Photons
in
Large Momentum Transfer Processes

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Outline

1. Introduction
    - Why photons?
    - Motivation for their study
2. Theory overview and comparison to selected data
3. Theoretical improvements
    - Resummation techniques
    - Some applications
4. Conclusions
5. Appendix: Notation, kinematics, and other useful information

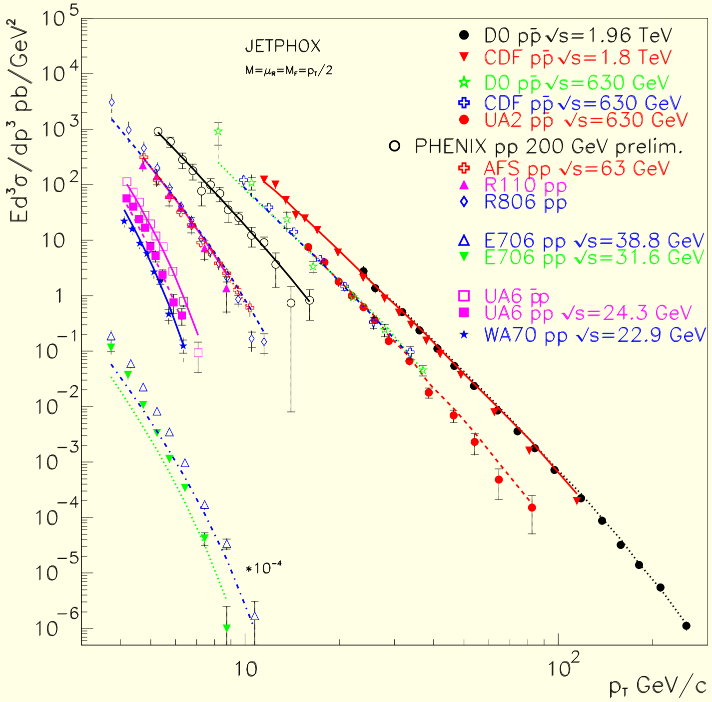
Resources - previous summer schools

- Direct photons have been covered in many previous CTEQ Summer Schools
  - The following are all from CTEQ Summer Schools (www.cteq.org)
  - DP = Direct Photon, VB = Vector boson and/or lepton pair production
- John Campbell 2013 (DP+VB)
- Jeff Owens 2012 (DP)
- Pavel Nadolsky 2011 (VB)
- Jeff Owens 2010 (DP + VB)
- Pavel Nadolsky 2007 (VB)
- Jeff Owens 2006 (DP)
- George Sterman 2005 (VB), Jeff Owens 2005 (DP)
- Fred Olness 2004 (VB), Jeff Owens 2004 (DP)

Why Photons?

- Well understood electromagnetic interaction
- Well defined probe of strong interaction dynamics
- Classic examples
  - Deep inelastic scattering
  - Lepton pair production
  - $e^+e^- \rightarrow$ hadrons
- Direct photons, photoproduction, and two photon processes continue this history
- Note: "Direct" photons are those that originate in the hard scattering subprocess and not from particle decays

Does QCD describe the data? YES!



- Inclusive and isolated photon data from Aurenche *et al.* hep-ph/0602133
- Agreement over 9 orders of magnitude (on a semi-log scale)
- Will look in more detail later...

Reasons to study photon production

- Gluon PDF is rather indirectly constrained
  - Momentum sum rule
  - Q^2 dependence of PDFs in DIS via DGLAP equations
  - Jet production at colliders via $gg \rightarrow gg$ and $gq \rightarrow gq$, but $qq \rightarrow qq$ dominates at high values of p_T
- QCD Compton process $gq \rightarrow \gamma q$ appears at lowest order only with $q\overline{q} \rightarrow \gamma g$ so looks to provide an ideal way to constrain the gluon PDF
- Intrinsic interest in seeing if QCD properly describes the photon production mechanisms - offers a new way of looking at QCD dynamics
- Photons are essential for certain search strategies at the LHC, *e.g.*, $H \rightarrow 2\gamma$, so photon production must be understood in order to control the backgrounds for such searches

