# Parton Distribution Functions

for beginners

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# Outline

## Lecture 1

- Introduction to PDFs - recap of DIS , parton model
- Properties of PDFs - sum rules, scale dependance
- Fitting PDFs - data, theory & the art of fitting

## Lecture 2

- Error PDFs - Hessian and NNPDF approach to errors
- Miscellaneous topics in PDFs - strong coupling constant, strange quark PDF etc...
- Review of current proton PDFs

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# Lecture 1

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# Intro to PDFs

# Fitting PDFs

# Recap of DIS

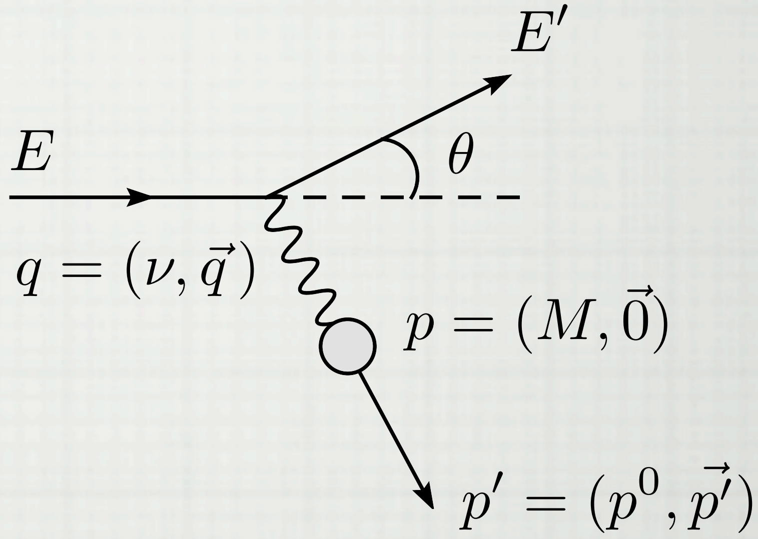
- Scattering - *THE tool to study inner structure of atoms, nuclei & proton*
- Parton distribution functions & the inner structure of the proton
  - direct descendants of Rutherford's experiments

- ***Rutherford's scattering***

  spin-0 non-relativistic projectile with very heavy target (no recoil)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Ruth.}} = \frac{(\alpha Z)^2}{4E^2 \sin^4 \frac{\theta}{2}} .$$

- ***Kinematics of elastic scattering***

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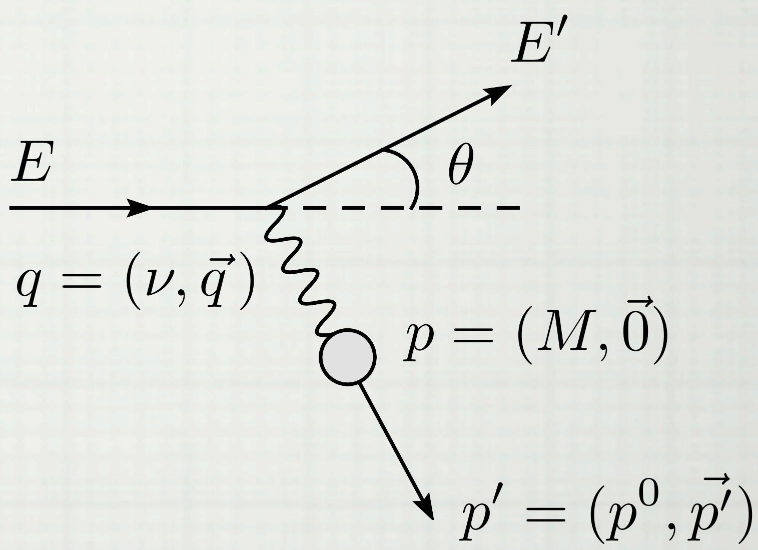
$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Ruth.}} = \frac{(\alpha Z)^2}{4E^2 \sin^4 \frac{\theta}{2}} \,.$$

- ***Mott's scattering***

  spin-0 relativistic projectile with recoil of the target

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} = \frac{(\alpha Z)^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \cos^2 \frac{\theta}{2}$$

- ***Kinematics of elastic scattering***

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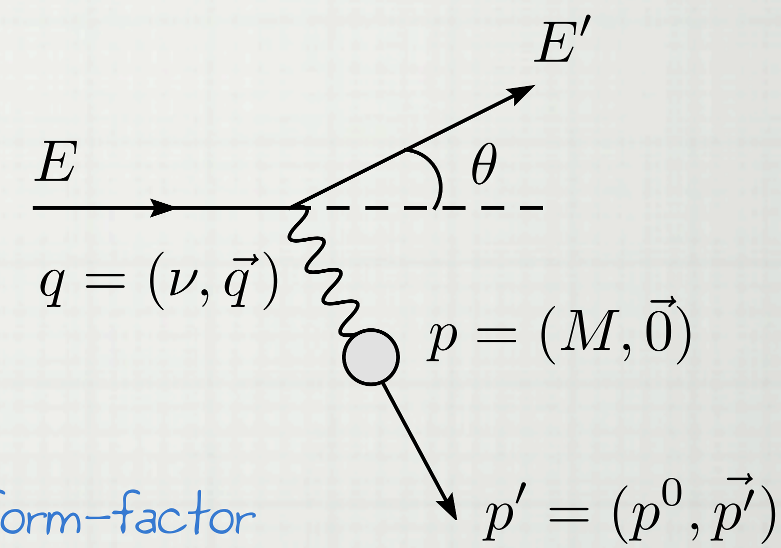
- ***Scattering of electrons on muons***

  spin-1/2 relativistic projectile scattering on muons

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\,\frac{E'}{E}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right]$$

- ***Kinematics of elastic scattering***




- ***Rosenbluth's scattering***

  spin-1/2 relativistic projectile scattering on non-point like target with spin-1/2 (proton)

magnetic form-factor

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\,\frac{E'}{E}\left[\left(F_1(q)^2 - \frac{\kappa^2 q^2}{4M^2}F_2(q)^2\right)\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\left(F_1(q) + \kappa F_2(q)\right)^2\sin^2\frac{\theta}{2}\right]$$

electric form-factor

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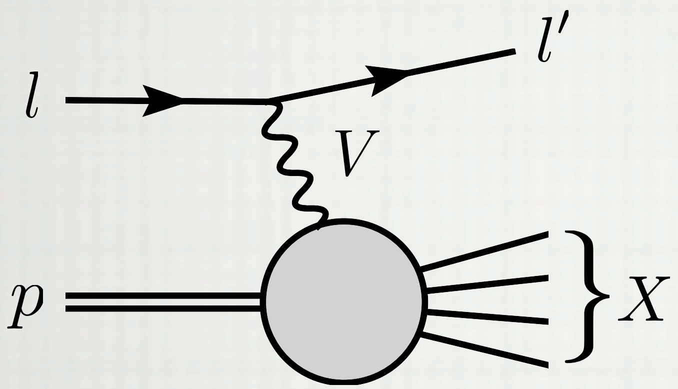
# Recap of DIS

