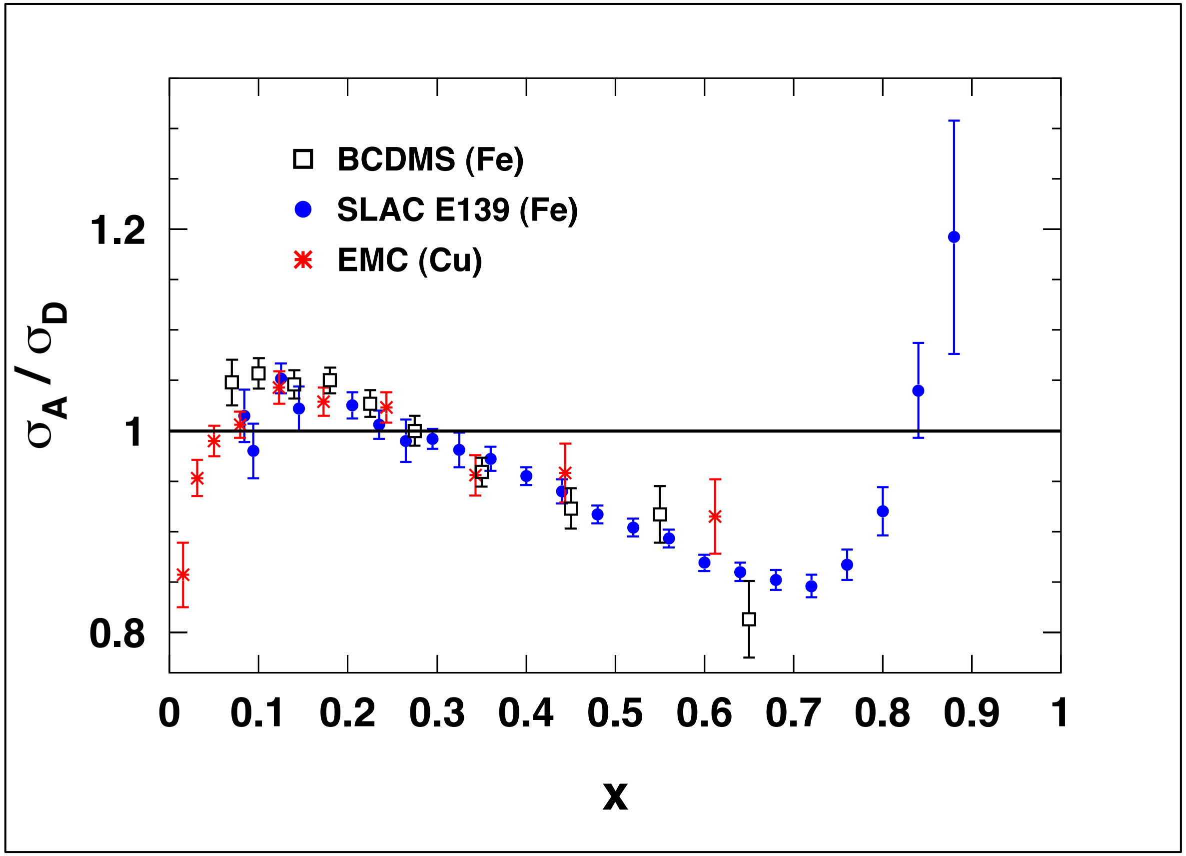
SLAC re-analysis of old solid target data used for measurements of cryotarget wall backgrounds

→Effect for *x>0.3* confirmed
→No large excess at very low *x*



Bodek et al, PRL 50, 1431 (1983) and PRL 51, 534 (1983)

Subsequent Measurements



A program of dedicated measurements quickly followed

The resulting data is remarkably consistent over a large range of beam energies and measurement techniques

Why is the EMC Effect Important?

- Neutron structure functions
  - Almost all the information we have on neutron structure functions comes from deuterium data
  - Nuclear effects in deuterium relevant for extraction of neutron information – directly impacts PDFS
- Neutrino experiments
  - Neutrino experiments need nuclear targets
  - Extraction of information for nucleons requires understanding nuclear effects
- Understanding QCD
  - Understanding the structure of the nucleon is obviously a key goal
  - Understanding the force between nucleons and how nuclei are held together also crucial
  - Why do “effective theories” work so well? At what point do quarks and gluons become relevant?

Nuclear dependence of structure functions

Experimentally, we measure cross sections (and the ratios of cross sections)

$$\frac{d\sigma}{d\Omega dE^{/}} = \frac{4\alpha^2(E^{/})^2}{Q^4\nu}\left[F_2(\nu,Q^2)\cos^2\frac{\theta}{2} + \frac{2}{M\nu}F_1(\nu,Q^2)\sin^2\frac{\theta}{2}\right]$$

$$F_2(x) = \sum_i e_i^2 x q_i(x)$$

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_2}{2xF_1}\left(1+4\frac{M^2x^2}{Q^2}\right)-1$$

$$\epsilon = \left[1+2\left(1+\frac{Q^2}{4M^2x^2}\right)\tan^2\frac{\theta}{2}\right]^{-1}$$

$$\frac{\sigma_A}{\sigma_D} = \frac{F_2^A(1+\epsilon R_A)(1+R_D)}{F_2^D(1+R_A)(1+\epsilon R_D)}$$

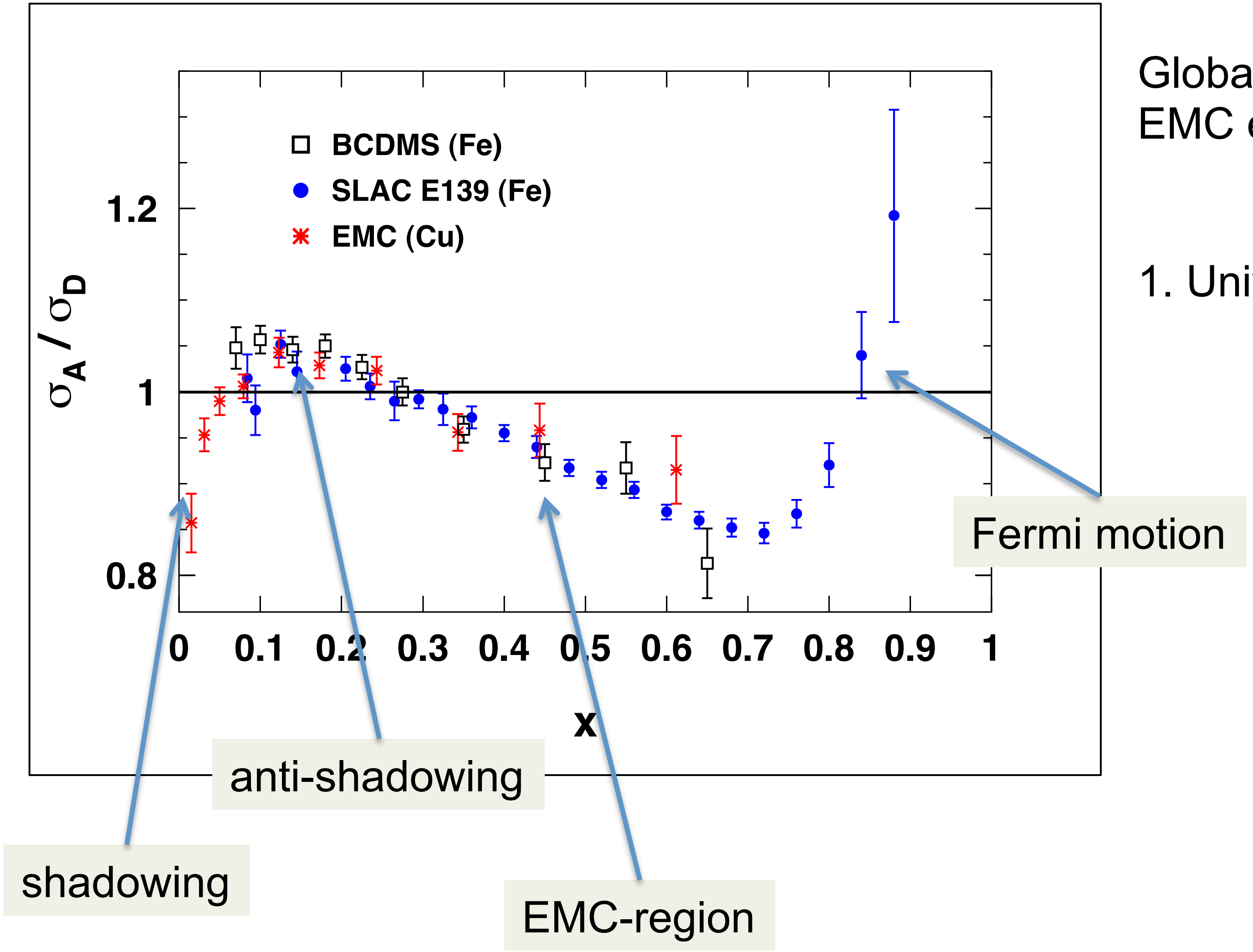
In the limit R_A = R_D or ε=1 $\longrightarrow$ $\boxed{\sigma_A/\sigma_D = F_2^A/F_2^D}$

Experiments almost always display cross section ratios, σ_A/σ_D

→Often these ratios are labeled or called F_2^A/F_2^D

→ Sometimes there is an additional uncertainty estimated to account for the $\sigma \rightarrow F_2$ translation. Sometimes there is not.