Direct Photon Data

- Initial measurements were done at fixed target energies: $\sqrt{s} = 20 - 40$ GeV
  - With p_T in the 3-10 GeV range this extended the measurements to high values of $x_T = 2p_T/\sqrt{s}$ which is a measure of the parton x being probed in the process
  - Ideal for constraining the gluon PDF, but problems in describing the data surfaced
- Data became available from colliders
  - pp at the ISR with $\sqrt{s} = 44 - 62$ GeV
  - $\overline{p}p$ at CERN and Fermilab with $\sqrt{s} = 540 - 1960$ GeV
  - pp at RHIC with $\sqrt{s} = 200$ GeV
  - pp at the LHC
- Wide kinematic range covered

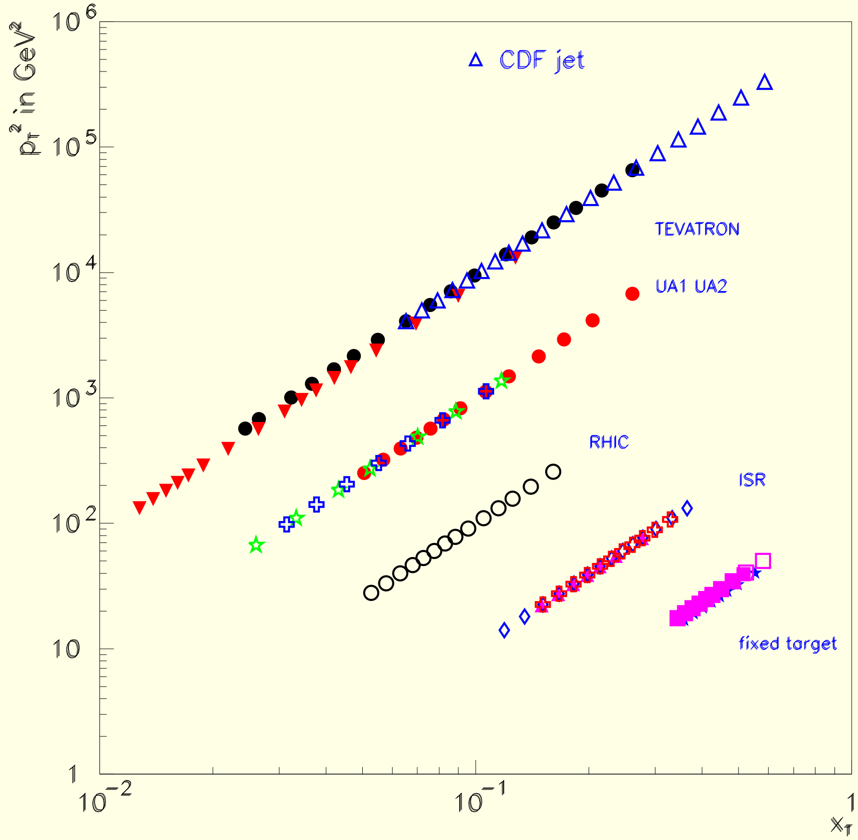
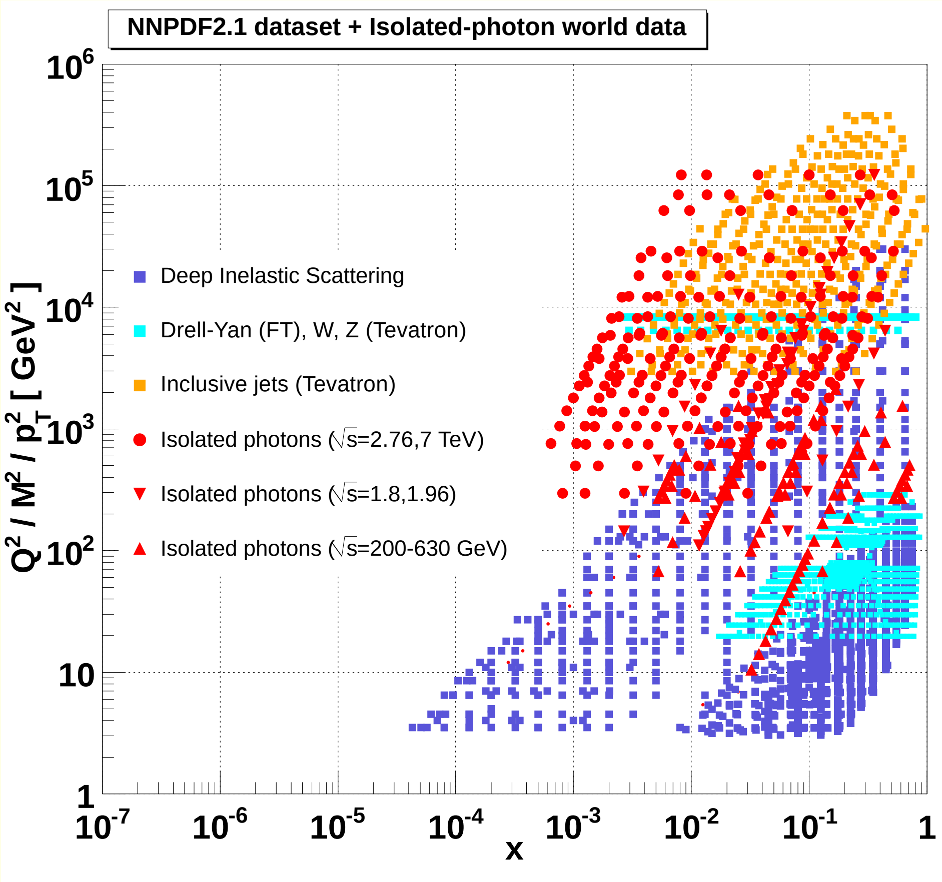
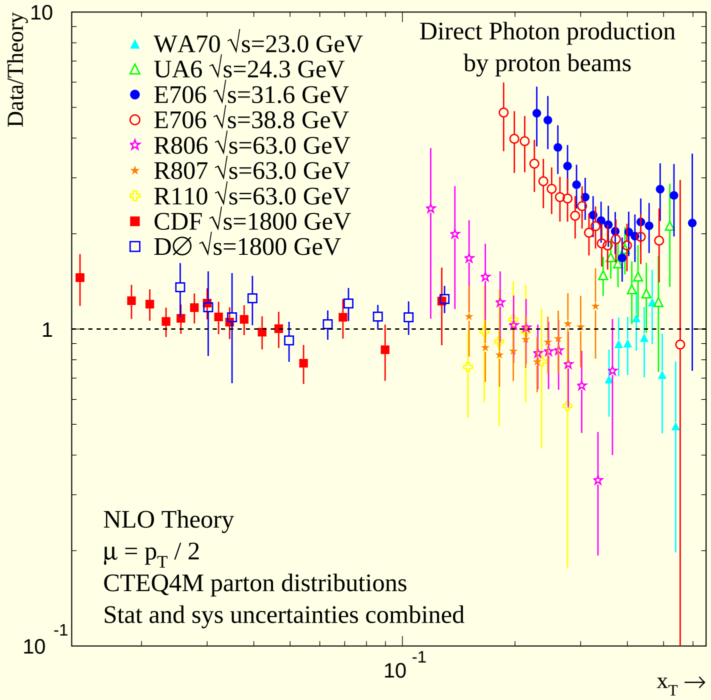