- *Deep Inelastic Scattering*




$$l(k) + p(p) \to l'(k') + X$$

- Kinematic variables
  - $E', \theta$ experimentally easy to understand
  - $x, Q^2$ Lorentz invariant

$$q = k - k' \qquad x = \frac{Q^2}{2p.q} \qquad x \in (0,1)$$

$$Q^2 = -q^2$$

Bjorken-x

- Distinguishing between elastic & inelastic scattering

$$W^2 = (p'_1 + p'_2 + \ldots + p'_n)^2$$

- if $W^2 = m_p^2$ elastic scattering
- if $W^2 \gg m_p^2$ inelastic scattering

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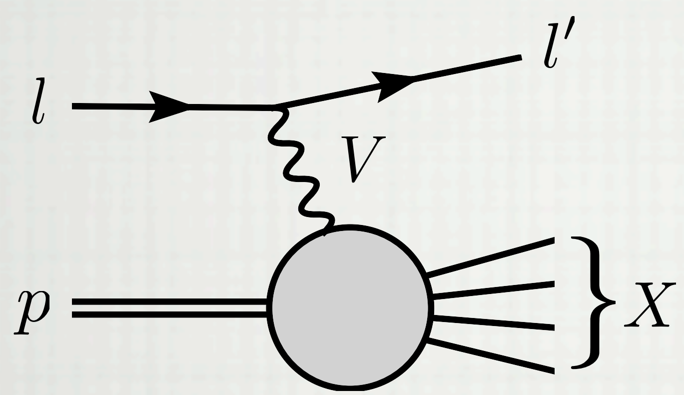
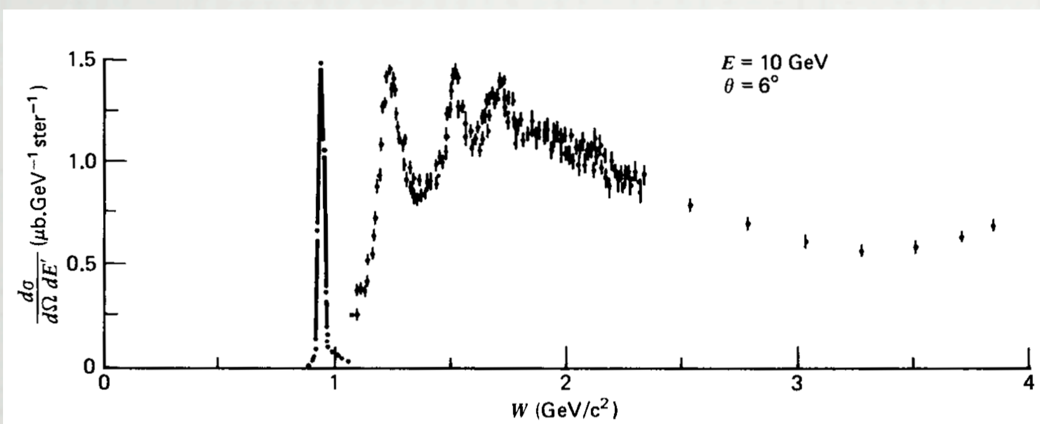
# Recap of DIS

- *Deep Inelastic Scattering*




$$l(k) + p(p) \rightarrow l'(k') + X$$

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- DIS cross-section & structure functions

$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left( \frac{2F_1(x,Q^2)}{M} \sin^2 \frac{\theta}{2} + \frac{F_2(x,Q^2)}{E - E'} \cos^2 \frac{\theta}{2} \right)$$

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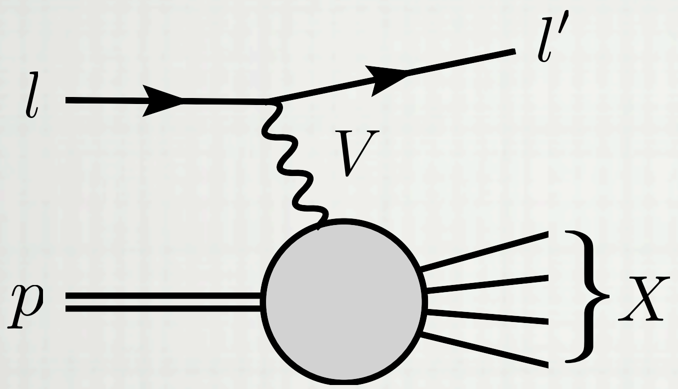
# Recap of parton model

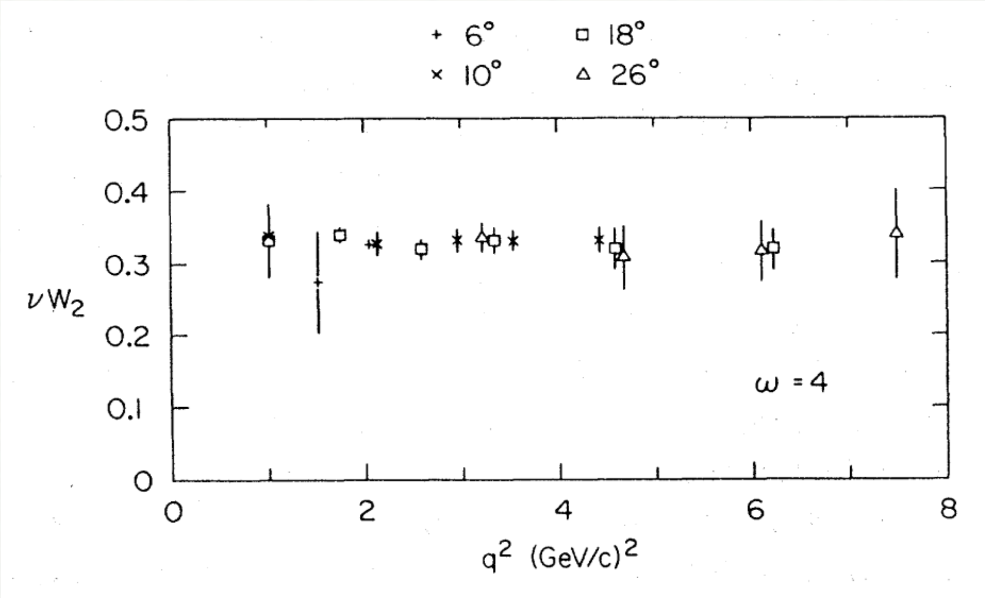
- *Deep Inelastic Scattering*




$$l(k) + p(p) \to l'(k') + X$$

- Biggest puzzle - Bjorken scaling
  - structure function $F_2(E', \theta)$ depends on $E', \theta$ only in one particular combination

$$x = \frac{Q^2}{2p.q} \stackrel{\text{lab}}{=} \frac{2EE' \sin^2 \frac{\theta}{2}}{M(E - E')}$$



# Recap of parton model

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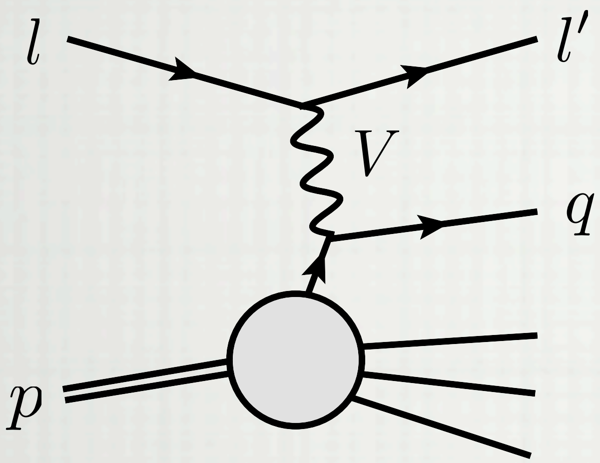