Properties of the EMC Effect



shadowing

anti-shadowing

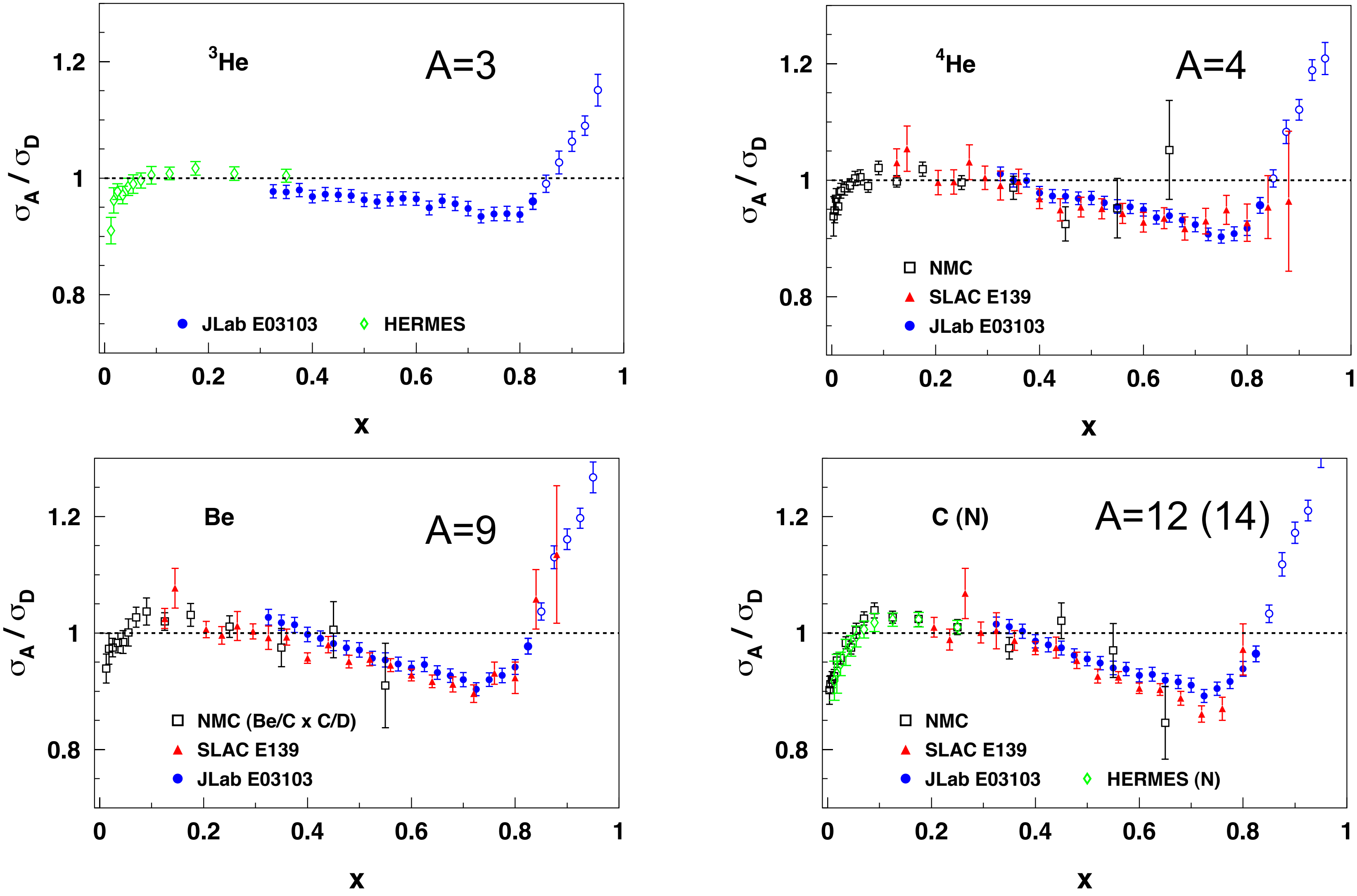
EMC-region

Fermi motion

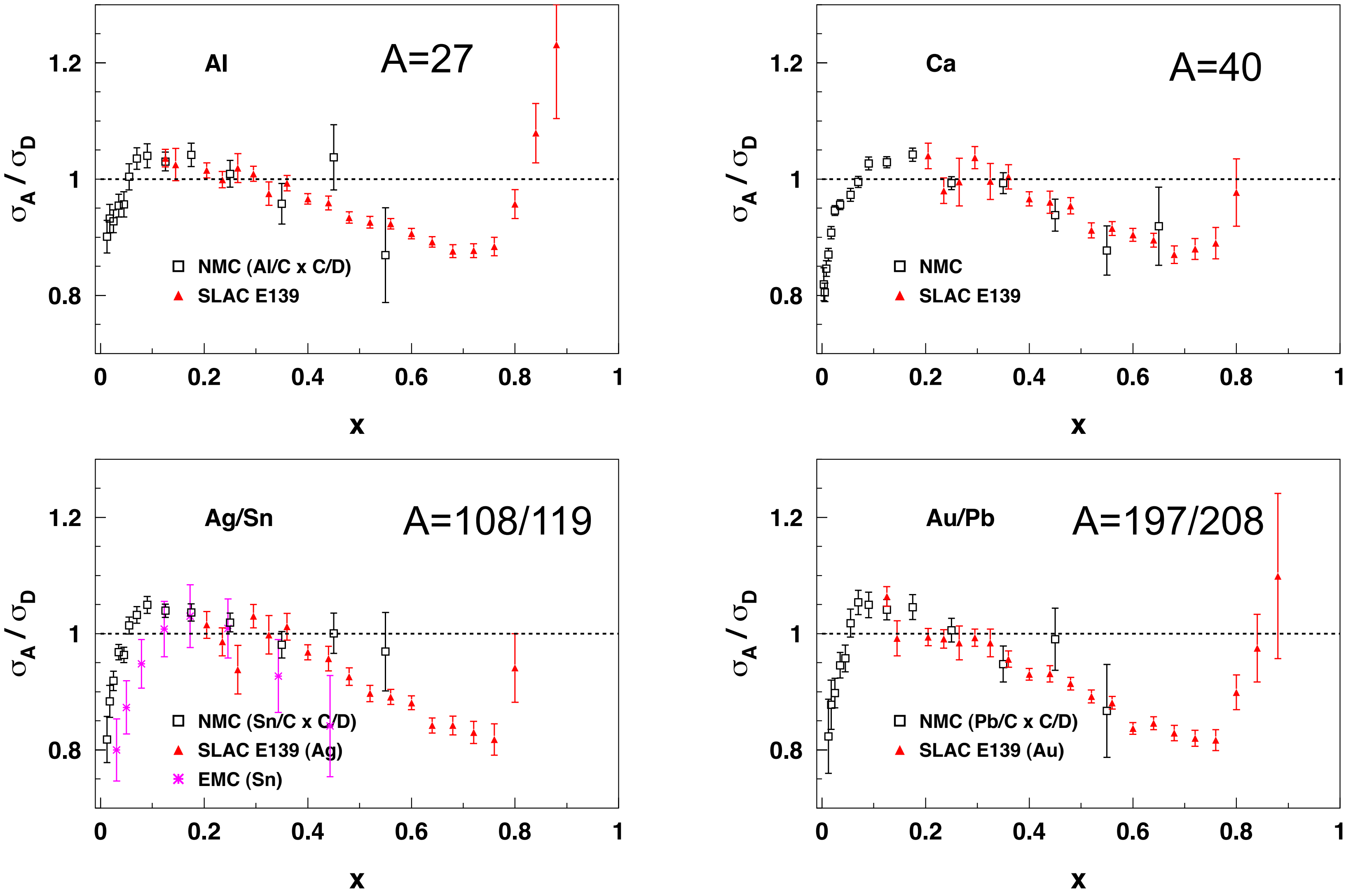
Global properties of the EMC effect

1. Universal x-dependence

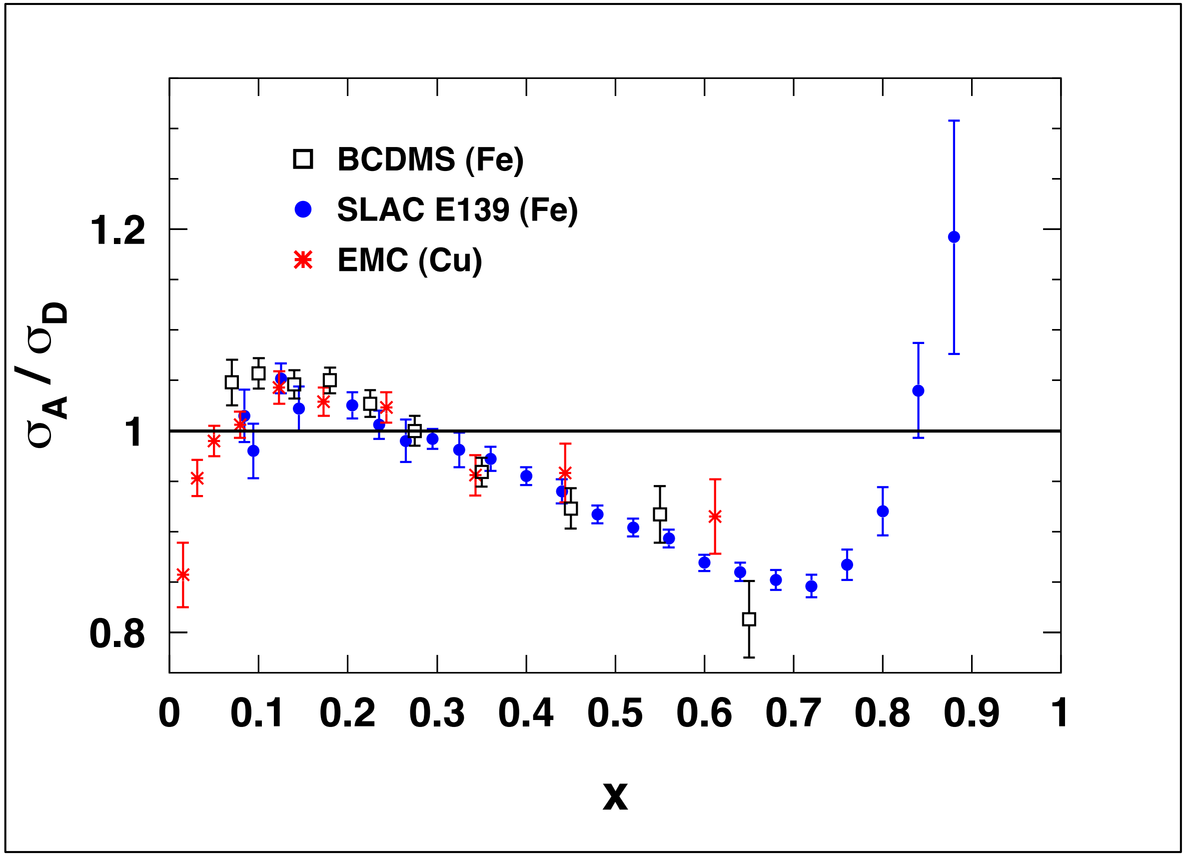
x Dependence



x Dependence



Properties of the EMC Effect



Global properties of the EMC effect

1. Universal x-dependence
2. Little Q^2 dependence*

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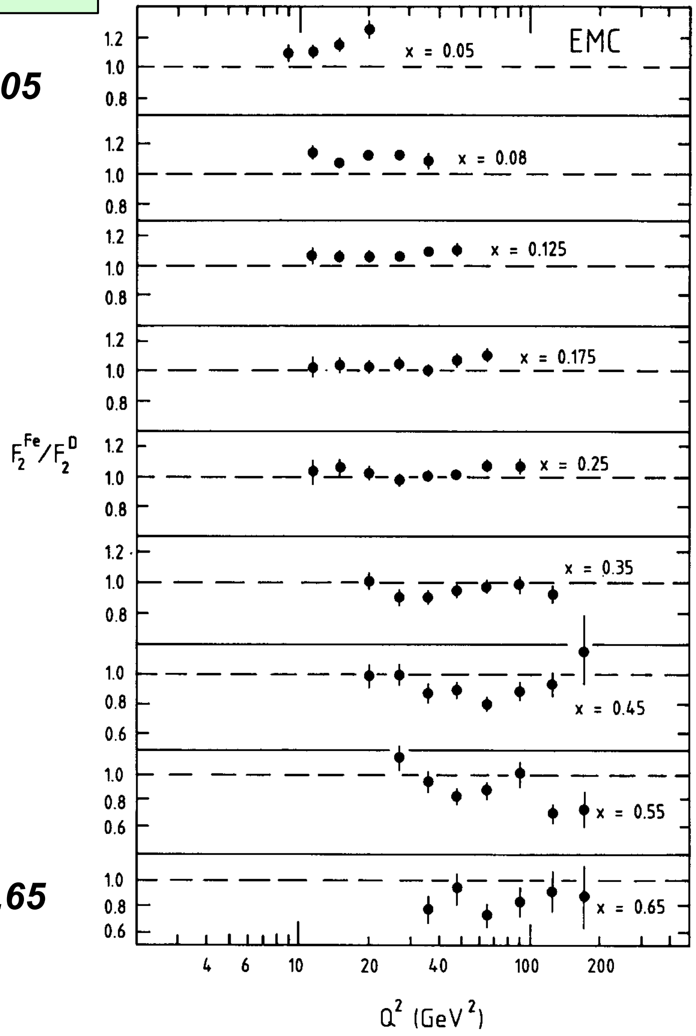
Q^2 Dependence of the EMC Effect

EMC $Q^2=10\text{-}200$ GeV^2

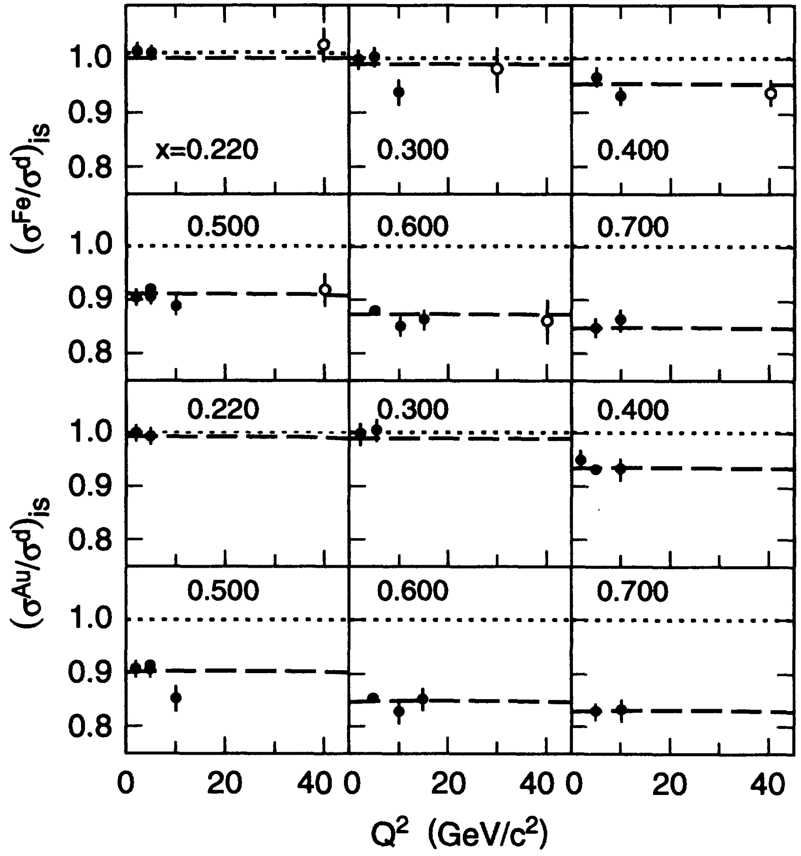
x=0.05

x=0.65



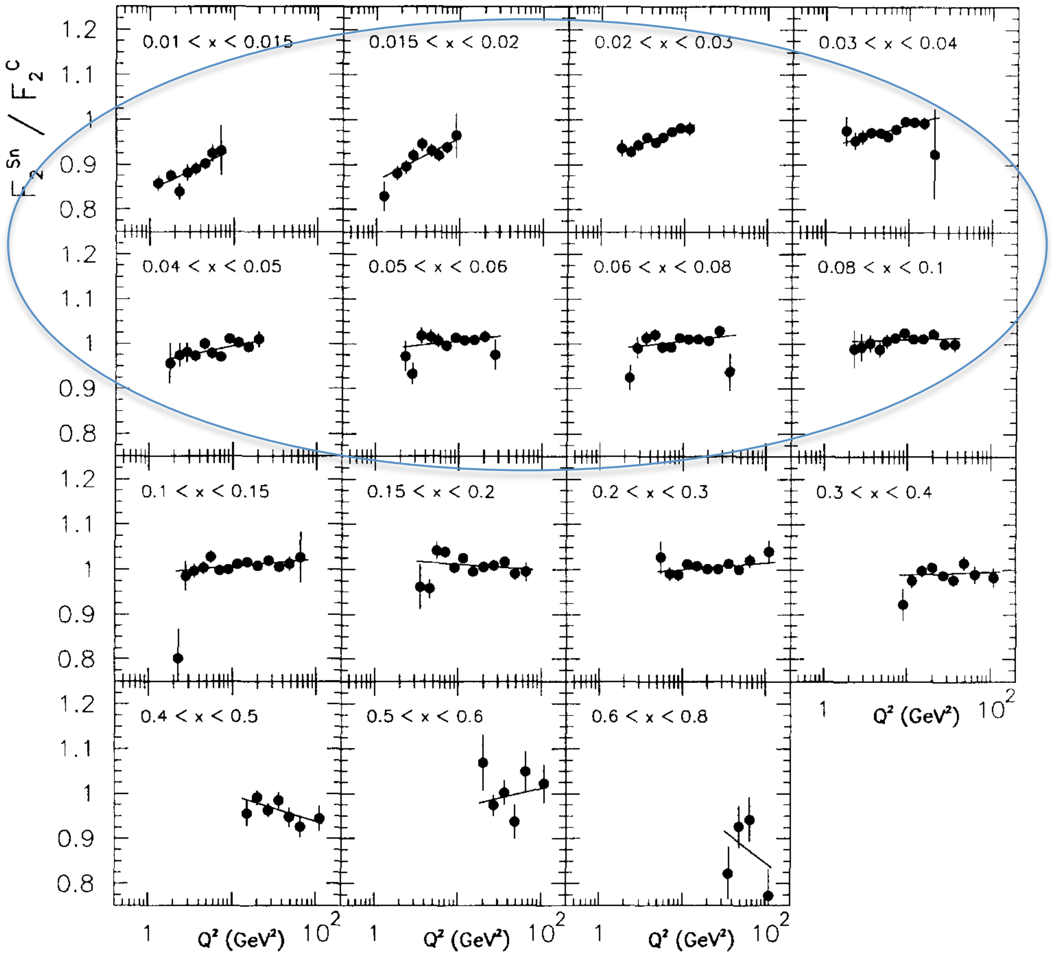
Aubert et al, Nucl. Phys. B293, 740 (1987)

SLAC E139 $Q^2=1\text{-}10$ GeV^2



Gomez et al, Phys. Rev. D 49, 4348 (1994)

(*) Q^2 Dependence of Sn/C



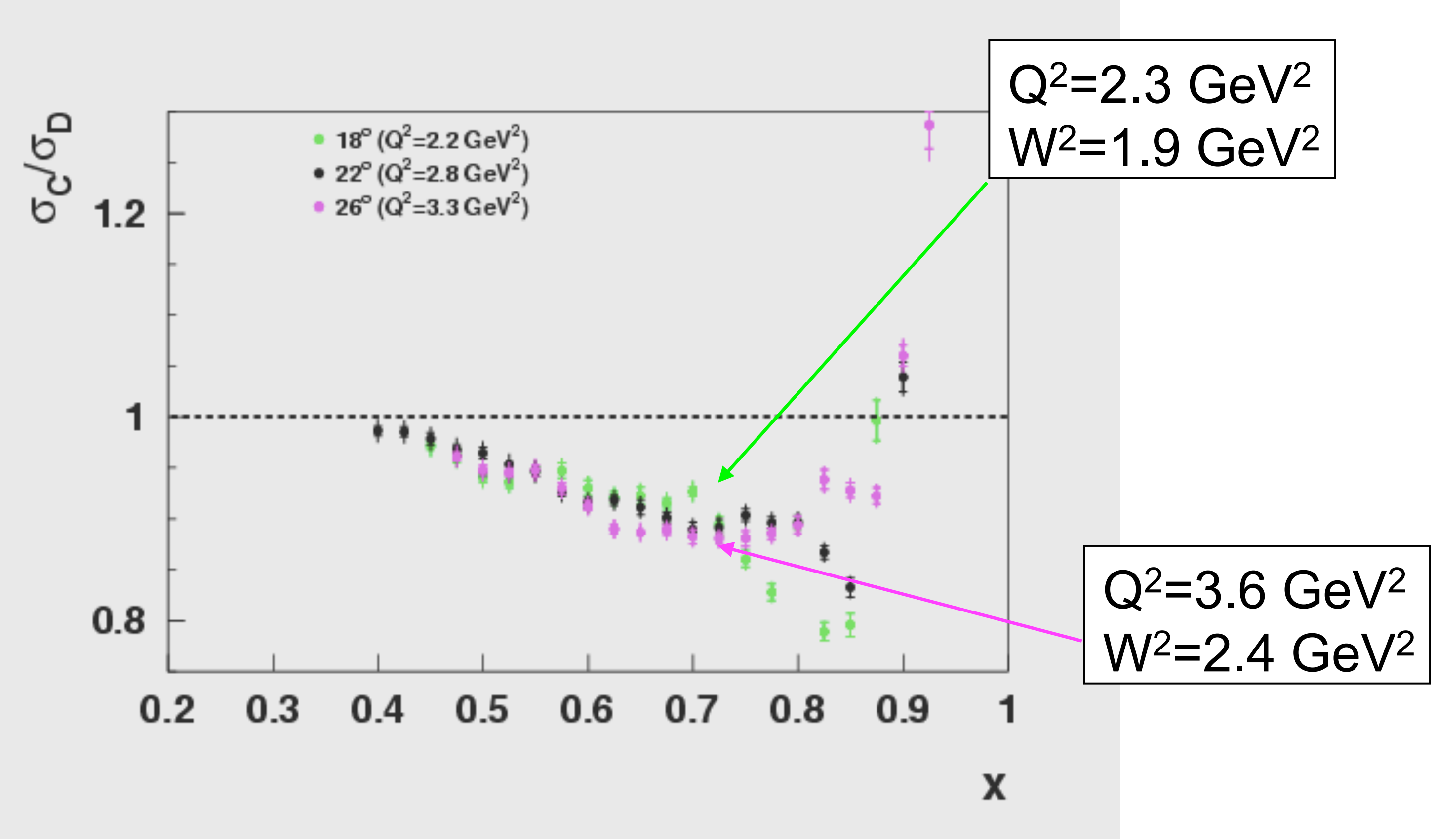
Arneodo et al, Nucl. Phys. B 481, 23 (1996)

NMC measured non-zero Q^2 dependence in Sn/C ratio at small x

→ This result is in some tension with other NMC C/D and HERMES Kr/D results

Q^2 Dependence at Large x

JLab Results from Hall C



Q^2=2.3 GeV^2
W^2=1.9 GeV^2

Q^2=3.6 GeV^2
W^2=2.4 GeV^2

Small angle, low Q^2 → clear scaling violations for $x>0.7$, but surprisingly good at lower x