Figure compares kinematic coverage for various direct photon experiments to that for jets at the Tevatron (Aurenche et al, hep-ph/0602133)

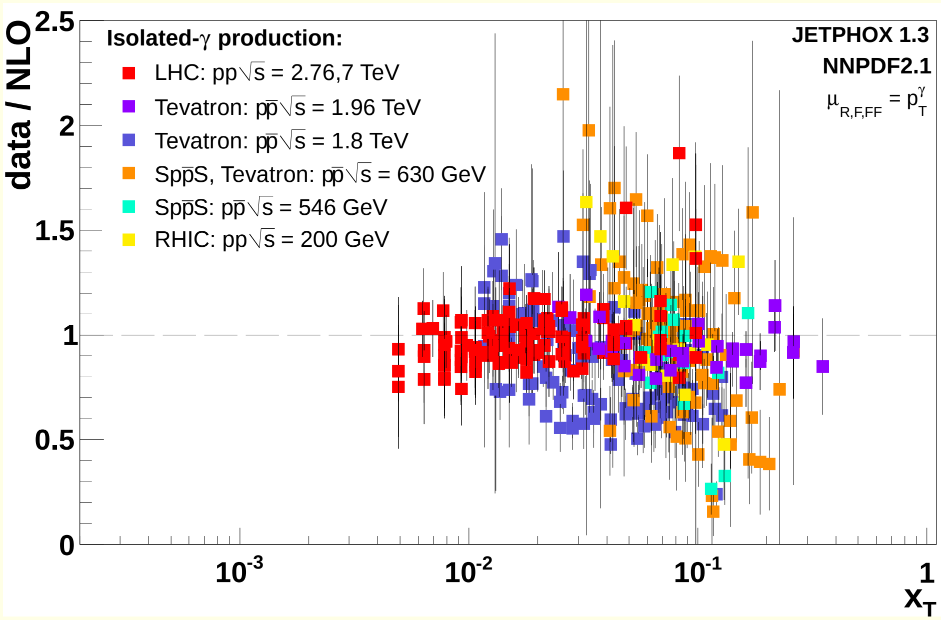


- Figure shows kinematic coverage compared to other hard scattering processes
- Complements jets for constraining the gluon PDF

- Data Sources
  - Data review by W. Vogelsang and M.R. Whalley, J. Phys. **G23**, Suppl. 7A, A1-A69 (1997)
  - Online database at http://durpdg.dur.ac.uk/HEPDATA
- Usual direct photon talk emphasizes problems describing p_T distributions



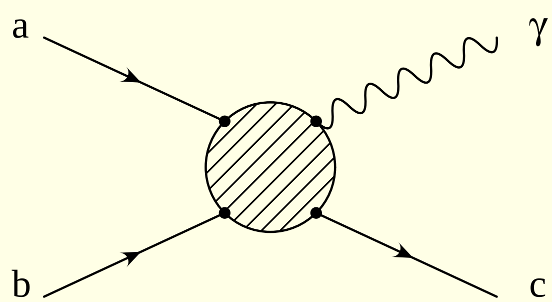
- Scatter in data/theory plot perhaps suggests that there is something wrong with direct photon theory!
- But here is another plot emphasizing newer higher energy data (from d'Enterria and Rojo, arXiv:1202.1762[hep-ph])



- My goal is to examine both the theory and the data to see what works and where improvements are needed

Theory Overview

- Lowest Order: $\mathcal{O}(\alpha\alpha_s)$
  1. $qg \rightarrow \gamma g$ QCD Compton
  2. $q\overline{q} \rightarrow \gamma g$ annihilation
- The single photon invariant cross section is given by a convolution with the beam and target parton distribution functions



$$
\begin{aligned}
d\sigma(AB \rightarrow \gamma + X) \quad &= \quad G_{a/A}(x_a, \mu_F)\, dx_a\, G_{b/B}(x_b, \mu_F)\, dx_b \\
&\quad \frac{1}{2\hat{s}} \overline{\sum_{ab}} |M(ab \rightarrow \gamma c)|^2 d^2 PS
\end{aligned}
$$

- d^2PS denotes two-body phase space and μ_F is the factorization scale

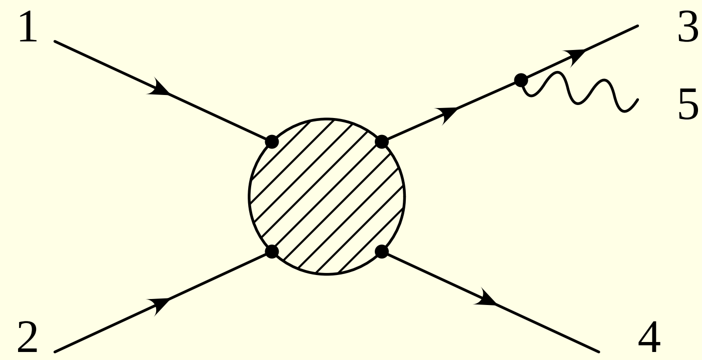
- Important point: the lowest order configuration corresponds to an isolated photon recoiling against a jet
- See the appendix for more details about variables and four-vectors
- Also see the Handbook of Perturbative QCD on the CTEQ web site http://www.cteq.org. The appendix has additional information on how to calculate cross sections for hadronic processes starting at the parton level.