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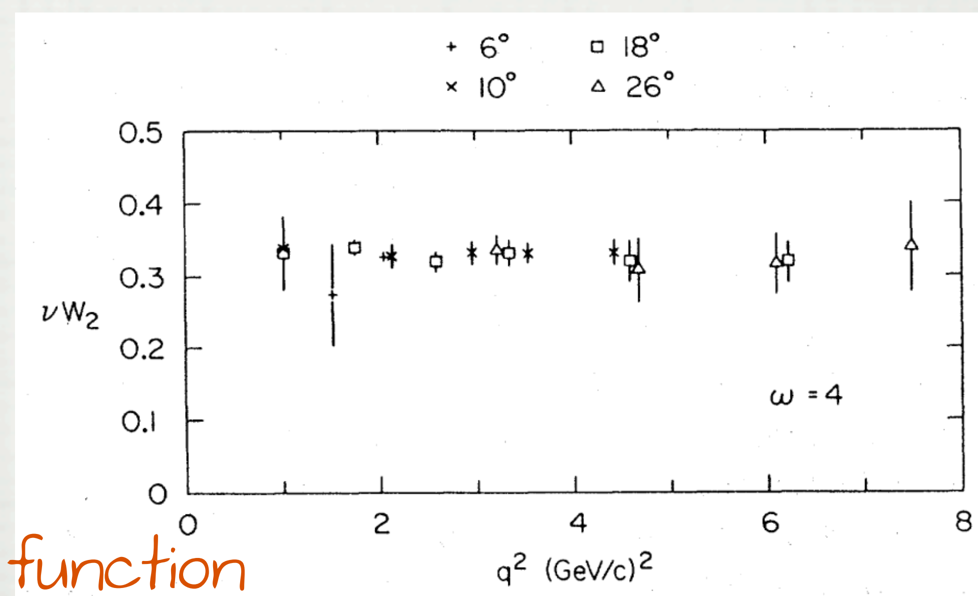


- Solution - Parton model
  - inelastic scattering an incoherent sum of elastic scatterings on proton constituents

$$\frac{d\sigma}{dx\, dQ^2} = \sum_q \int \mathrm{d}\xi\, f_q(\xi) \left( \frac{d\sigma^{eq}}{dx\, dQ^2} \right)_{\text{el.}}$$

parton distribution function

Structure function in parton model at leading order

$$F_2(x, Q^2) = \sum_q e_q^2 \int \mathrm{d}\xi\, x\, f_q(\xi)\, \delta\left( \xi - \frac{Q^2}{2p.q} \right)$$

mom. fraction $\xi$ = Bjorken-x

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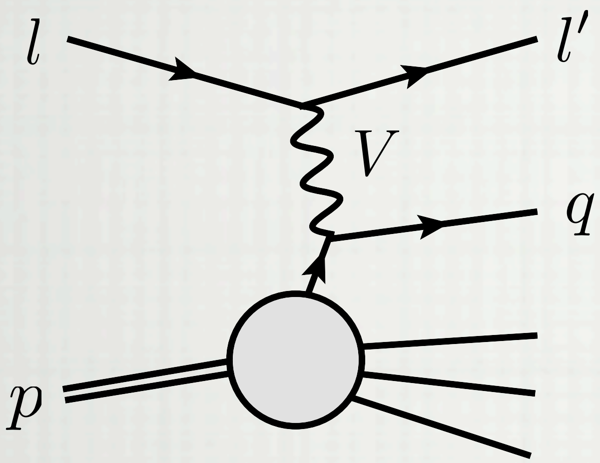
# Recap of parton model

- *Deep Inelastic Scattering*

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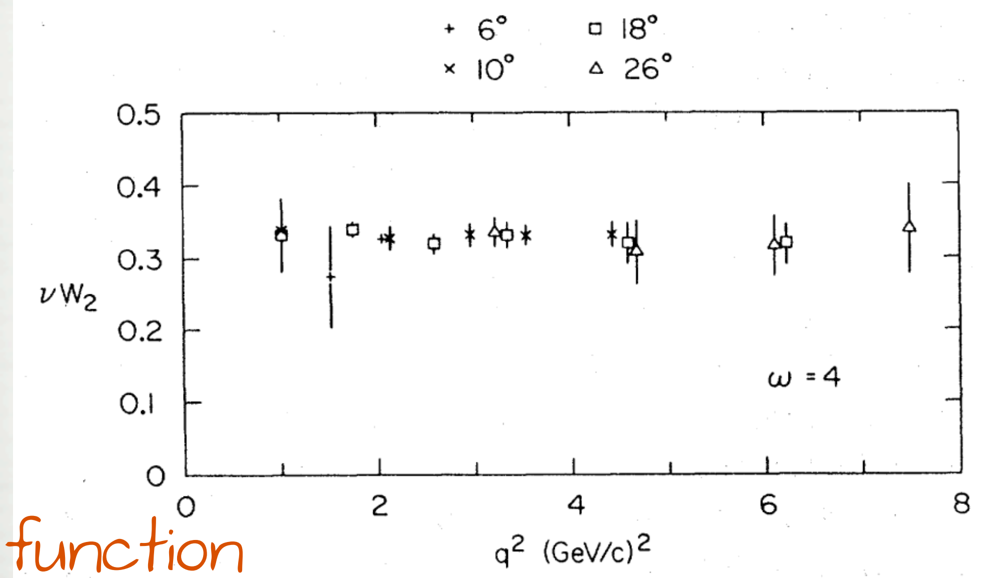


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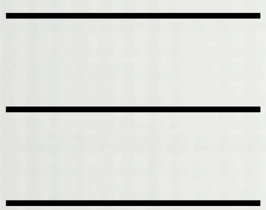
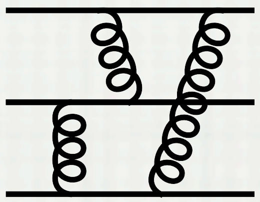
# Properties of PDFs

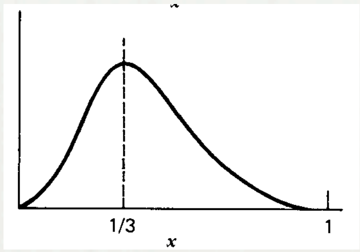
- *PDF sum rules*
  - PDFs at leading order - number densities of partons

free quarks

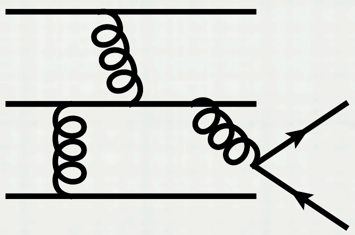


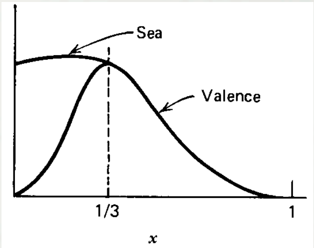

bound quarks






bound quarks + QCD effects






  - PDF sum rules - connect partons to quarks from SU(3) flavour symmetry of hadrons
    proton (uud), neutron (udd)

for proton

$$\int_0^1 dx\,[u(x) - \bar{u}(x)] = 2 \qquad \int_0^1 dx\,[d(x) - \bar{d}(x)] = 1$$

valence quark sum rules

  - Momentum sum rule - connects all PDF flavours including gluon

$$\sum_i \int_0^1 dx\, x f_i(x) = 1$$

momentum sum rule

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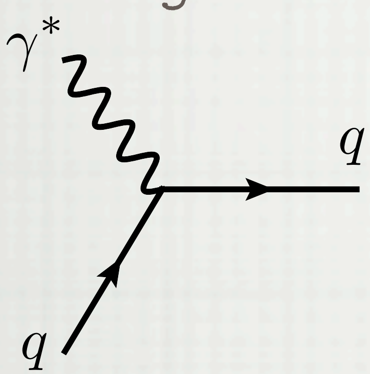
# Properties of PDFs

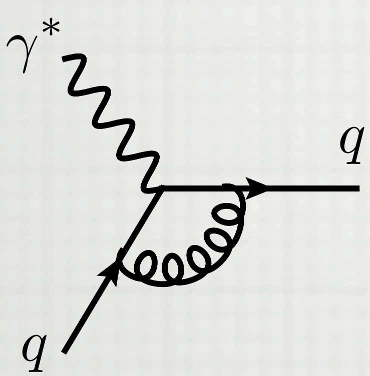
- *Scale dependance of PDFs*
  - Scale dependance connected to DIS at next-to-leading order