Scaling at Large x

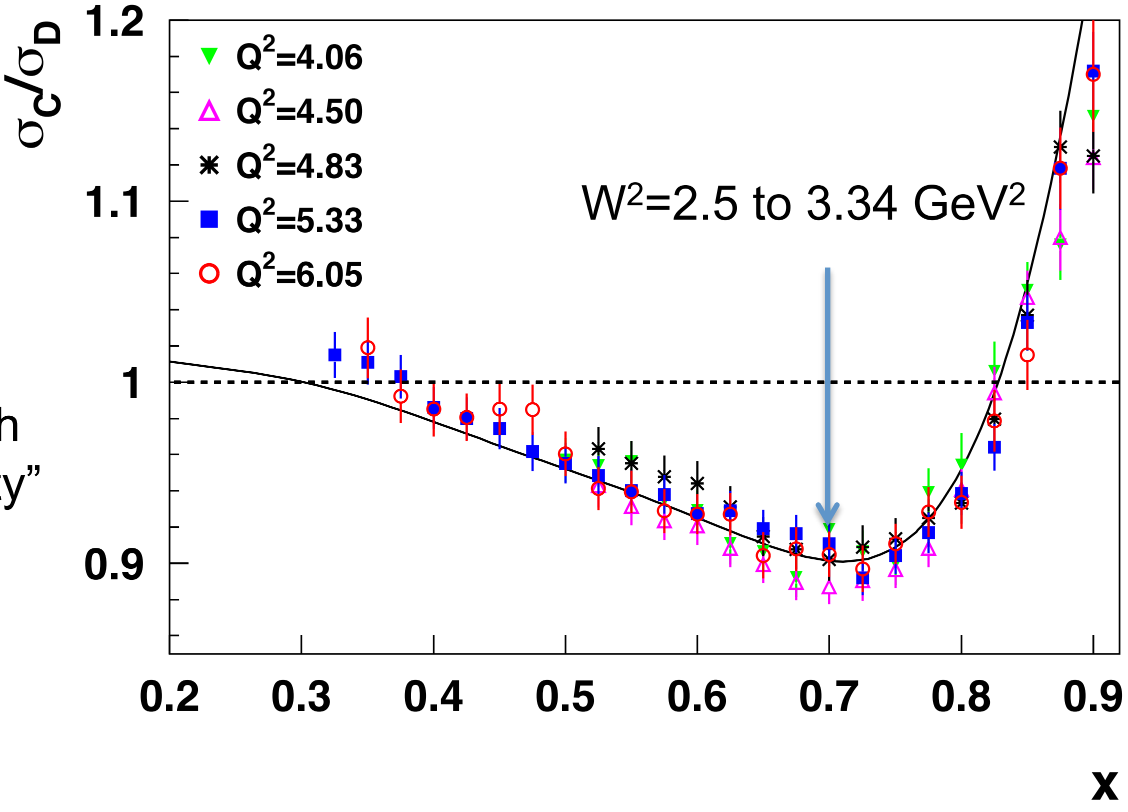
JLab Results from Hall C

JLab data from Hall C

→ C/D ratio constant even at large x for W<2 GeV

→ The nuclear wave function smears the cross section enough to mimic "local duality"

→ Need to avoid the Delta resonance



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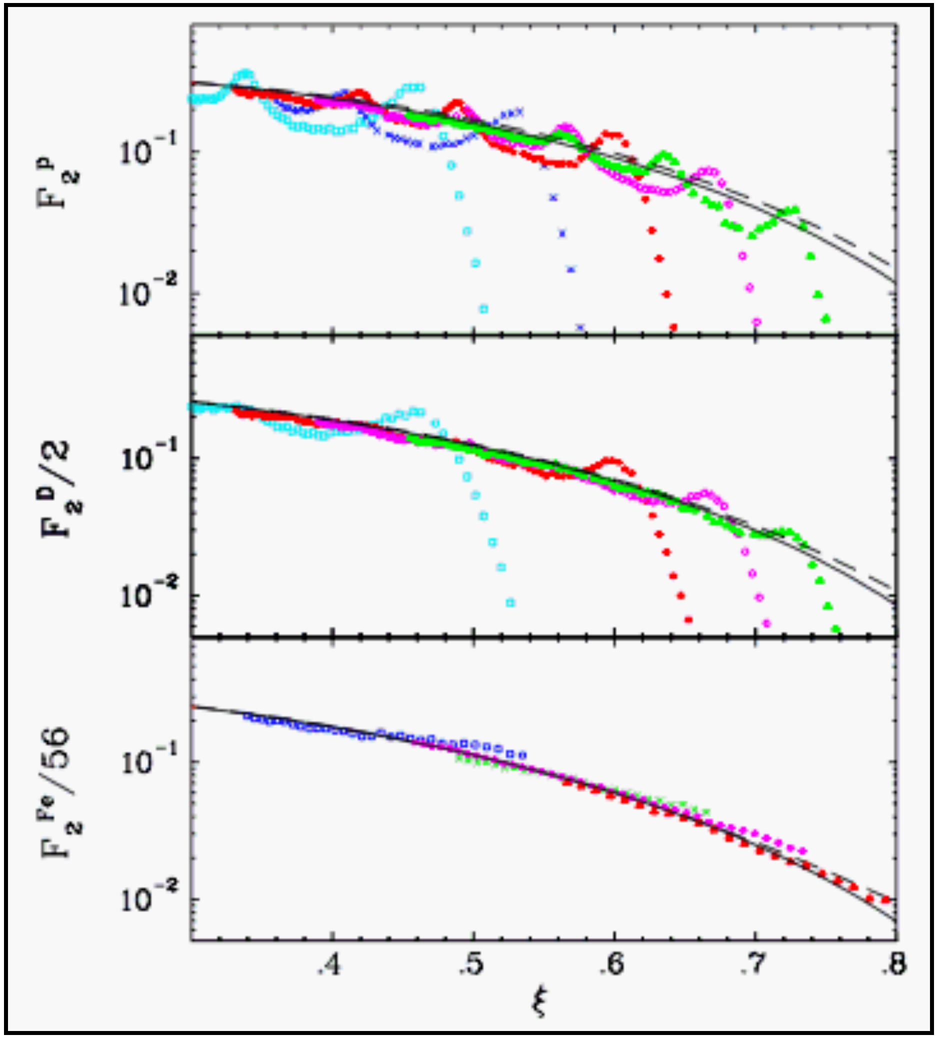
Quark-Hadron Duality in Nuclei

- Free nucleon
  - average over resonance region =DIS scaling limit
- Bound nucleon
  - Fermi motion does the averaging for us
  - Resonances much less prominent in nuclear structure functions
- Nuclear structure functions appear to "scale" to lower Q^2 than their free nucleon counterparts with no explicit resonance averaging

J. Arrington, et al., PRC73:035205 (2006)

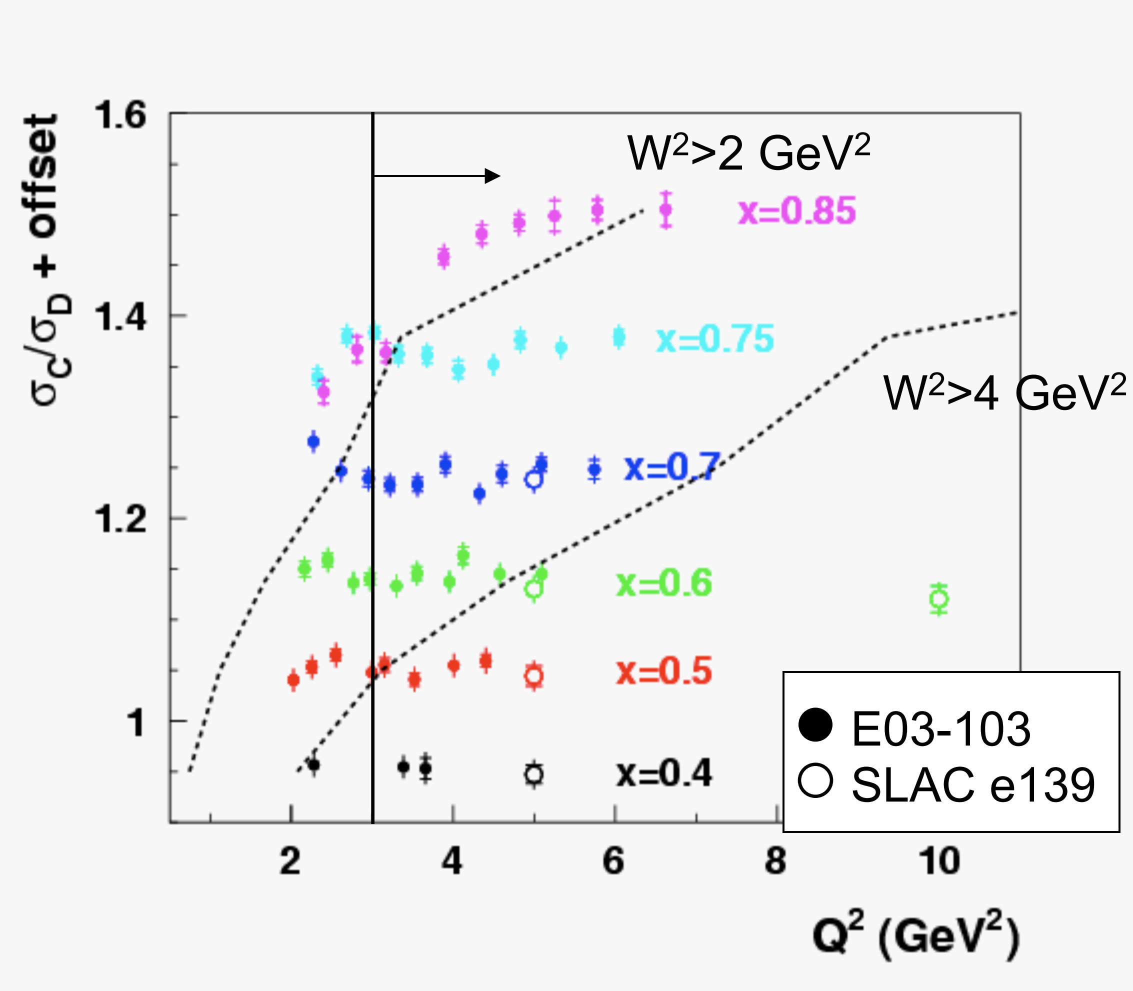


More detailed look at scaling

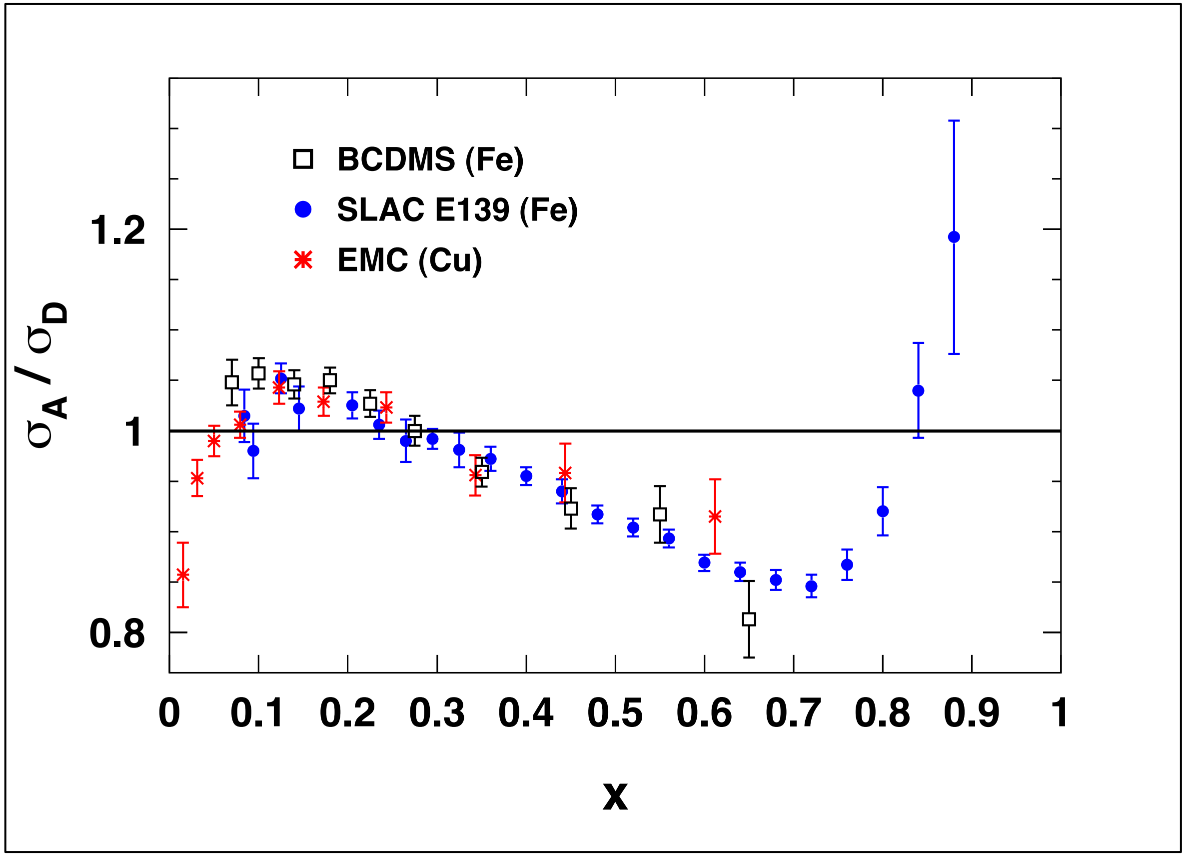
C/D ratios at fixed x are Q^2 independent for

W^2>2 GeV^2 and *Q^2*>3 GeV^2

JLab 6 GeV EMC data scale up to *x*=0.85



Properties of the EMC Effect

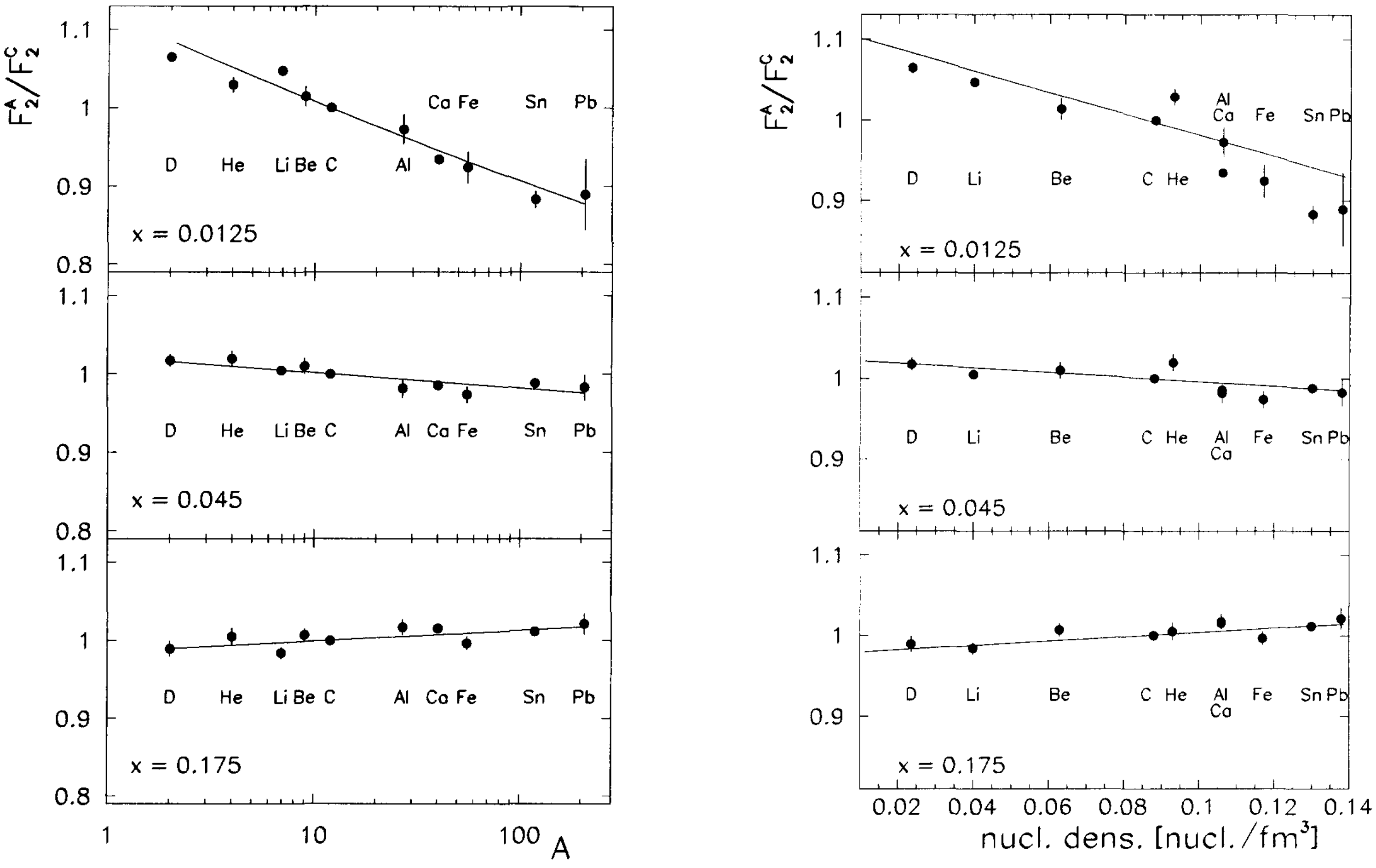


Global properties of the EMC effect

1. Universal x-dependence
2. Little Q^2 dependence
3. EMC effect increases with *A*

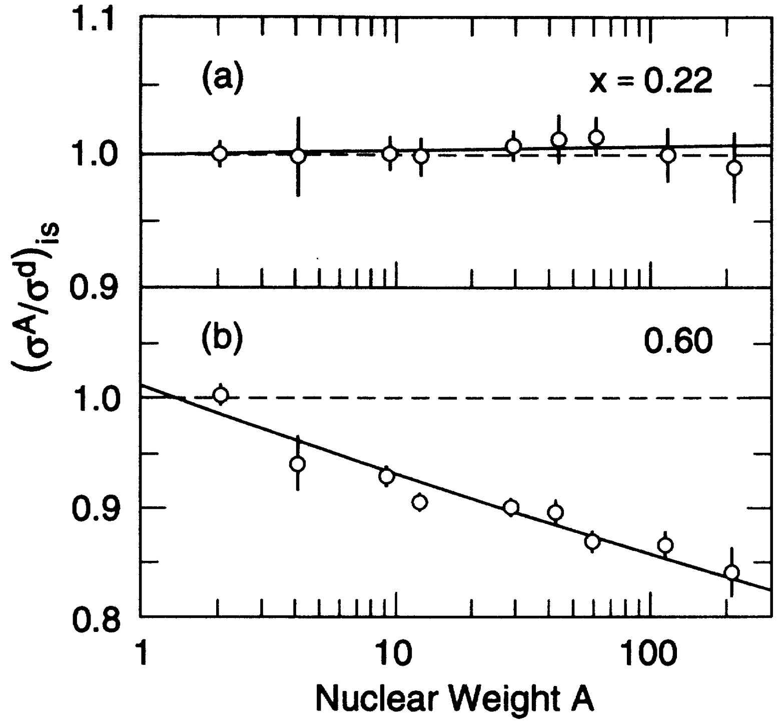
→ *Anti-shadowing region shows little nuclear dependence*