Next-to-Leading Order: $\mathcal{O}(\alpha\alpha_s^2)$

1. one-loop virtual contributions
2. $q\overline{q} \rightarrow \gamma g g$
3. $gq \rightarrow \gamma q g$
4. $qq' \rightarrow \gamma q q'$ plus related subprocesses

- In the next order one sees a new configuration wherein the photon is no longer isolated. Instead, it may be radiated off a high-p_T quark produced in the hard scattering process.

- Consider the subprocess $q(1)q(2) \to q(3)q(4)\gamma(5)$
- Examine the region where $s_{35} = (p_3 - p_5)^2 \approx 0$



$$\overline{\sum}|M(qq \to qq\gamma)|^2 \approx \frac{\alpha}{2\pi}\, P_{\gamma q}(z)\, \frac{1}{s_{35}} \overline{\sum}|M(qq \to qq)|^2$$

- An internal quark line is going on-shell signalling long distance physics effects
- Gives rise to a collinear singularity
- Can factorize the singularity by introducing a *photon fragmentation function*

Photon Fragmentation

- Photon is accompanied by jet fragments on the *same* side
- Factorize the singularity and include it in the bare photon fragmentation function
- Sum large logs with modified Altarelli-Parisi equations

$$Q^2 \frac{dD_{\gamma/q}(x,Q^2)}{dQ^2} = \frac{\alpha}{2\pi} P_{\gamma q} + \frac{\alpha_s}{2\pi} \left[D_{\gamma/q} \otimes P_{qq} + D_{\gamma/g} \otimes P_{gq} \right]$$

$$Q^2 \frac{dD_{\gamma/g}(x,Q^2)}{dQ^2} = \frac{\alpha_s}{2\pi} \left[\sum_q D_{\gamma/q} \otimes P_{qg} + D_{\gamma/g} \otimes P_{gg} \right]$$

- As with hadron PDFs and fragmentation functions, can't perturbatively calculate the fragmentation functions, but the scale dependence is perturbatively calculable
- Note the $P_{\gamma q}$ splitting function - represents the pointlike coupling of the photon to the quark in $q \to \gamma q$

Fragmentation Component

- The situation has become more complex
- Expect to see two classes of events
    1. Direct (or pointlike) - no hadrons accompanying the photon
    2. Fragmentation (or bremsstrahlung) - photon is a fragment of a high-p_T jet. Part of the fragmentation function is perturbatively calculable.
- Expect (1) to dominate at high-p_T since the energy is not shared with accompanying hadrons.
- The $P_{\gamma q}$ splitting function gives rise to the leading high Q^2 behavior going as $\alpha \log(Q^2/\Lambda^2) \sim \frac{\alpha}{\alpha_s}$ (see the Appendix for a derivation)

So, to our list of contributions add those involving photon fragmentation functions

- $\mathcal{O}(\alpha\alpha_s) : \frac{d\sigma}{d\hat{t}}(ab \to cd) \otimes D_{\gamma/c}$ or γ/d
- $\mathcal{O}(\alpha\alpha_s^2) : \frac{d\sigma}{d\hat{t}}(ab \to cde) \otimes D_{\gamma/c}$ or γ/d or γ/e

Some Comments

- Photons can be produced as fragments of jets, as is also the case for particles
- Photon production therefore involves all of the subprocesses relevant for jet or particle production
- In addition, one also has the pointlike production processes

Photon production is *more* complicated than jet production, not *less*

Next-to-leading-order Calculations

- Have to integrate over unobserved partons. There are regions of phase space where partons can become parallel to each other (collinear) or soft. Both regions are singular.
- Usually use dimensional regularization to regulate the divergences

Two types of programs exist

1. Phase space integrations done symbolically so expressions for the integrated parton-level subprocess cross sections exist. Integrations over the parton momentum fractions x_a, x_b, and z_c done numerically. This approach is suitable for the single photon inclusive cross section.
2. All integrations done via Monte Carlo
   - Phase space slicing method
   - Subtraction method

With Monte Carlo programs one can examine correlations between the photon and other partons in the final state.