DIS leading order




$$\frac{F_2(x,Q^2)}{x} = \int \mathrm{d}\xi \sum_q e_q^2 f_q(\xi)\, \delta\left(\xi - \frac{Q^2}{2p.q}\right) = \sum_q e_q^2 f_q(x)$$

DIS next-to-leading order (virtual + renormalization)




$$\frac{F_2(x,Q^2)}{x} = \int \mathrm{d}z\, \mathrm{d}\xi \sum_q e_q^2 f_q(\xi)\, \frac{\alpha_s}{(2\pi)} C_F \left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$\left\{\left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 - \frac{\pi^2}{3}\right)\delta(1-z)\right\}\delta(x - z\xi)$$

infrared soft & coll. divergence

ultraviolet divergence cancelled

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# Properties of PDFs

- *Scale dependance of PDFs*
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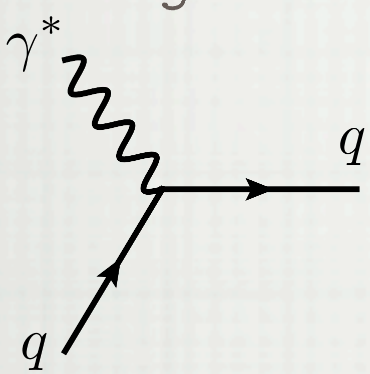
DIS leading order

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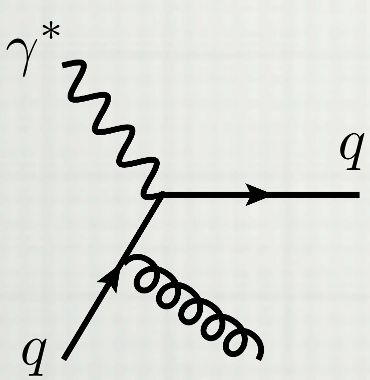
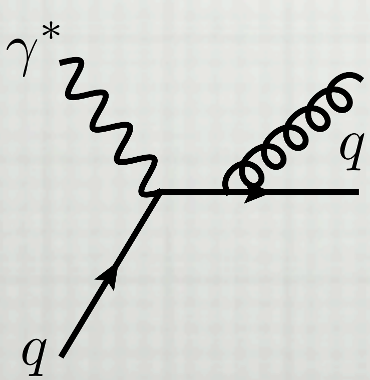
DIS next-to-leading order (real radiation)




$$\frac{F_2(x,Q^2)}{x} = \int \mathrm{d}z\, \mathrm{d}\xi \sum_q e_q^2 f_q(\xi)\, \frac{\alpha_s}{(2\pi)} C_F \left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$\left\{ \left(\frac{2}{\varepsilon^2} + \frac{3}{2}\frac{1}{\varepsilon} + \frac{7}{2}\right)\delta(1-z) + 3 + 2z - \frac{1+z^2}{1-z}\ln z \right.$$

$$\left. - \left(\frac{1+z^2}{\varepsilon} + \frac{3}{2}\right)\left[\frac{1}{1-z}\right]_+ + (1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ \right\} \delta(x - z\xi)$$

infrared soft + coll. divergence

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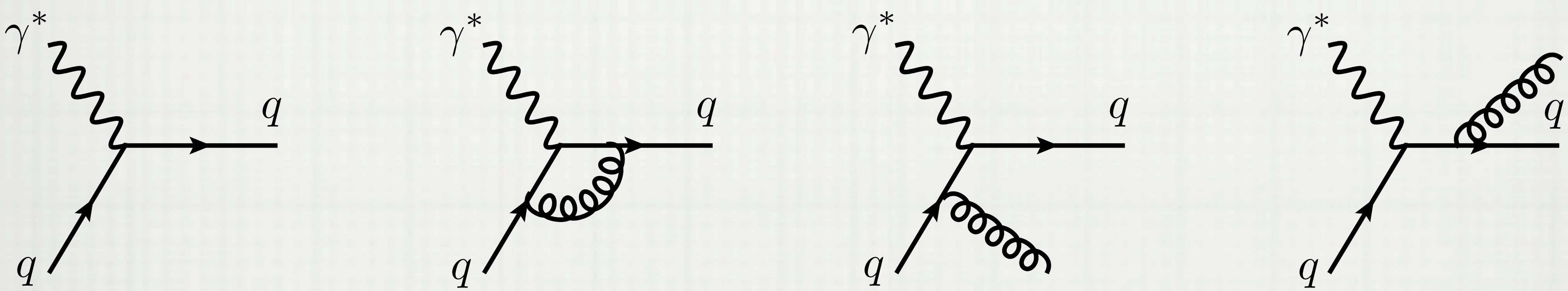
# Properties of PDFs

- *Scale dependance of PDFs*
  - Scale dependance connected to DIS at next-to-leading order

DIS leading + next-to-leading order




$$\frac{F_2(x,Q^2)}{x} = \int_x^1 \frac{\mathrm{d}\xi}{\xi} \sum_q e_q^2 f_q(\xi) \left\{ \delta\left(1 - \frac{x}{\xi}\right) - \frac{\alpha_s}{(2\pi)} P_{qq}\left(\frac{x}{\xi}\right) \left(\frac{1}{\varepsilon} + \ln\frac{\mu^2}{Q^2} + \text{finite}\right)\right\}$$

uncancelled infrared coll. divergence

splitting function

$$P_{qq}(z) = C_F \left\{ (1+z^2)\left[\frac{1}{1-z}\right]_+ + \frac{3}{2}\delta(1-z)\right\}$$

- Similar trick to renormalization - divergence disappears in differences

$$\frac{F_2(x,Q^2)}{x} - \frac{F_2(x,Q_0^2)}{x} = \int_x^1 \frac{\mathrm{d}\xi}{\xi} \sum_q e_q^2 f_q(\xi) \frac{\alpha_s}{(2\pi)} P_{qq}\left(\frac{x}{\xi}\right) \ln\frac{Q^2}{Q_0^2} = \text{finite}$$

# Properties of PDFs

- *Scale dependance of PDFs*
    - Scale dependance connected to DIS at next-to-leading order

PDF definition beyond leading order (one possibility)

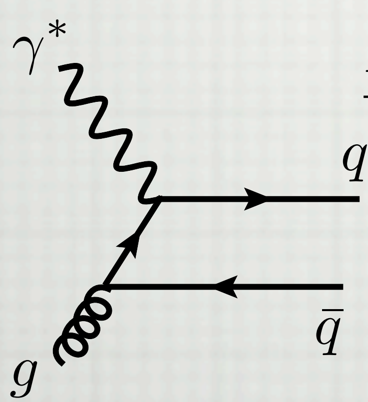
$$\frac{F_2(x,Q^2)}{x} \equiv \sum_q e_q^2\, f_q(x,Q^2)$$

Q-dependent PDF

Scale dependance of PDF

$$\frac{d\, f_q(x,Q^2)}{d \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{\mathrm{d}\xi}{\xi}\, f_q(\xi,Q^2)\, P_{qq}\left(\frac{x}{\xi}\right)$$