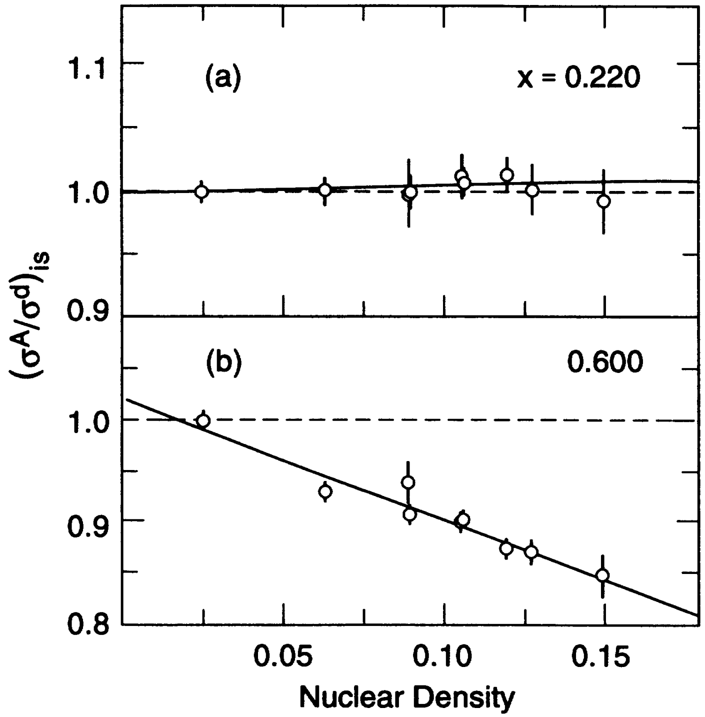
A-Dependence of EMC Effect



NMC: *Arneodo et al, Nucl. Phys. B 481, 3 (1996)*

A-Dependence of EMC Effect





$\rho=3A/4\pi R_e^3$ $R_e^2=5\langle r^2\rangle/3$

$<r^2>$=RMS electron scattering radius

SLAC E139: *Gomez et al, PRD 49, 4348 (1992)*

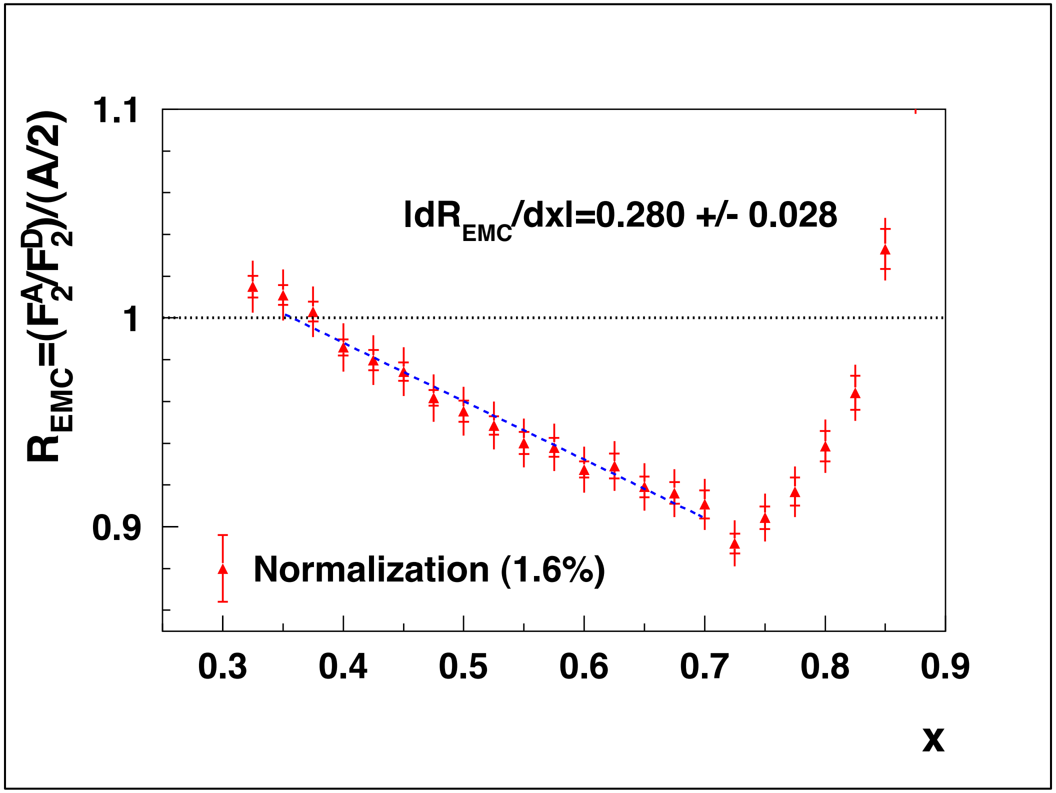
JLab E03103

E03103 in Hall C at Jefferson Lab ran Fall 2004

→ Measured EMC ratios for light nuclei (^{3}He, ^{4}He, Be, and C)

→ Results consistent with previous world data

→ Examined nuclear dependence a la E139



New definition of “size” of the EMC effect

→Slope of line fit from x=0.35 to 0.7

Definition assumes shape of the EMC effect is universal for nuclei

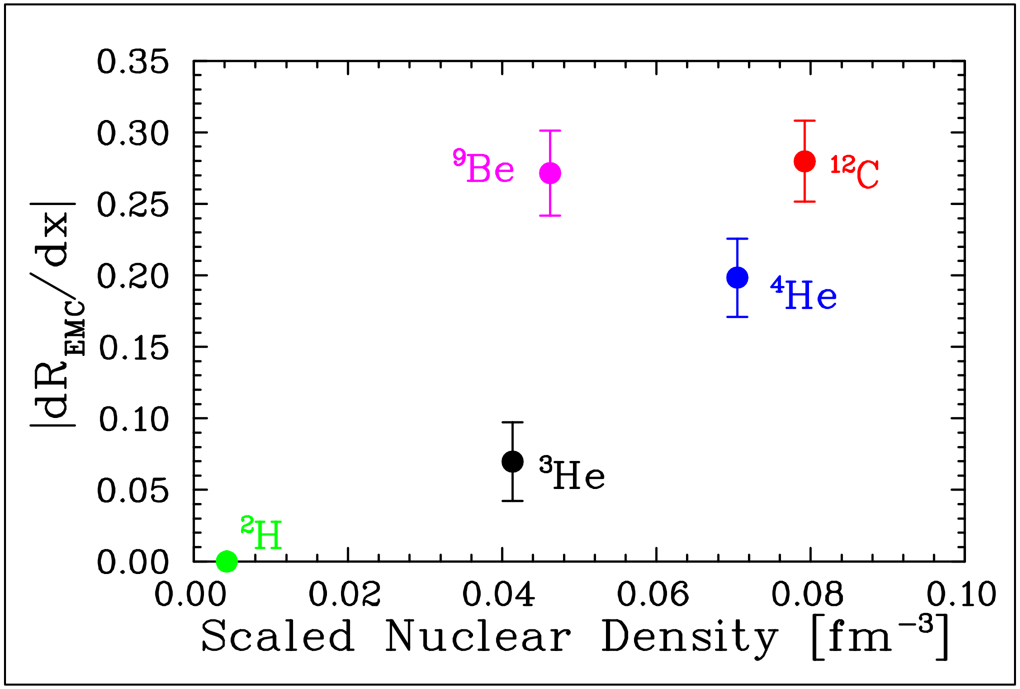
→Data *consistent* with this assumption

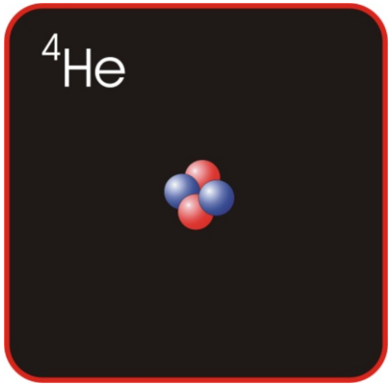
→ Normalization errors mean we can only confirm this at 1-1.5% level

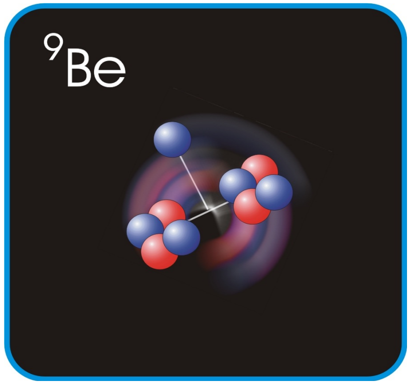
EMC Effect and Local Nuclear Density

^{9}Be has low average density
→ Large component of structure is ***2α+n***
→ Most nucleons in tight, *α*-like configurations

EMC effect driven by ***local*** rather than *average* nuclear density





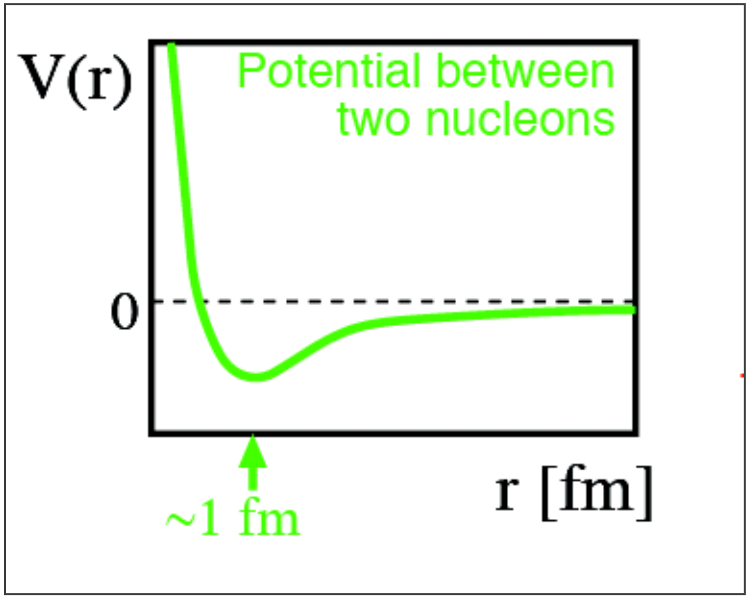


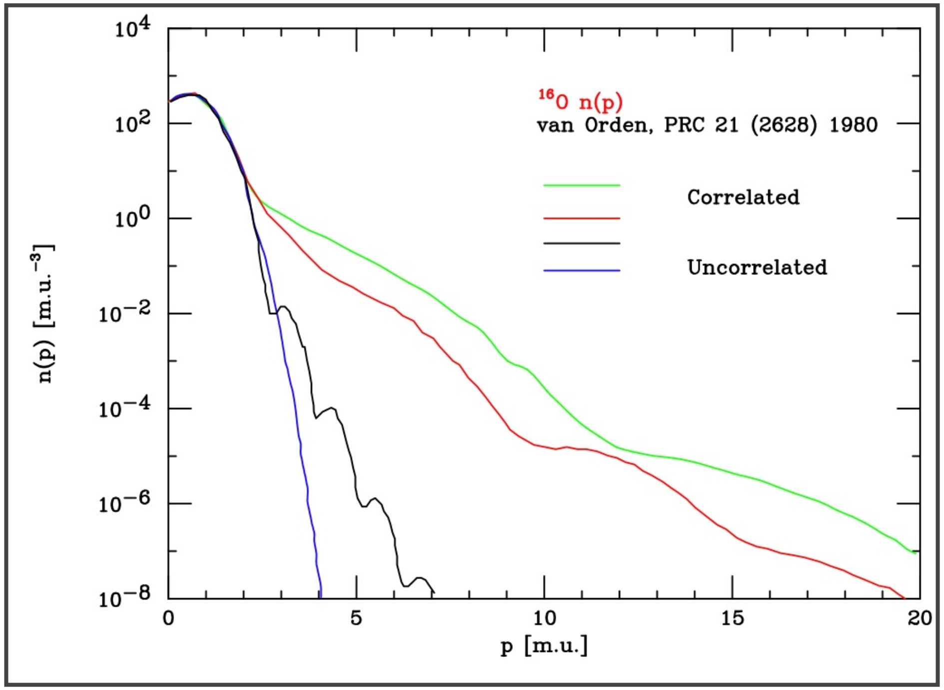
"Local density" is appealing in that it makes sense intuitively – can we make this more quantitative?

Local Density → Short Range Correlations

What drives high "local" density in the nucleus?