Short summary of two-body kinematics

The following (LO) relations are useful when trying to understand the regions of the parton variables which are important for specific observables. More information is available in the appendix.

Consider a photon and a jet produced with approximately balancing p_T and (pseudo)rapidities η_γ and η_{jet} in the hadron-hadron center-of-mass system.

γ-jet invariant mass M: $M^2 = x_a x_b s$ γ-jet rapidity Y: $Y = \frac{1}{2} \ln \frac{x_a}{x_b}$

Scattering angle in the γ-jet rest frame: $\cos\theta^* = \tanh\left(\frac{\eta_\gamma - \eta_{jet}}{2}\right)$

Parton momentum fractions

$$x_a = x_T(e^{\eta_\gamma} + e^{\eta_{jet}})/2 = Me^{Y}/\sqrt{s}$$
$$x_b = x_T(e^{-\eta_\gamma} + e^{-\eta_{jet}})/2 = Me^{-Y}/\sqrt{s}$$

Note: for $Y \approx 0$ one is sensitive to $x_a \approx x_b \approx M/\sqrt{s}$. For $\eta_\gamma \approx \eta_{jet} \approx 0$ one has $x_a \approx x_b \approx x_T$, but this is only a guide, since the cross sections will involve some integrations.

Comparison to Data

- First examine data which yield information on the underlying parton subprocesses
- Distinctive predictions occur for
  - γ-jet angular distribtutions
  - Photon cross section scaling behavior

γ-jet angular distributions

- QCD Compton and annihilation subprocess both behave as

$$\frac{d\sigma}{d\hat{t}} \sim (1 - \cos(\theta^*))^{-1} \text{ as } \cos(\theta^*) \to 1$$

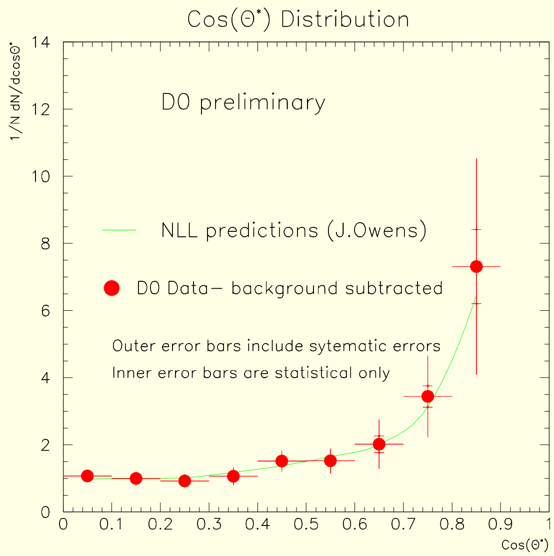
- Other parton-parton scattering subprocesses ($qq \to qq, qg \to qg, gg \to gg$, etc.) behave as

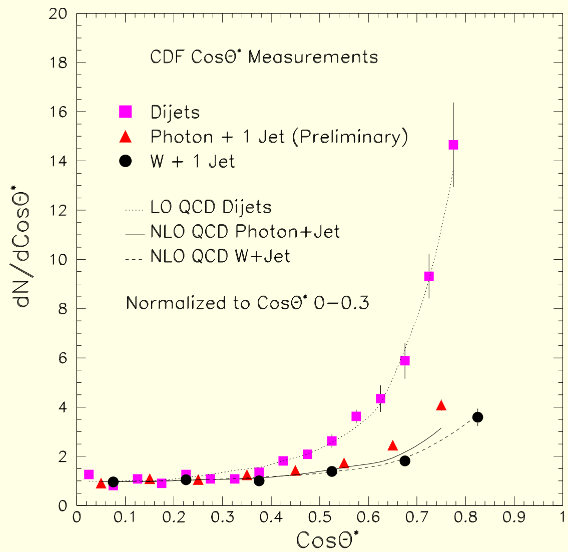
$$(1 - \cos(\theta^*))^{-2}$$

- This means that the γ-jet angular distribution should be flatter than that observed in jet-jet final states.
- See the appendix for a derivation of these relations
- Also, see the appendix of the Handbook of Perturbative QCD - available on the CTEQ website www.cteq.org
- Exercise: derive the previously given relations for the partonic variables based on two-body kinematics