- Oops - forgot one contribution at NLO




$$\frac{F_2(x,Q^2)}{x} = \int \mathrm{d}z\, \mathrm{d}\xi\, f_g(\xi) \left\{ \frac{\alpha_s}{2\pi} \left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left(-\frac{1}{\varepsilon}\right) \left(P_{qg}(z) + P_{\bar{q}g}(z)\right) + \frac{\alpha_s}{2\pi} \left[ \left(z^2 + (1-z)^2\right) \ln\frac{1-z}{z} + 6z(1-z) \right] \right\} \delta(x - z\xi)$$

splitting function

$$P_{qg}(z) = \frac{1}{2}(z^2 + (1-z)^2)$$

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# Properties of PDFs

- *Scale dependance of PDFs - DGLAP equations*
  - all contributions together mix different PDF via DGLAP equations

$$\frac{d\,f_q(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}\xi}{\xi}\left[P_{qq}\left(\frac{x}{\xi}\right)\,f_q(\xi,Q^2) + P_{qg}\left(\frac{x}{\xi}\right)\,f_g(\xi,Q^2)\right]$$

$$\frac{d\,f_g(x,Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}\xi}{\xi}\left[P_{gg}\left(\frac{x}{\xi}\right)\,f_g(\xi,Q^2) + P_{gq}\left(\frac{x}{\xi}\right)\,f_q(\xi,Q^2)\right]$$

  - $2n_f + 1$ set of coupled differential equations with initial conditions determined through the fit to data (!)
  - decoupling some equations by working in a different basis

$$g(x,Q^2)$$

gluon

$$q_S(x,Q^2) = \sum_{i=1}^{n_f}\left(q_i(x,Q^2) + \bar{q}_i(x,Q^2)\right)$$

singlet

$$q_V(x,Q^2) = \sum_{i=1}^{n_f}\left(q_i(x,Q^2) - \bar{q}_i(x,Q^2)\right)$$

$$q^{\pm}_{ij}(x,Q^2) = (q_i \pm \bar{q}_i) - (q_j \pm \bar{q}_j)$$

non-singlet

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# Properties of PDFs

- *Scale dependance of PDFs - DGLAP equations*
  - $2n_f + 1$ set of coupled differential equations with initial conditions determined through the fit to data (!)

$$\frac{d}{d\log Q^2}\begin{pmatrix} q_S \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q_S \\ g \end{pmatrix}$$

$$\frac{dq_{ij}^{\pm}}{d\log Q^2} = P_{\pm} \otimes q_{ij}^{\pm} \qquad \frac{dq_V}{d\log Q^2} = P_v \otimes q_V$$

  - splitting functions have perturbative expansions in $\alpha_s$

$$P_{ij}(z) = P_{ij}^{(0)}(z) + \frac{\alpha_s}{2\pi} P_{ij}^{(1)}(z) + \dots$$

  - non-singlet splitting functions @ LO identical

$$P_{\pm}^{(0)}(z) = P_v^{(0)}(z) = P_{qq}^{(0)}(z)$$

  - 2 approaches to solutions of DGLAP equations

| x-space | Mellin space |
|---|---|
| Hoppet, QCDnum, ... | Pegasus, ... |
| **arXiv:0804.3755** **arXiv:1005.1481** | **hep-ph/0408244** |

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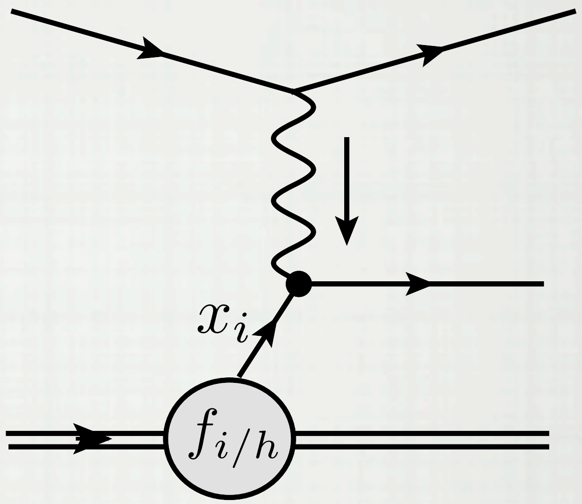
# Properties of PDFs

- *Universality of PDFs*
  - QCD factorization - proof that parton splittings universal - allows a separation of hadronic cross-sections into **process-dependent** partonic cross-section and **non-perturbative process-independent parton distributions**




$$F_i(x, Q^2) = x \sum_i \int_x^1 \frac{dz}{z} C_i\left(\frac{x}{z}, \alpha_s(Q^2)\right) f_i(z, Q^2)$$

partonic cross-section

parton distribution

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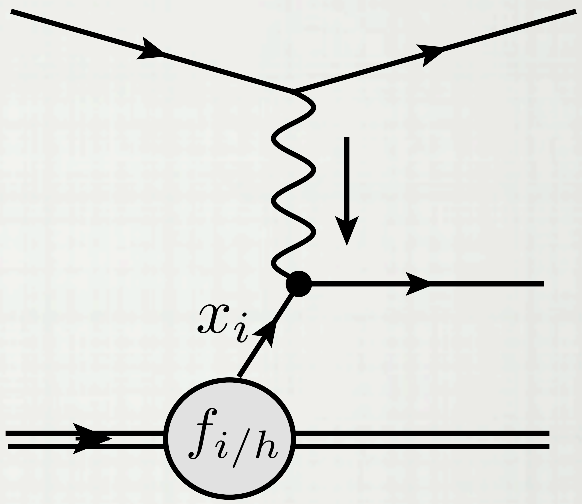


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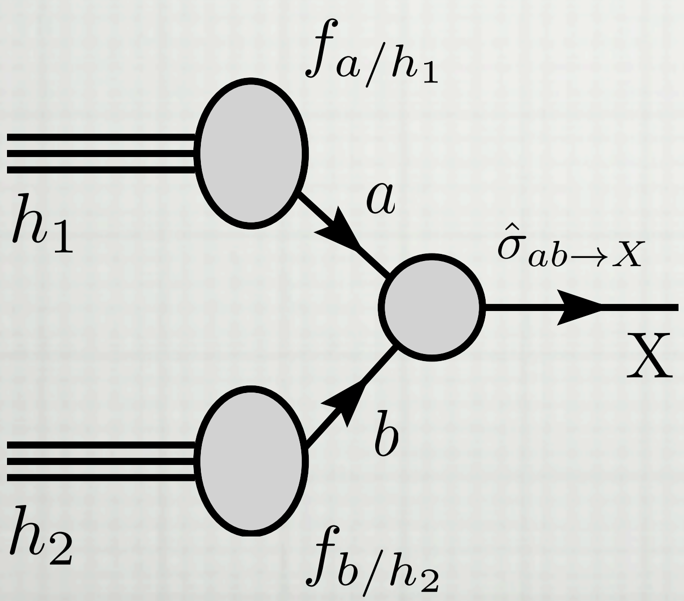
partonic cross-section    parton distribution