More complex calculations start from realistic NN potentials



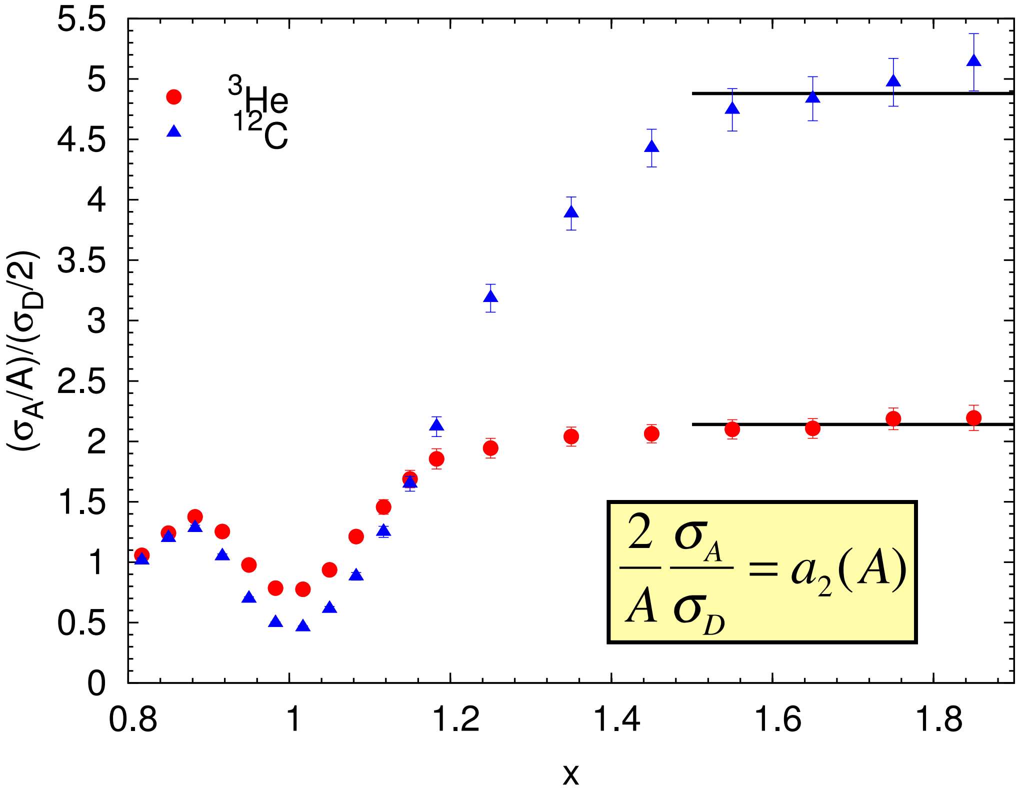


Tensor interaction and short range repulsive core lead to high momentum tail in nuclear wave function → correlated nucleons

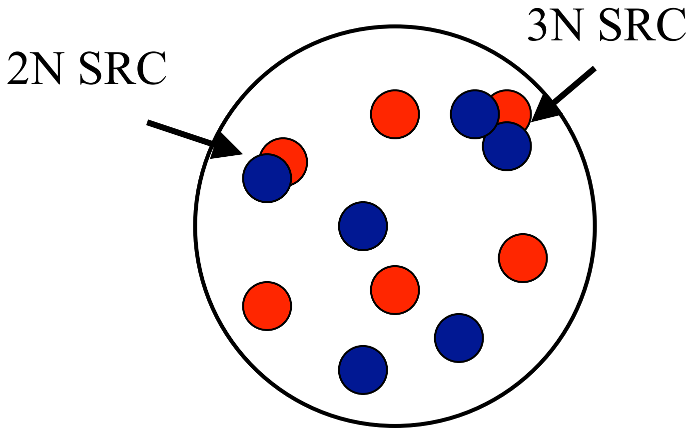
Measuring Short Range Correlations

To measure the (relative) probability of finding a correlated pair, ratios of heavy to light nuclei are taken at ***x>1*** → QE scattering

If high momentum nucleons in nuclei come from correlated pairs, ratio of A/D should show a plateau (assumes FSIs cancel, etc.)



$$\frac{2}{A}\frac{\sigma_A}{\sigma_D} = a_2(A)$$



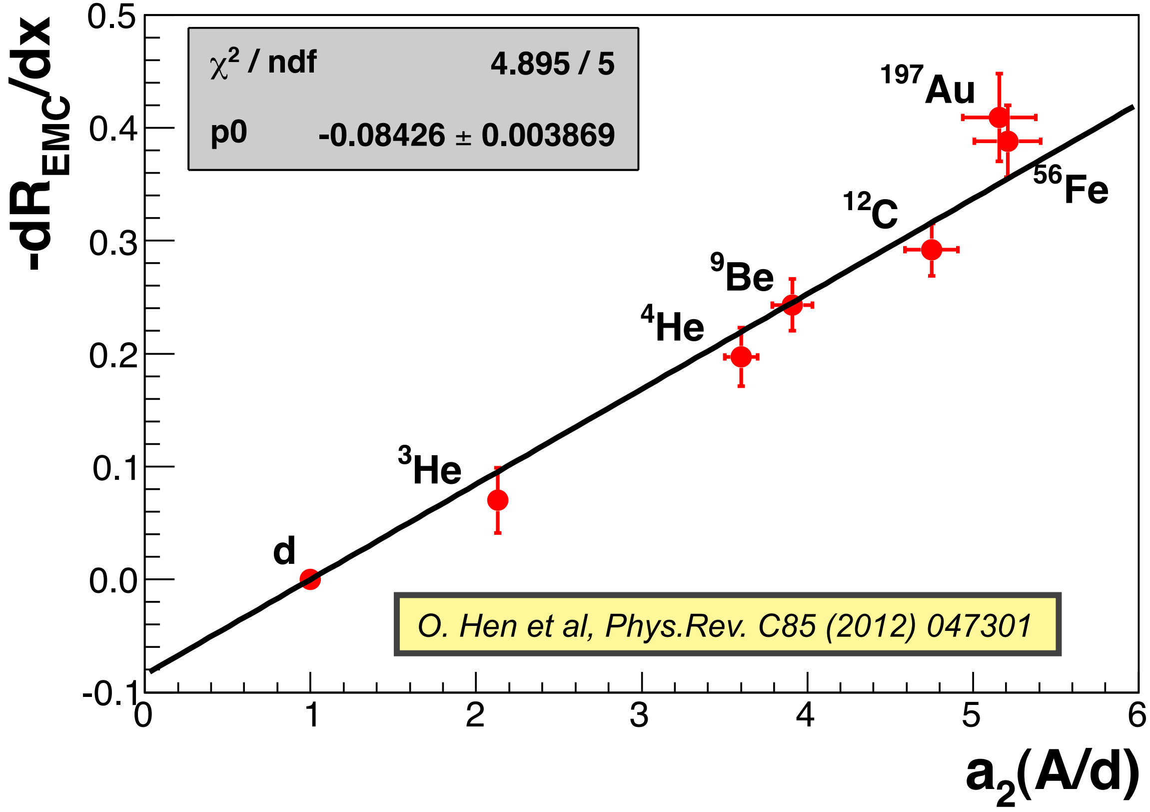
1.4<x<2 => 2 nucleon correlation

2.4<x<3 => 3 nucleon correlation

EMC Effect and SRC

Weinstein *et al* first observed linear correlation between size of EMC effect and Short Range Correlation "plateau"

Correlation strengthened with addition of Beryllium data



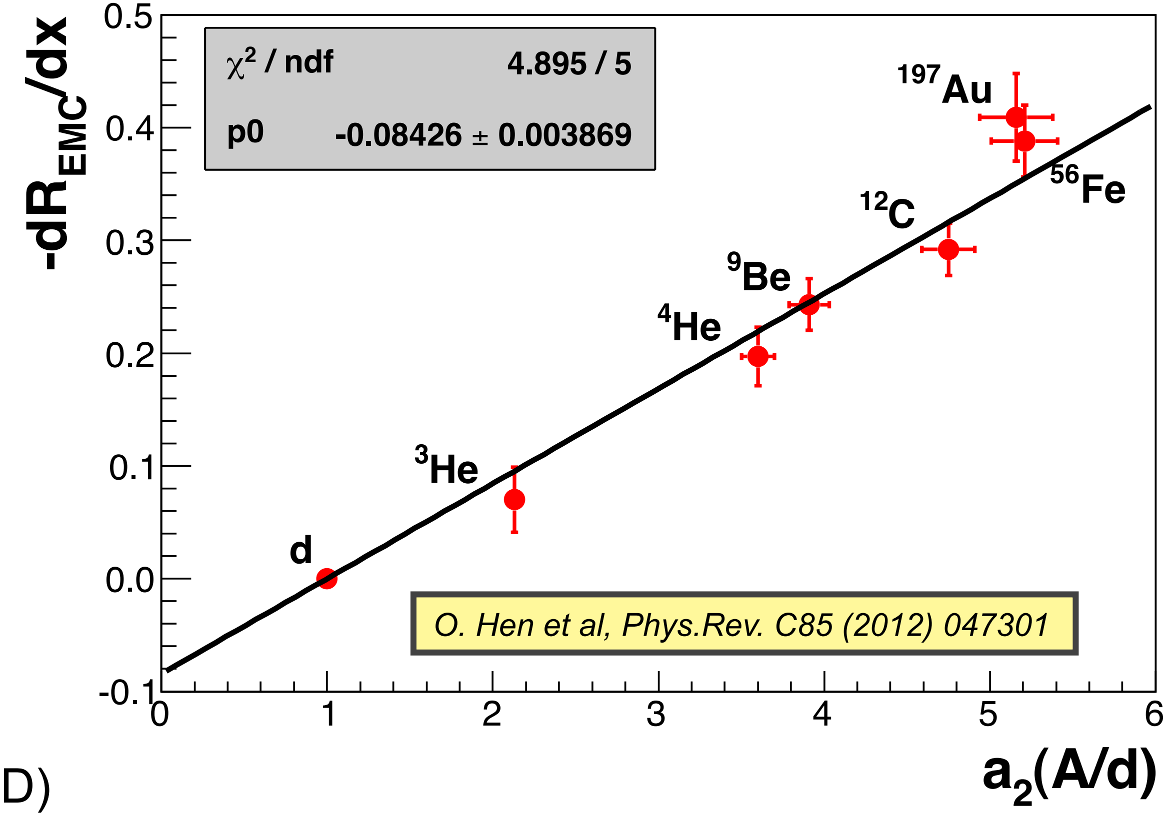
This result provides a ***quantitative*** test of level of correlation between the two effects

Origin of the EMC Effect

- Observation of correlation between size of EMC effect and SRCs interesting – but still does not explain the origin of the EMC effect
- Seems odd that an effect observed in QE scattering perhaps has common origin with modification of quark distributions
- Nonetheless, this correlation can be used to glean information about nucleon structure

EMC Effect in Deuteron from SRCs

EMC-SRC correlation used by Weinstein et al to extract the “in-medium correction” by extrapolating to $a_2=0$



IMC → $\frac{2}{A}\frac{\sigma^A}{\sigma_n+\sigma_p}$

IMC(D) = EMC(N) – EMC(D)

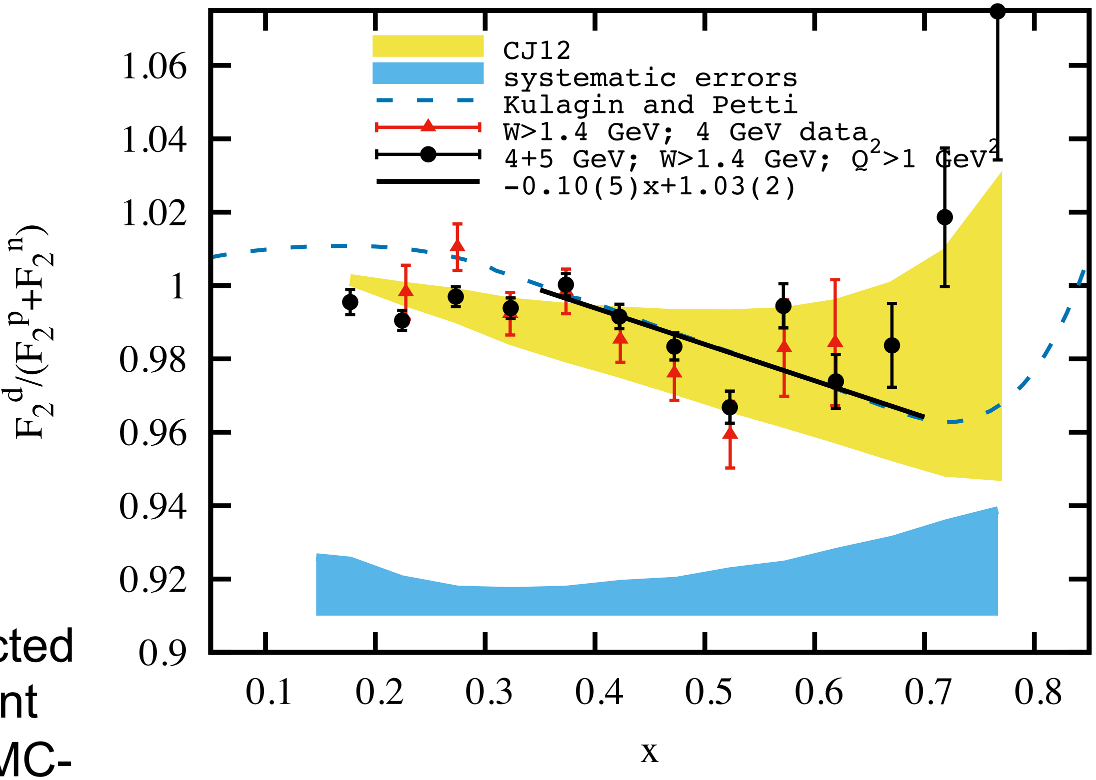
$$\left|\frac{dR_{IMC}}{dx}\right| = 0.079 \pm 0.006$$

EMC Effect in Deuterium

EMC effect in deuterium also recently extracted using data from BONUS experiment (see yesterday)

From BONUS: $F^2{}_n/F^2{}_p$
Use world data from $F^D{}_2$ and $F^2{}_p$

Size of EMC "slope" extracted from this analysis consistent with that extracted from EMC-SRC correlation



Griffioen et al, arXiv:1506.00871 [hep-ph]

Future of the EMC Effect

- A key question moving forward is exploration of the role of SRCs in the EMC effect
- New observables may be needed to gain further insight
  - Flavor dependence of the EMC effect (valence and sea quarks)
  - EMC effect in polarized quark distributions?
- Quark-meson inspired coupling models of the EMC effect seem to bridge the gap between quark and nucleon degrees of freedom
  - These models do not include or give rise to SRCs

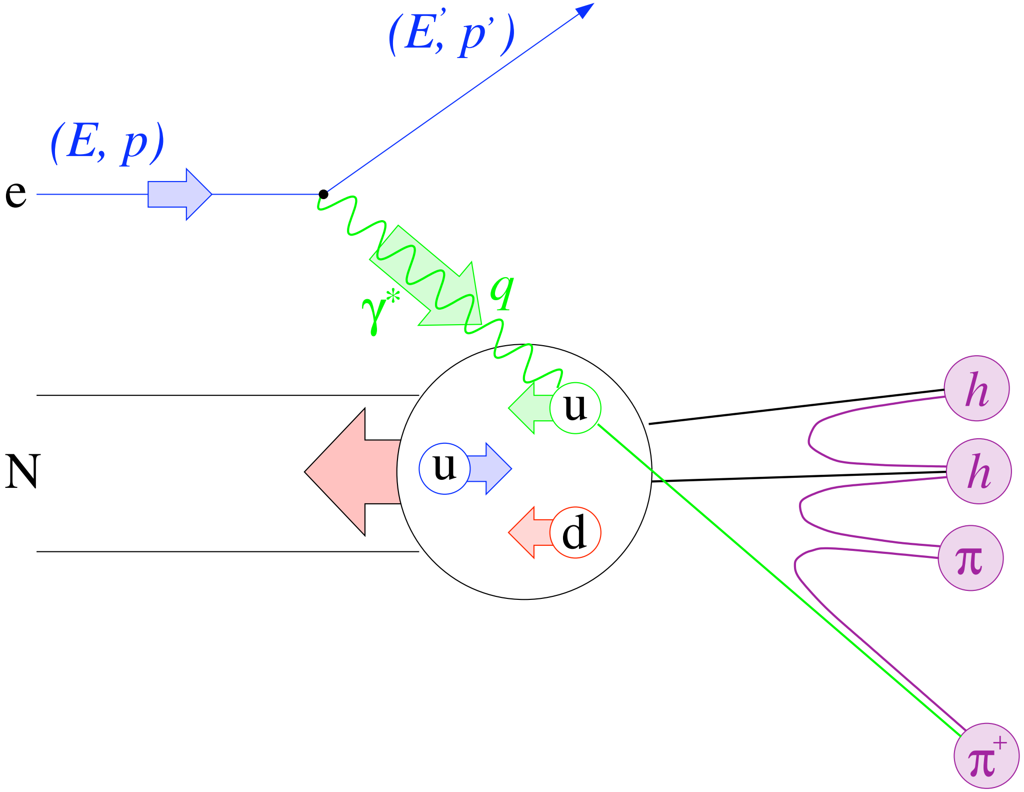
Semi-inclusive DIS

SIDIS → production of one or more hadrons in DIS reaction

Simple picture:

1. Electron scatters from quark in nucleon
2. Quark is kicked out → subsequently hadronizes, ending up in bound state

In this simple picture, SIDIS can be used to “tag” the flavor of the struck quark in DIS process