Direct Measurement of the γ-jet angular distribution

- Measuring both η_γ and η_{jet} allows one to reconstruct $\cos\theta^* = \tanh\left(\frac{\eta_\gamma - \eta_{jet}}{2}\right)$
- Both DØ and CDF have measured the γ-jet angular distribution





- Both experiments observe a shape consistent with expectations
- Direct photon production is dominated by subprocesses which yield a flatter angular distribution than is observed for dijet production

Event Structure

- Lowest order Compton and annihilation subprocesses correspond to an isolated photon recoiling against a jet
- Fragmentation contributions add a component where the photon is accompanied by the hadronic fragments of the parent jet
- Expect to see fewer hadrons on the photon side of the event than would be the case with a hadronic trigger
- Results from UA-2 (R. Ansari et al., Z. Phys. **C41**, 395(1988)) show the reduced number of same side hadrons for photon triggers
- Note that the amount of hadronic activity for photon triggers is larger than that for W production, while being smaller than that for hadronic triggers
- Consistent with a superposition of direct and fragmentation components

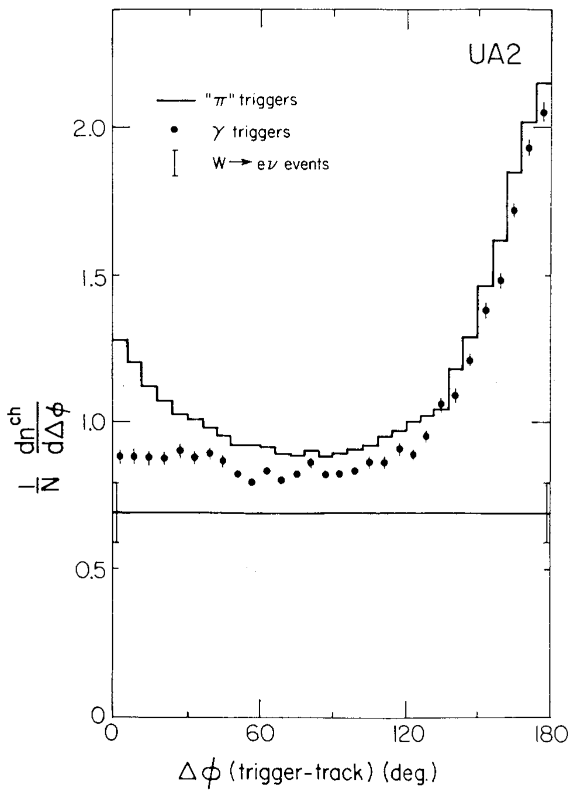
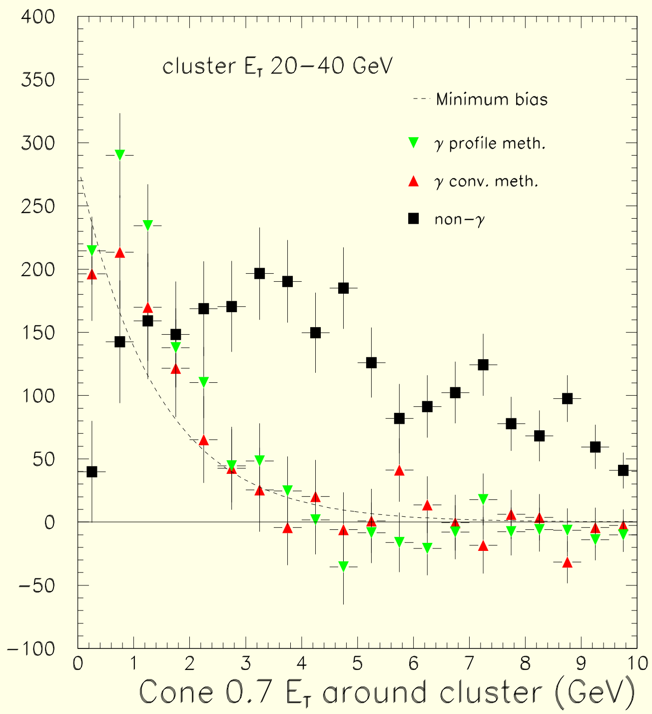