  - The same factorization allows predictions at the LHC, using the same **universal** PDF obtained elsewhere e.g. @ HERA

$$\sigma = \sum_{a,b} \int dx_1 dx_2 \; f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \; \hat{\sigma}_{ab\to X}(x_1 x_2 s)$$

from experiment    from pQCD

Intro to PDFs

# Fitting PDFs

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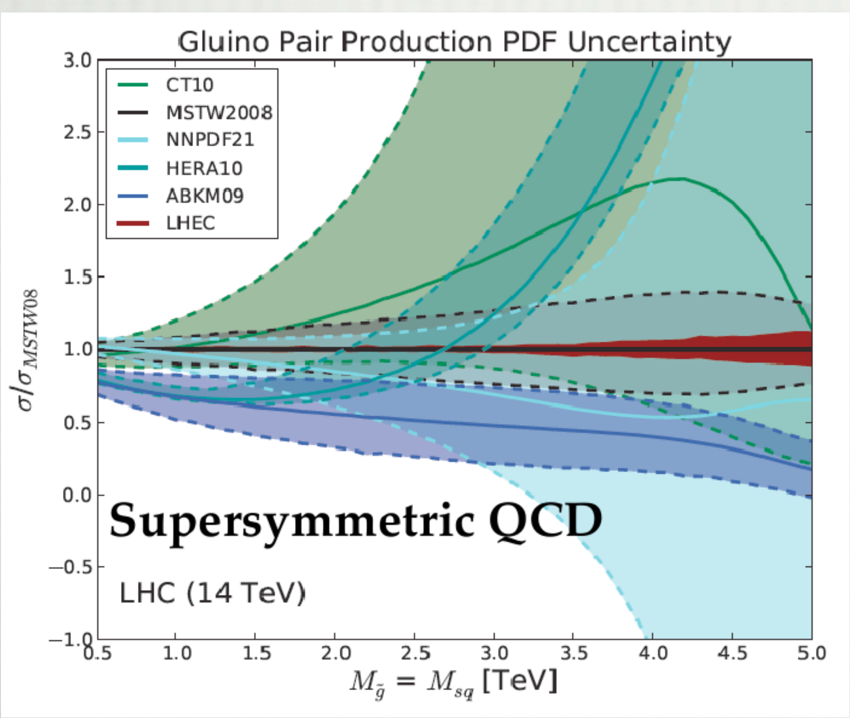
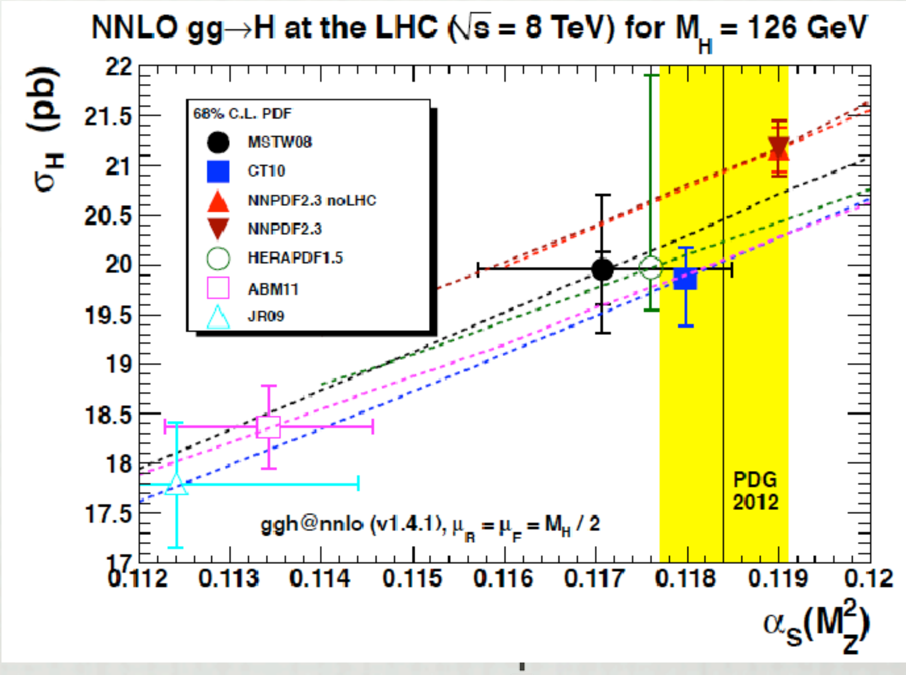
# Fitting PDF

- *Motivation to fit PDFs from data*
  - need experimental input to get rid of IR collinear divergencies (much like renormalization)
  - use data and pQCD to gain information on the proton structure - intrinsically non-perturbative
- *PDFs as tools in particle physics*
  - Open questions in proton structure - anti-symmetric sea quarks, intrinsic charm ...
  - PDF uncertainties limit predictions of Higgs production ⇒ limit measurement of Higgs coupling
  - New physics production probes PDF at high-x




$$x \approx \frac{M}{\sqrt{s}}\, e^{y}$$

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# Fitting PDF

- *Parton Distribution Functions*
    - how do we determine them ? What are the moving parts in a typical PDF fitting-machine ?

Choose PDF input parameters

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# Fitting PDF

- *Parton Distribution Functions*
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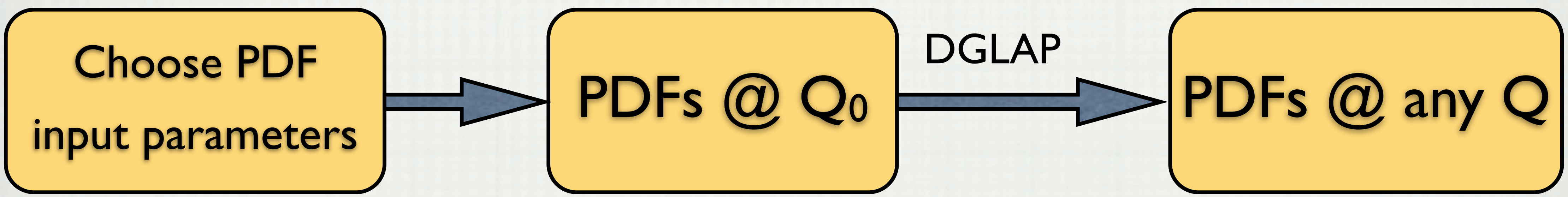


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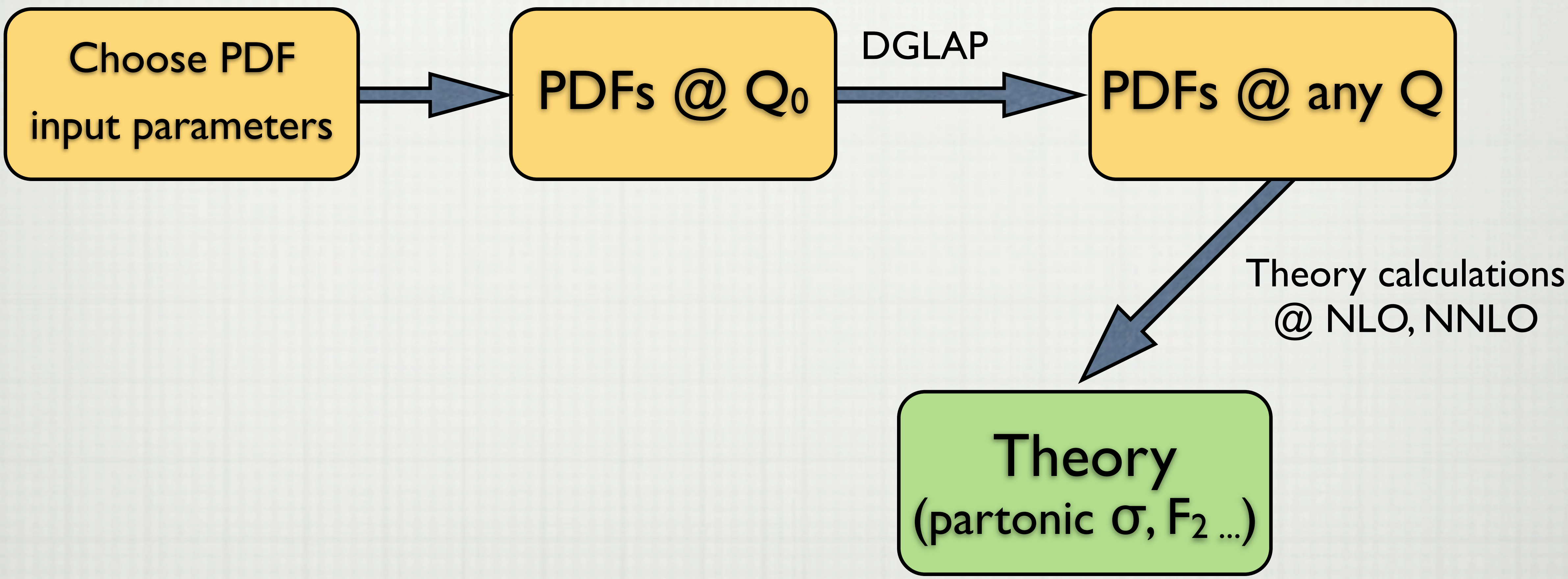