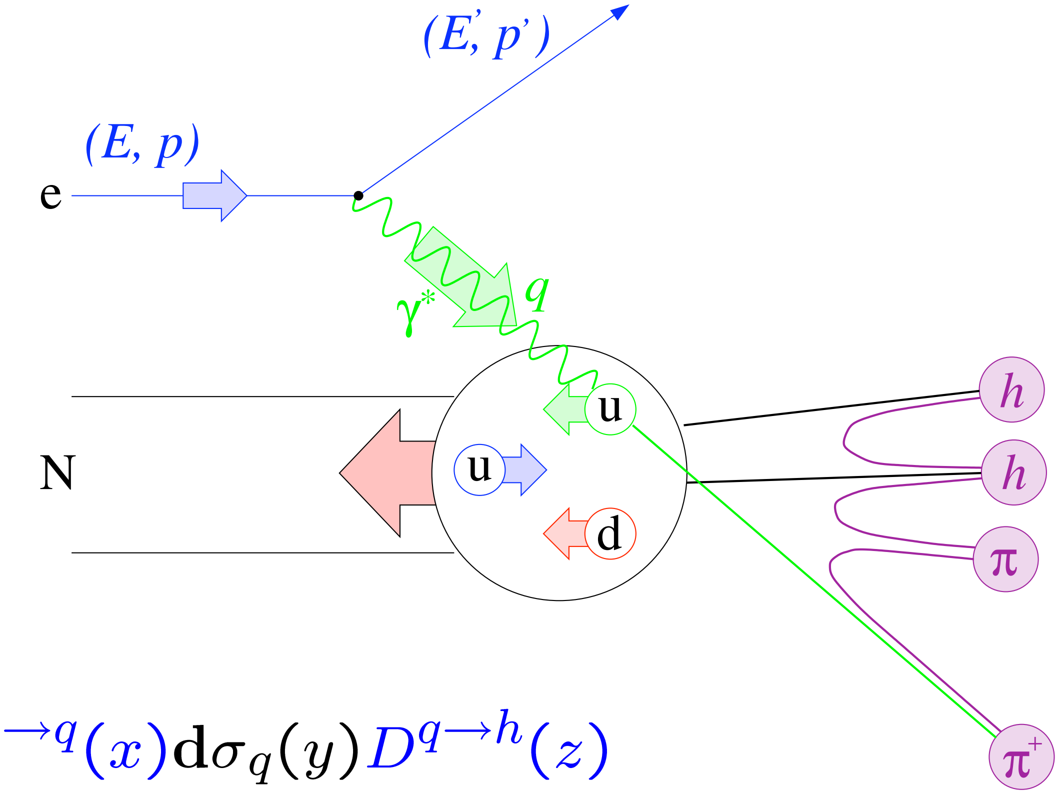
$$d\sigma^h \propto \sum f^{H\to q}(x)\,\mathrm{d}\sigma_q(y)\,D^{q\to h}(z)$$

Fragmentation function

Semi-inclusive DIS

In principle SIDIS can also be used to gain information about spatial distributions of quarks in nucleons

→ This requires measurements/ observation of transverse degrees of freedom



$$d\sigma^h \propto \sum f^{H\to q}(x)\mathrm{d}\sigma_q(y)D^{q\to h}(z)$$

$$\downarrow$$

$$d\sigma^h \propto \sum f^{H\to q}(x,k_T)\otimes \mathrm{d}\sigma_q(y)\otimes D^{q\to h}(z,p_\perp)$$

SIDIS Kinematics

Useful kinematics related to outgoing hadron:

$$z = \frac{q \cdot p}{q \cdot P} = \frac{E_h}{\nu}$$

Fraction of virtual photon energy transferred to hadron

$$p_{\|} = \frac{p \cdot q}{|q|} \qquad p_T = (p^2 - p_{\|}^2)^{\frac{1}{2}}$$

Components of hadron momentum relative to q

$$\cos\phi = \frac{(-\vec{q} \times \vec{k}) \cdot (-\vec{q} \times \vec{p_h})}{|\vec{q} \times \vec{k}||\vec{q} \times \vec{p_h}|}$$

Azimuthal angle between electron scattering plane and hadron reaction plane

SIDIS Cross Section

General form of unpolarized cross section:

$$\frac{d\sigma}{dxdydzdp_T^2d\phi} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}\left[F_T + \epsilon F_L + \sqrt{2\epsilon(1+\epsilon)}\cos\phi F_{LT} + \epsilon\cos 2\phi F_{TT}\right]$$

Integrate over p_T and ϕ, cross section can be expressed in terms of quark-parton model:

$$\frac{d\sigma}{dxdydz} = \sigma_{DIS}\frac{d\sigma}{dz}$$

where

$$\frac{1}{\sigma_{DIS}}\frac{d\sigma}{dz} = \frac{\sum_f e_q^2 q_f(x,Q^2)D_f^h(z)}{\sum_f e_f^2 q_f(x,Q^2)}$$

Fragmentation Functions

$D_q^h(z,Q^2)$ → probability to form hadron (h) after quark of flavor (q) is struck

In Quark-Parton Model, fragmentation functions Q^2 independent
→ In reality, evolve with Q^2 like quark PDFs

Fragmentation functions can be measured in e+e- reactions → no complication due to hadron structure
→ Only average FF accessible – need other information for flavor dependence

For pion production, charge and isospin symmetry reduce number of FF's needed

$$D^+ = D_u^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+}$$

$$D^- = D_u^{\pi^-} = D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_{\bar{d}}^{\pi^-}$$

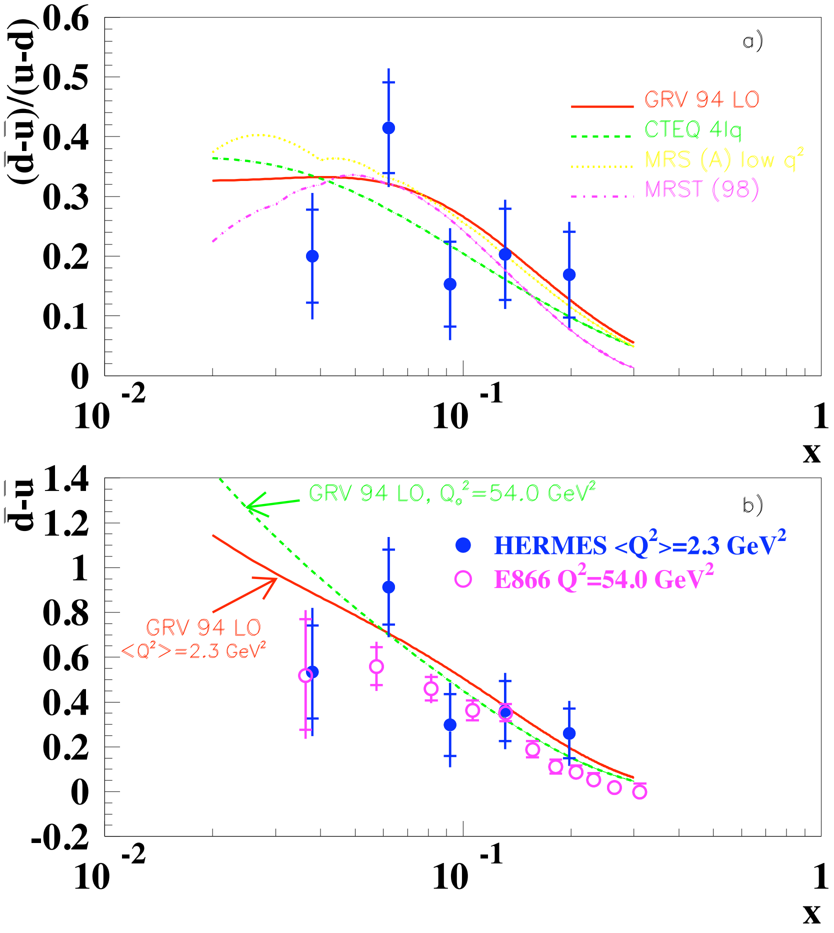
SIDIS Examples

Light quark sea flavor asymmetry can be extracted using semi-inclusive pion production

→ Assumes leading order factorization

→ Do not need knowledge of absolute fragmentation functions, but do need FF ratio: D^-/D^+

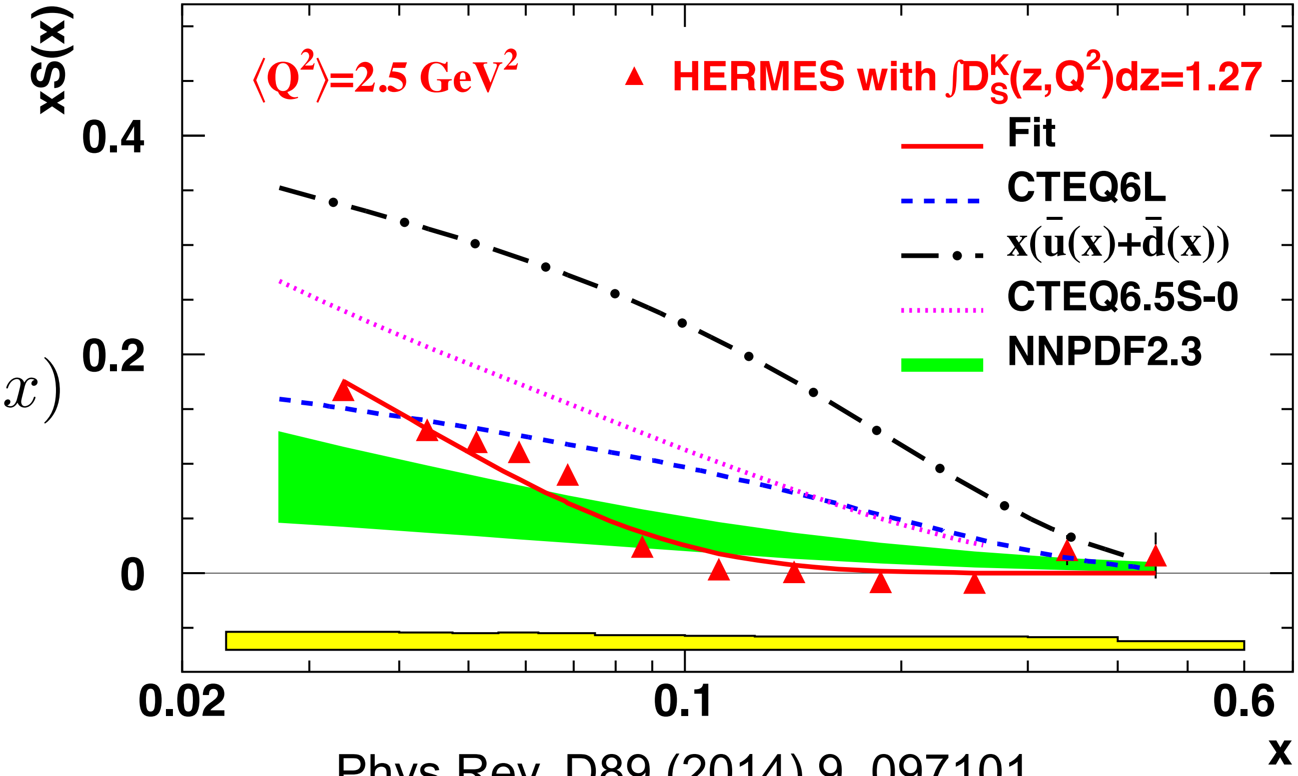
D+ = favored fragmentation function (u→ π+)
D- = unfavored fragmentation function (u→ π-)



SIDIS Examples

Strange quark distributions from (K⁺+K⁻) production

$$S(x) = s(x) + \bar{s}(x)$$



Phys.Rev. D89 (2014) 9, 097101

$$S(x)\int D_S^K(z,Q^2)dz \approx Q(x,Q^2)\left[5\frac{dN^K(x,Q^2)}{dN^{DIS}(x,Q^2}-\int D_Q^K(z,Q^2)dz\right]$$

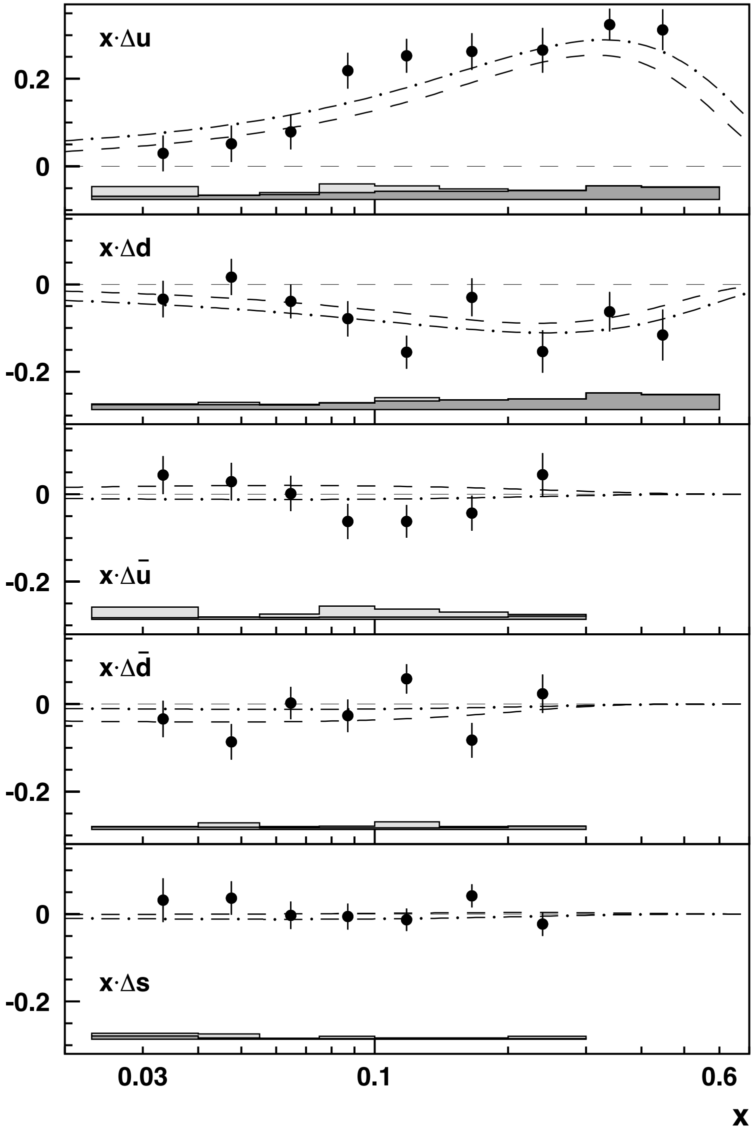
SIDIS Examples

Polarized quark distributions

$$\frac{d\sigma^h_{\frac{1}{2}(\frac{3}{2})}}{dx\,dQ^2\,dz} \propto \sum_q e_q^2 q^{+(-)} D_q^h(z,Q^2)$$

$$\Delta q(x) = q^+(x) - q^-(x)$$

HERMES used a “purity” analysis so they wouldn’t have to measure the absolute magnitude of fragmentation function



HERMES, Phys.Rev. D71 (2005) 012003

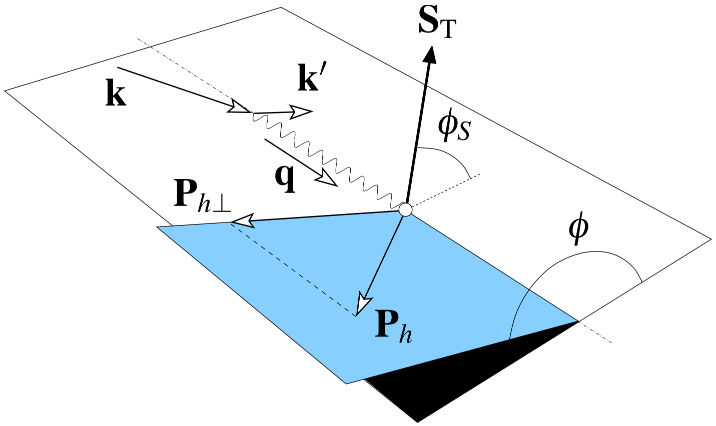
SIDIS with Transverse Degrees of Freedom

In the more general case, allowing for target and beam polarizations, the cross section is a bit more complicated

→ Measurement of the transverse momentum of the hadron also allows for access to information regarding the initial transverse momentum of the quark

→ Transverse momentum dependent distributions – TMDs

→ Azimuthal asymmetries key to accessing TMDs



Example: Transversely polarized target, unpolarized beam

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dp_{h\perp}^2} = \sigma_{unpol} + \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\epsilon)}|\mathbf{S}_\perp|[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\epsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)} + \epsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)}\sqrt{2\epsilon(1+\epsilon)}\sin\phi_S F_{UT}^{\sin\phi_S} +$$

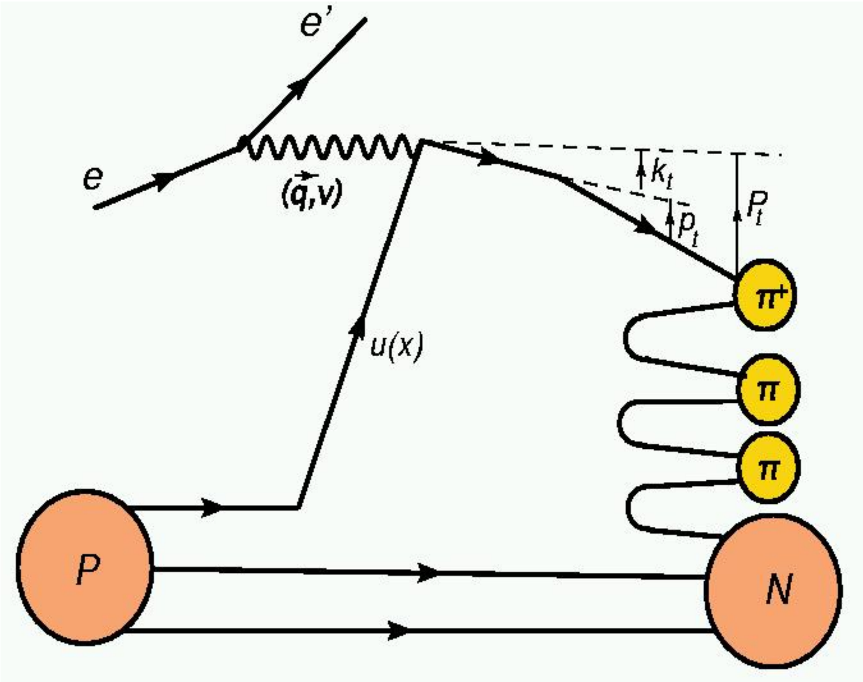
$$\sqrt{2\epsilon(1+\epsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}]$$

TMDs

In the inclusive/forward hadron production case, we sample one-dimensional parton distribution functions

TMDs allows us to explore the distributions of partons in the transverse direction

$$q(x) \rightarrow q(x, k_T)$$



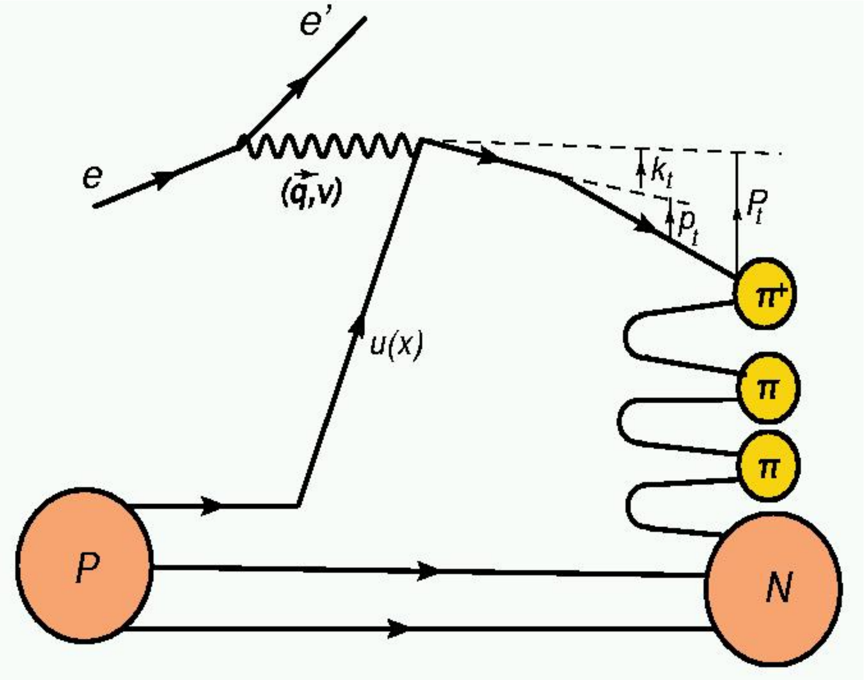
Transverse momentum also generated during fragmentation

$$D(z) \rightarrow D(z, p_T)$$

TMDs

$$q(x) \to q(x, k_T) \qquad D(z) \to D(z, p_T)$$

Probability of producing hadron with transverse momentum P_T comes from a convolution of k_T dependent parton distribution and p_T dependent fragmentation function



$$P_T = p_T + zk_T + O(k_T^2/Q^2)$$

SIDIS and TMDs

D_1 Unpolarized fragmentation function

$H_1^{\perp}$ Collins fragmentation function

Beam polarization

Target polarization

$F_{UU} \rightarrow f_1 D_1$	Unpolarized TMD
$F_{UU}^{\cos(2\phi_h)} \rightarrow h_1^{\perp} H_1^{\perp}$	Boer-Mulders TMD
$F_{UL}^{\sin(2\phi_h)} \rightarrow h_{1L}^{\perp} H_1^{\perp}$	Worm gear TMD
$F_{LL} \rightarrow g_{1L} D_1$	HelicityTMD
$F_{UT,T}^{\sin(\phi_h-\phi_S)} \rightarrow f_{1T}^{\perp} D_1$	Sivers TMD
$F_{UT}^{\sin(\phi_h+\phi_S)} \rightarrow h_1 H_1^{\perp}$	Transversity TMD
$F_{UT}^{\sin(3\phi_h-\phi_S)} \rightarrow h_{1T}^{\perp} H_1^{\perp}$	Pretzelosity TMD
$F_{LT}^{\cos(\phi_h-\phi_S)} \rightarrow g_{1T} D_1$	Worm gear TMD

Distribution Functions

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1 h_{1T}^\perp$

quarks

nucleon

U=unpolarized
L=long. polarized
T=trans. polarized

Diagonal elements = usual PDFs

Off-diagonal elements = transverse momentum distributions, require non-zero angular momentum

$f_{1T}^\perp$ → Sivers function, describes unpolarized quark in trans. pol. nucleon

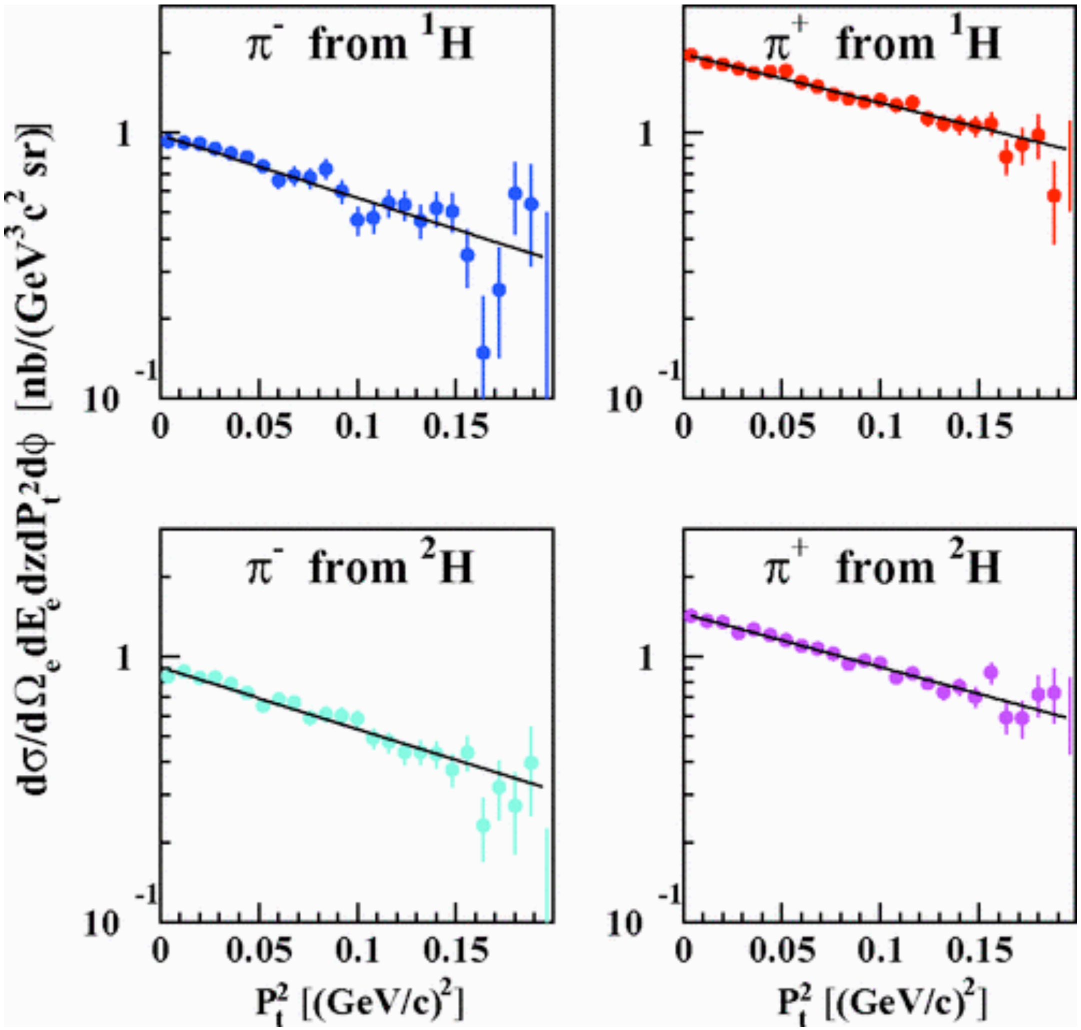
$h_1^\perp, h_{1L}^\perp, h_{1T}^\perp$ → Boer-Mulders functions describe transversely polarized quarks in un/long./trans./polarized nucleon

Unpolarized SIDIS

Hall C @ JLAB: E00-108

Measured P_T dependence of unpolarized SIDIS cross sections for:

π$^+$ and π$^-$ from H and D



Model P_T dependence of SIDIS

Gaussian distributions for P_T dependence, no sea quarks, and leading order in (k_T/q)

$$\sigma_p^{\pi+} = C\left[4c_1(P_t)e^{-b_u^+ P_t^2} + (d/u)\left(D^-/D^+\right)c_2(P_t)e^{-b_d^- P_t^2}\right],$$

$$\sigma_p^{\pi-} = C\left[4\left(D^-/D^+\right)c_3(P_t)e^{-b_u^- P_t^2} + (d/u)c_4(P_t)e^{-b_d^+ P_t^2}\right],$$

$$\sigma_n^{\pi+} = C\left[4(d/u)c_4(P_t)e^{-b_d^+ P_t^2} + \left(D^-/D^+\right)c_3(P_t)e^{-b_u^- P_t^2}\right],$$

$$\sigma_n^{\pi-} = C\left[4(d/u)\left(D^-/D^+\right)c_2(P_t)e^{-b_d^- P_t^2} + c_1(P_t)e^{-b_u^+ P_t^2}\right],$$

Inverse of total width for each combination of quark flavor and fragmentation function given by:

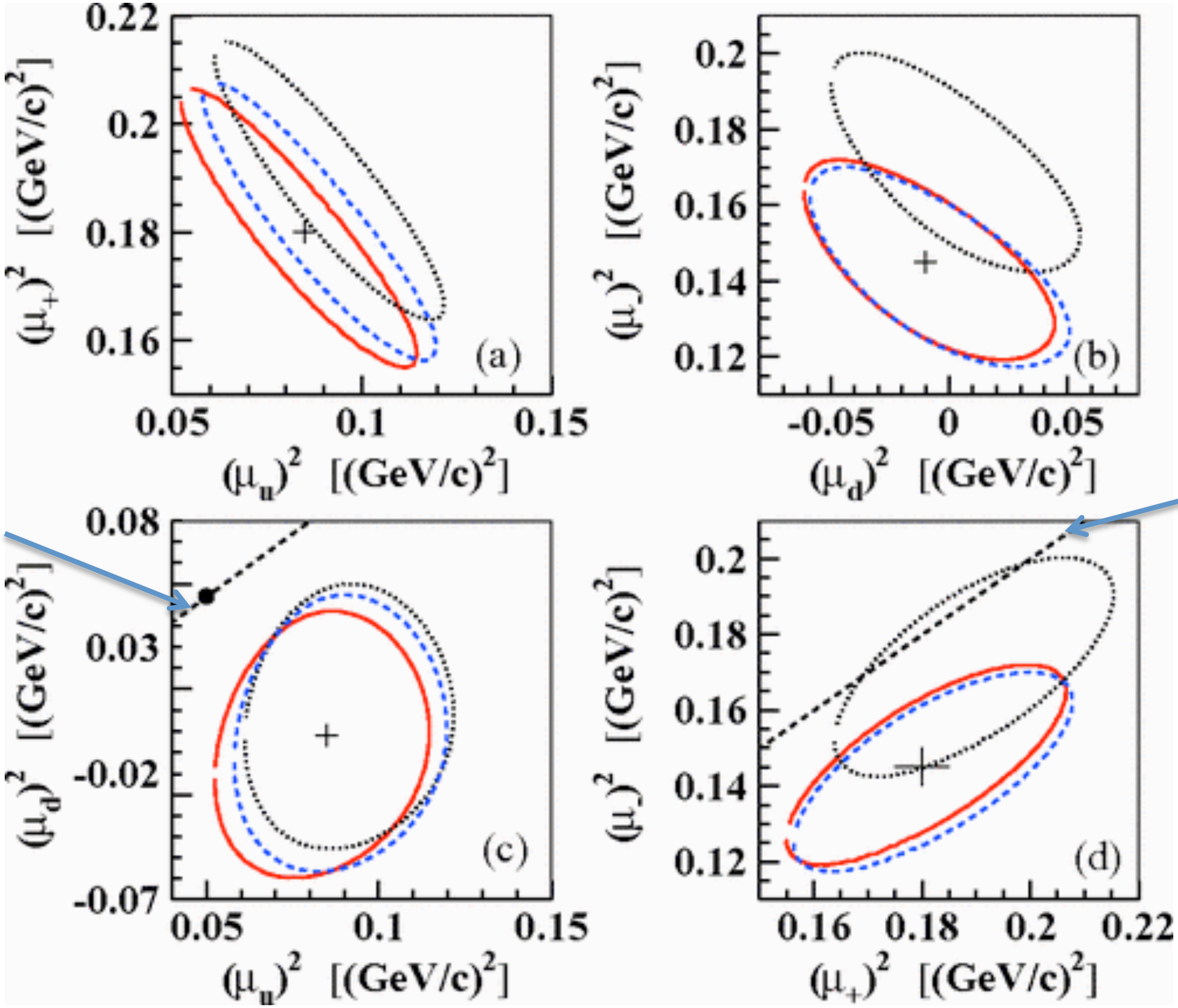
$$\boxed{b_u^\pm = \left(z^2\mu_u^2 + \mu_\pm^2\right)^{-1}}$$

Simple model, with several assumptions:
→ factorization valid
→ fragmentation functions do not depend on quark flavor
→ transverse momentum widths of quark and fragmentation functions are Gaussian and can be added in quadrature
→ more …

Unpolarized SIDIS

$\mu^2_u = 0.07 \pm 0.03\ (\text{GeV}/c)^2$

$\mu^2_d = -0.01 \pm 0.05\ (\text{GeV}/c)^2$



$\mu_u=\mu_d$

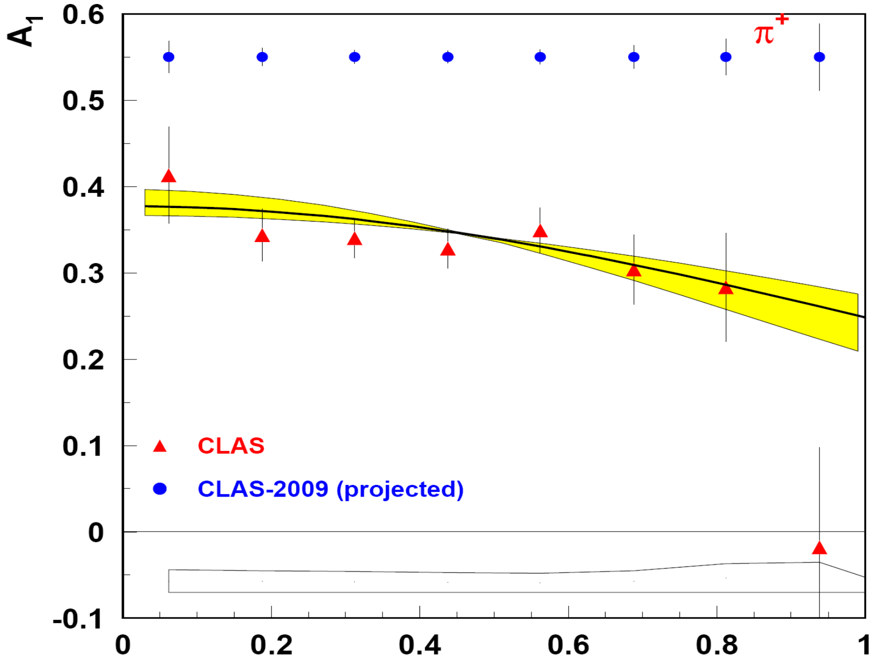
$\mu_+=\mu_-$

A_1 P_T-Dependence in SIDIS

$$A_1 \propto \frac{\sum_q e_q^2 g_1^q(x) D_1^{q\to\pi}(z)}{\sum_q e_q^2 f_1^q(x) D_1^{q\to\pi}(z)} \; e^{-z^2 P_T^2 \frac{(\mu_0^2-\mu_2^2)}{(\mu_D^2+z^2\mu_0^2)(\mu_D^2+z^2\mu_2^2)}}$$

m_0^2=0.25GeV^2
m_D^2=0.2GeV^2

M.Anselmino et al
hep-ph/0608048



$$f_1^q(x,k_T) = f_1(x)\frac{1}{\pi\mu_0^2}\exp\left(-\frac{k_T^2}{\mu_0^2}\right)$$

$$g_1^q(x,k_T) = g_1(x)\frac{1}{\pi\mu_2^2}\exp\left(-\frac{k_T^2}{\mu_2^2}\right)$$

$$D_1^q(z,p_T) = D_1(z)\frac{1}{\pi\mu_D^2}\exp\left(-\frac{p_T^2}{\mu_D^2}\right)$$