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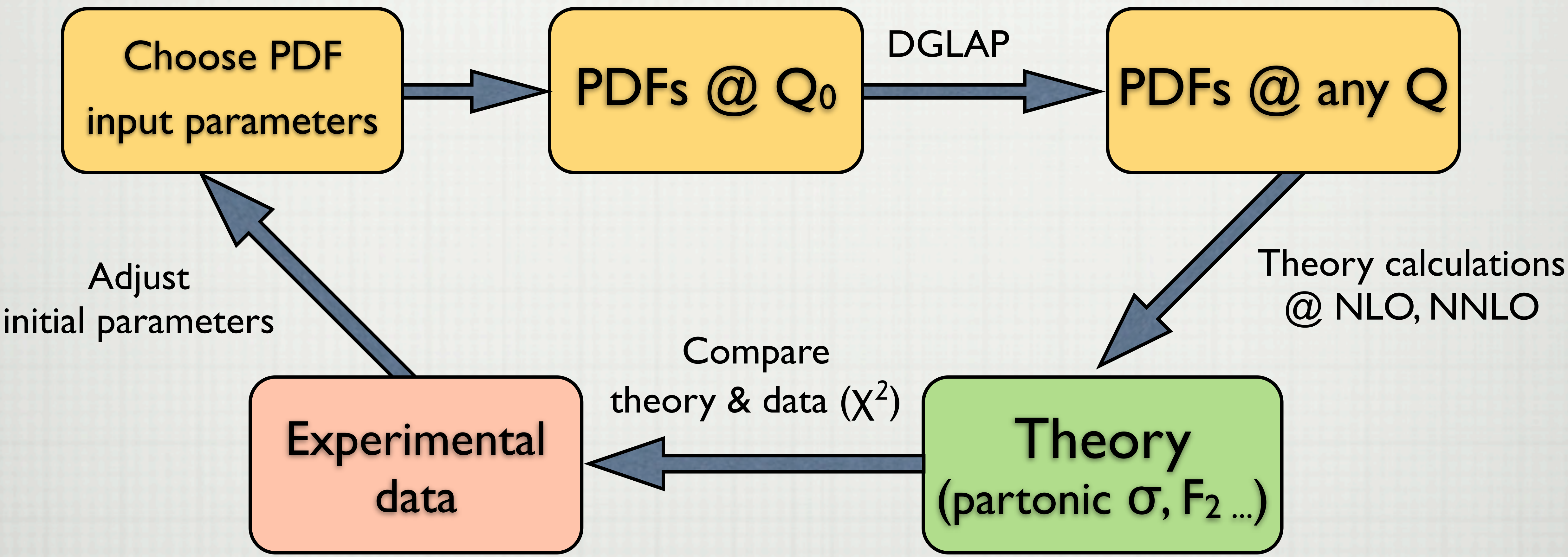

# Fitting PDF

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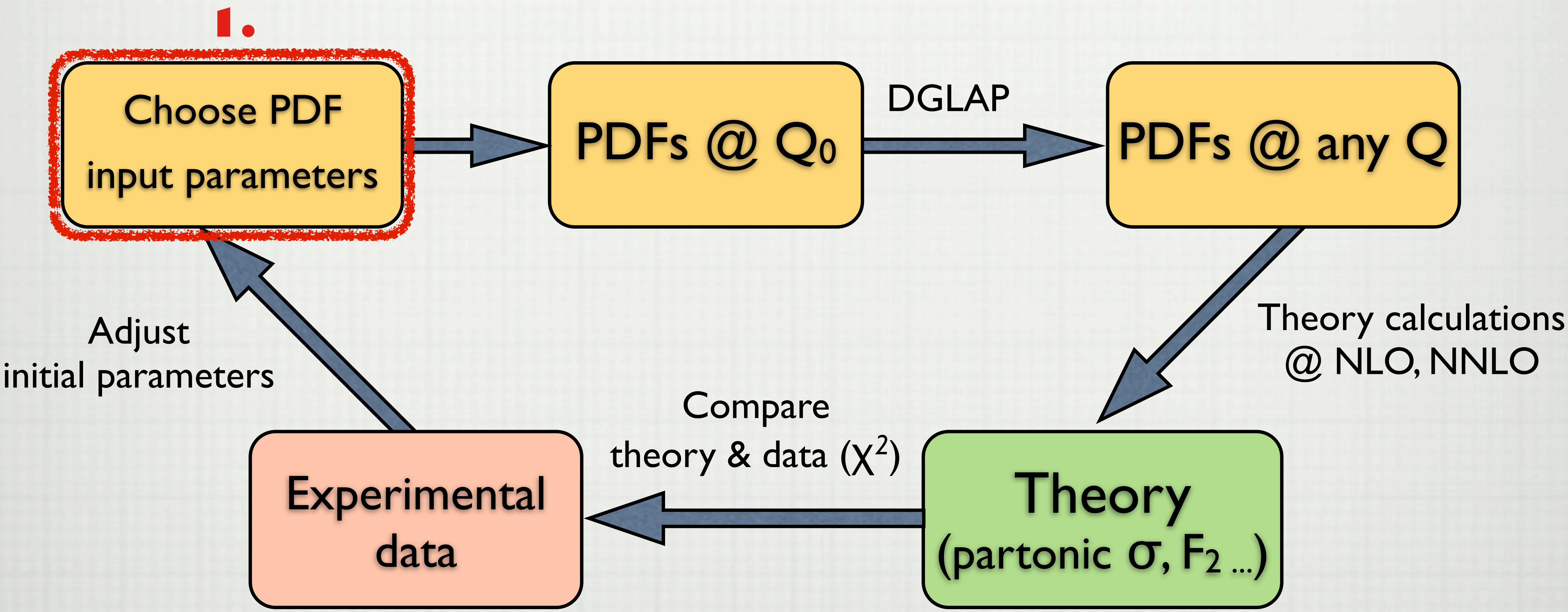

# Fitting PDF

- *Parton Distribution Functions - parameterisation @ initial scale*
  - dependance on Bjorken-x not predicted or constrained by pQCD - need to fit to data
  - every global PDF analysis starts with a flexible parameterisation chosen at the input scale (normally polynomials with ~ 30 free params)

- *Parameterisation caveats*
  - flexibility requires to use as many free parameters as possible (eliminates bias)
  - too many free parameters leads to false minima, unconstrained parameters, ...
  - some bias might be put in the parameterisation

Generic PDF parameterisation

$$xf_k(x, Q_0) = x^{c_1}(1-x)^{c_2} P_k(x)$$

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# Fitting PDF

- *Parton Distribution Functions - parameterisation @ initial scale*
    - different parameterisations possible and often lead to similar result

CTEQ 6

**hep-ph/0201195**

$$x\, f_k(x, Q_0) = c_0\, x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4} x)^{c_5} \qquad k = u_v, d_v, g, \bar{u} + \bar{d}$$

$$\bar{d}(x, Q_0)/\bar{u}(x, Q_0) = c_0\, x^{c_1} (1-x)^{c_2} + (1 + c_3 x)(1-x)^{c_4}$$

$$s = \bar{s} = 0.2\, (\bar{u} + \bar{d})$$

CTEQ 6.5

**hep-ph/0611254**

$$x\, f_k(x, Q_0) = c_0\, x^{c_1} (1-x)^{c_2} e^{-c_3 (1-x)^2 + c_4 x^2} \qquad k = u_v, d_v, g$$

$$x\, f_k(x, Q_0) = c_0\, x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4} x)^{c_5} \qquad k = \bar{u} + \bar{d}$$

$$\bar{d}(x, Q_0)/\bar{u}(x, Q_0) = c_0\, x^{c_1} (1-x)^{c_2} + (1 + c_3 x)(1-x)^{c_4}$$

$$s = \bar{s} = \frac{\kappa}{2}\, (\bar{u} + \bar{d})$$

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# Fitting PDF

- *Parton Distribution Functions - parameterisation @ initial scale*
  - different parameterisations possible and often lead to similar result
  - parameterisation might change with time to allow for flexibility where data is available

CT10

arXiv:1007.2241

$$x\, f_k(x, Q_0) = c_0\, x^{c_1}\, (1-x)^{c_2}\, e^{-c_3 (1-x)^2 + c_4 x^2} \qquad k = u_v, d_v$$

$$x\, g(x, Q_0) = c_0\, x^{c_1}\, (1-x)^{c_2}\, e^{c_3 x + c_4 x^2}\, e^{-c_6 x^{-c_7}}$$

CT14

arXiv:1506.07443

$$x\, f_k(x, Q_0) = x^{c_1}\, (1-x)^{c_2}\, \left(c_3 + c_4 \sqrt{x} + c_5\, x + c_6\, x^{3/2} + c_7\, x^2\right)$$

$$k = u_v, d_v, g$$

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# Fitting PDF

- *Parton Distribution Functions - parameterisation @ initial scale*
  - different parameterisations possible and often lead to similar result
  - parameterisation might change with time to allow for flexibility where data is available

MSTW

arXiv:0901.0002

$$x\, f_k(x,Q_0) = A_k\, x^{\eta_1^k}(1-x)^{\eta_2^k}(1+\epsilon_k\sqrt{x}+\gamma_k x) \qquad k = u_v, d_v, 2(\bar{u}+\bar{d})+s+\bar{s}, s+\bar{s}$$

$$x\, g(x,Q_0) = A_g\, x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\sqrt{x}+\gamma_g x) + A_{g'}\, x^{\delta_{g'}}(1-x)^{\eta_{g'}}$$

CTEQ-JLAB '12

arXiv:1212.1702

$$x\, f_k(x,Q_0) = c_0\, x^{c_1}(1-x)^{c_2}(1+c_3\sqrt{x}+c_4\, x) \qquad k = u_v, d_v, g, \bar{u}+\bar{d}, \bar{d}-\bar{u}$$

$$d_v \to c_0^{d_v}\left(\frac{d_v}{c_0^{d_v}} + b\, x^c u_v\right)$$

- input scale choice arbitrary (some choices more practical than others - mostly $Q_0 = m_c$)

| $Q_0 = 1.3\,\mathrm{GeV}$ | $Q_0 = 1.0\,\mathrm{GeV}$ | $Q_0 = \sqrt{2}\,\mathrm{GeV}$ |
|---|---|---|
| CTEQ | MSTW | NNPDF |

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# Fitting PDF

- *Parton Distribution Functions - parameterisation @ initial scale*
    - different parameterisations possible and often lead to similar result
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MSTW

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$$x\, f_k(x,Q_0) = A_k\, x^{\eta_1^k}(1-x)^{\eta_2^k}(1+\epsilon_k\sqrt{x}+\gamma_k x) \qquad k = u_v, d_v, 2(\bar{u}+\bar{d})+s+\bar{s}, s+\bar{s}$$

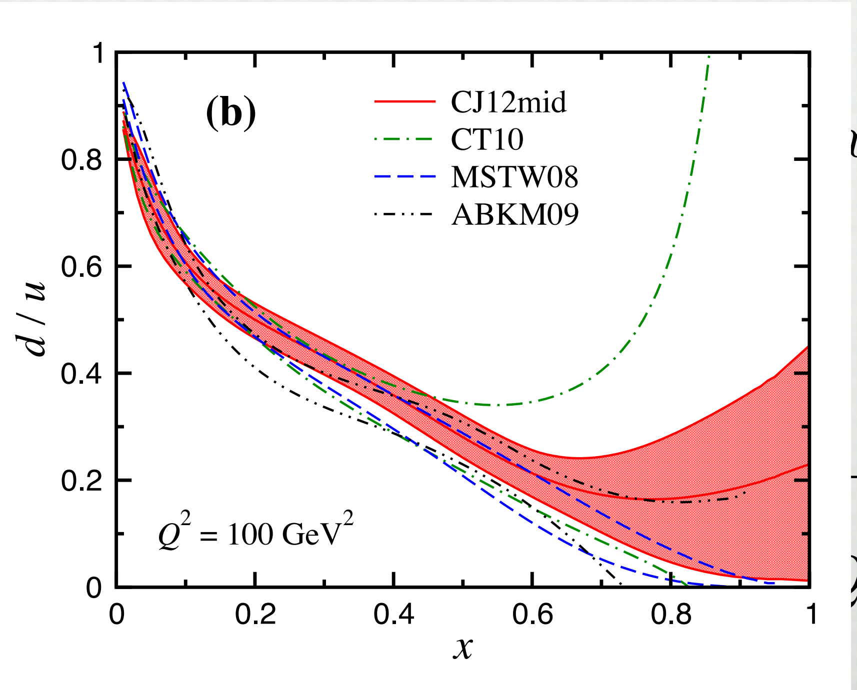
$$x\, g(x,Q_0) = A_g\, x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\sqrt{x}+\gamma_g x) + A_{g'}\, x^{\delta_{g'}}(1-x)^{\eta_{g'}}$$

CTEQ-JLAB '12

**arXiv:1212.1702**

$x\, f_k(x,Q_0) =$ … $x)$ $\qquad k = u_v, d_v, g, \bar{u}+\bar{d}, \bar{d}-\bar{u}$

$d_v$ -




- input scale choice arbi… than others - mostly $Q_0 = m_c$)

$Q_0 = 1.3\,\mathrm{GeV}$ … $Q_0 = \sqrt{2}\,\mathrm{GeV}$

CTEQ … NNPDF

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# Fitting PDF

- *Parton Distribution Functions - parameterisation @ initial scale*
  - different parameterisations possible and often lead to similar result
  - parameterisation might change with time to allow for flexibility where data is available

NNPDF 1.0

arXiv:0808.1231

- too many free parameters not a bug but a feature

$$\Sigma(x,Q_0^2) = (1-x)^{m_\Sigma}\, x^{-n_\Sigma}\, \mathrm{NN}_\Sigma(x)$$
$$V(x,Q_0^2) = (1-x)^{m_V}\, x^{-n_V}\, \mathrm{NN}_V(x)$$
$$g(x,Q_0^2) = A_g(1-x)^{m_g}\, x^{-n_g}\, \mathrm{NN}_g(x)$$
$$s(x) = \bar{s}(x) = \frac{1}{2}C_s(\bar{u}(x)+\bar{d}(x))$$

- different basis adapted to evolution

$$\Sigma(x) = \sum_{i=1}^{n_f}(q_i(x)+\bar{q}_i(x)) \qquad V(x) = \sum_{i=1}^{n_f}(q_i(x)-\bar{q}_i(x))$$
$$T_3(x) = (u(x)+\bar{u}(x)) - (d(x)+\bar{d}(x))$$

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# End of Lecture 1