New eg1dvcs data allow multidimensional binning to study k_T-dependence for fixed x

In perturbative limit predicted to be constant

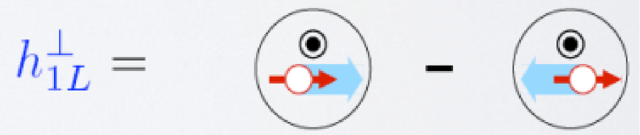
π+ A_{LL} can be explained in terms of broader k_T distributions for f_1 compared to g_1

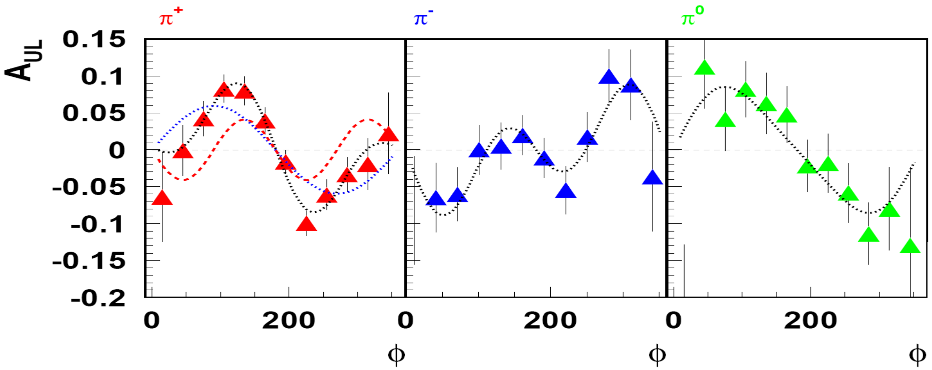
Proton Single-Spin Asymmetries with CLAS

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

$$\rightarrow A_{UL}^{\sin 2\phi} \sim h_{1L}^{\perp} H_1^{\perp}$$

Transversely polarized quarks in the longitudinally polarized nucleon





W²>4 GeV²

Q²>1.1 GeV²

y<0.85

0.4<z<0.7

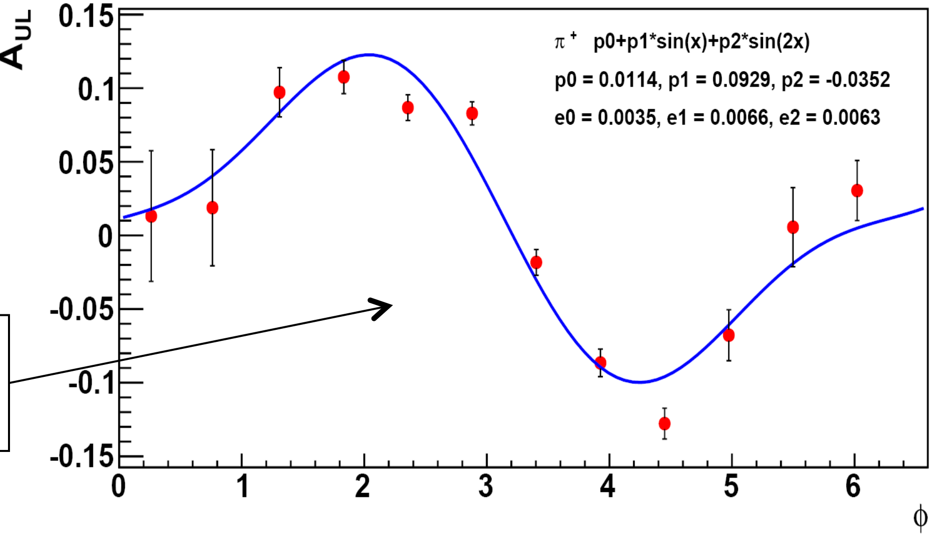
M_X>1.4 GeV

P_T<1 GeV

0.12<x<0.48

ep→e'pX

~10% of
E05-113 data



Much higher statistics from 2009 run

Transverse Target Asymmetries

Collins asymmetry $\sin\left(\phi_h + \phi_S\right) \rightarrow h_1 H_1^{\perp}$

Provides access to “transversity” distribution → linked to tensor charge of the proton

$$\delta q = \int_0^1 h_1^q(x)$$

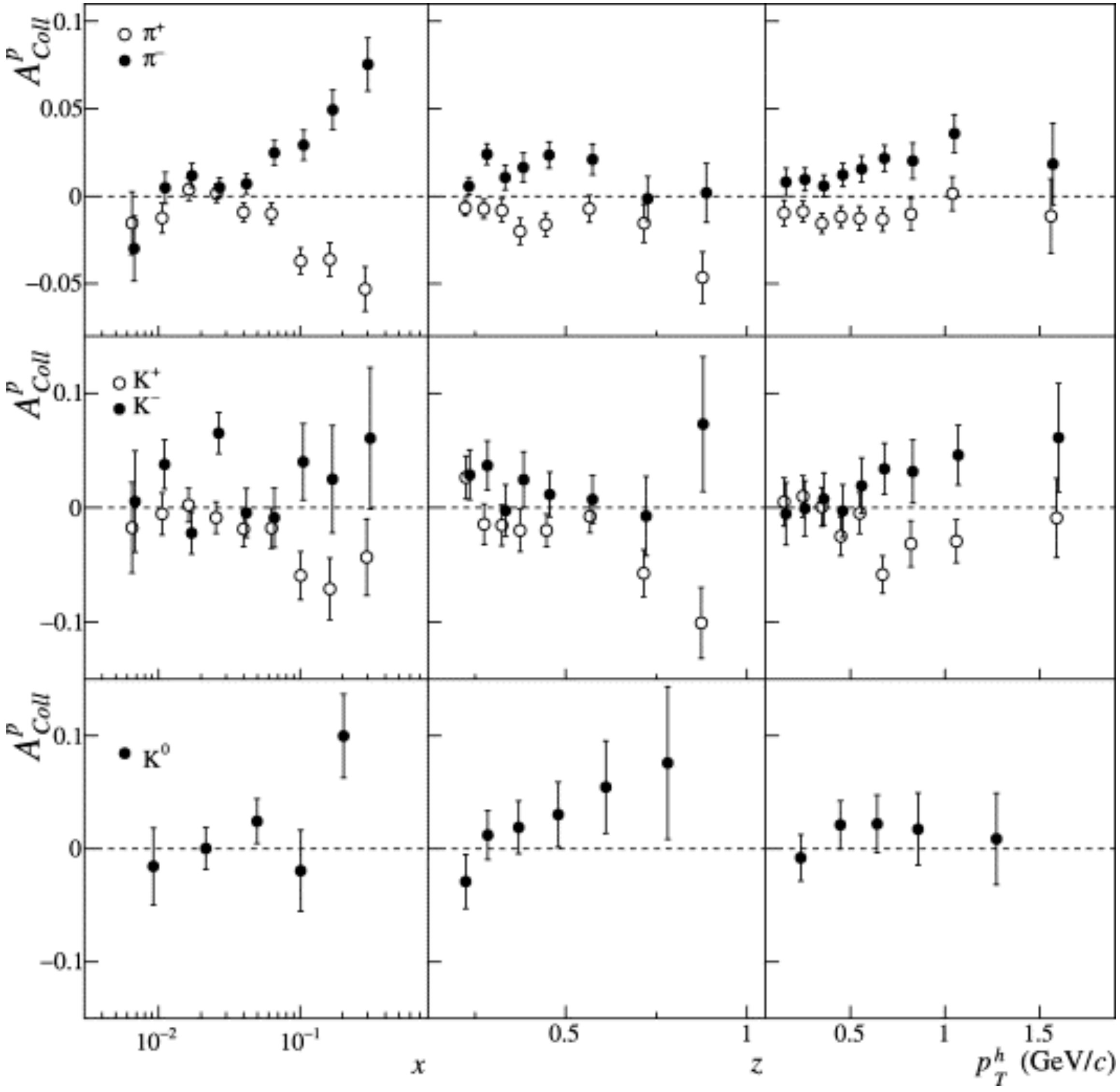
Fundamental property of nucleon,

Sivers asymmetry $\sin\left(\phi_h - \phi_S\right) \rightarrow f_{1T}^{\perp} D_1$

Quark distributions in a transversely polarized nucleon

Transverse Target asymmetries from COMPASS

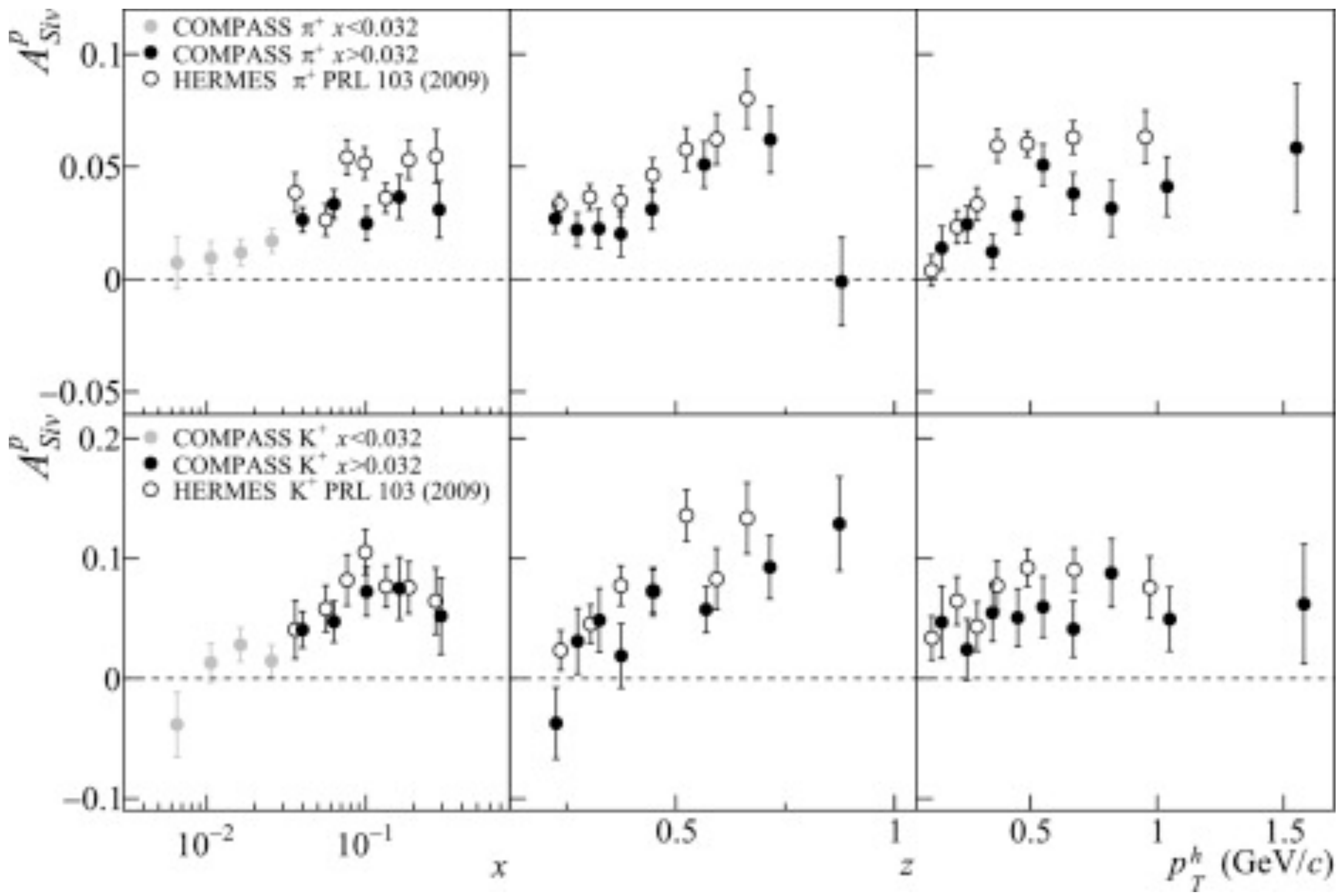
Collins asymmetries



PLB 744 (2015) 250

Transverse Target asymmetries from COMPASS and HERMES

Sivers asymmetries



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SIDIS Summary

- Semi-inclusive DIS a powerful tool for exploring how quarks are distributed in the nucleon
  - Flavor tagging for polarized and unpolarized PDFs
  - TMDs allow exploration of transverse structure → link to orbital angular momentum
- Most SIDIS data has been acquired at fixed target facilities → HERMES, JLab, COMPASS
  - JLab has a large SIDIS program planned for Halls A, B, and C as part of 12 GeV Upgrade
  - A future EIC would provide a huge amount of data in the "sea-quark" regime

EXTRA

DIS Cross Section

Reminder: Inclusive case

$$\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{xQ^2}\left[y^2 x F_1(x,Q^2) + (1-y) F_2(x,Q^2)\right]$$

Quark parton-model

$$F_2 = 2xF_1 \qquad F_2(x) = x\sum_f e_f^2 q_f(x)$$

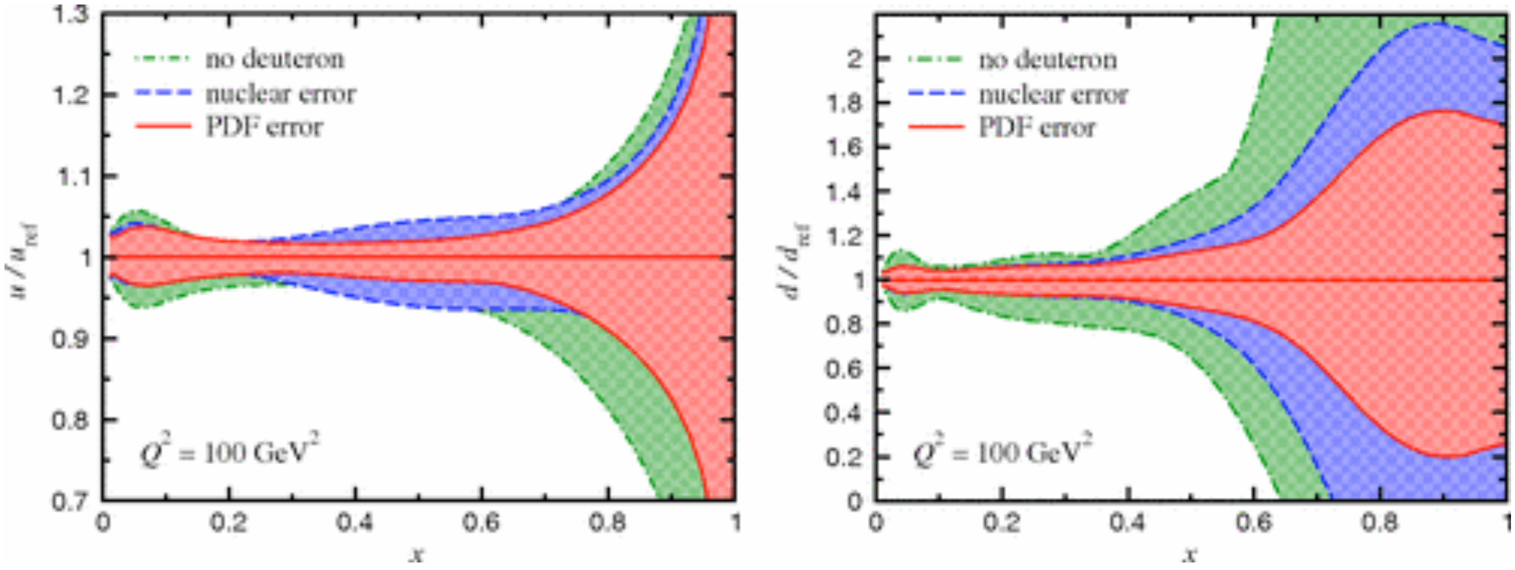
$$\frac{d\sigma}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4}[1+(1-y)^2]\sum_f e_f^2 q_f(x)$$

$$\frac{d\sigma}{dx dy} = \frac{2\pi\alpha^2 xs}{Q^4}[1+(1-y)^2]\sum_f e_f^2 q_f(x)$$

CJ12 PDFs

Nuclear effects in deuteron lead to significant uncertainties in quark PDFs at large *x*

→ This has been studied in some depth by the CTEQ-JLAB collaboration



J. F. Owens, A. Accardi and W. Melnitchouk, Phys. Rev. D 87, 094012 (2013)