UA2
"π" triggers
γ triggers
W → eν events
2.0
1.5
1.0
0.5
0
0
60
120
180
$\frac{1}{N}\frac{dn^{ch}}{d\Delta\phi}$
Δφ (trigger-track) (deg.)

Figure 12

Similar result seen in preliminary CDF data

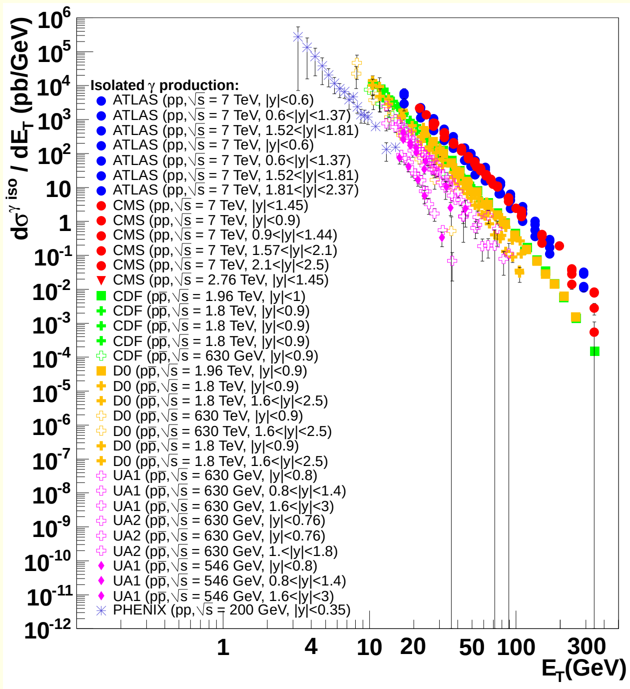


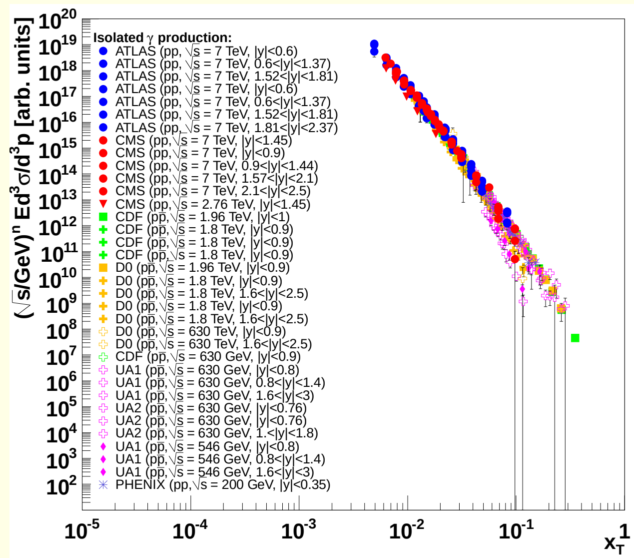
- E_T inside a cone of radius 0.7 about the photon looks just like minimum bias case - and not at all like that for hadronic events
- Difficult to identify fragmentation component due to background from beam fragments

Scaling

- Invariant cross section $E\frac{d^3\sigma}{d^3p}$ has dimensions of E^{-4} in natural units
- If scaling holds exactly, then mutiplying by p_T^4 should yield a dimensionless function which should depend only in dimensionless variables like $x_T = \frac{2p_T}{\sqrt{s}}$ and θ (or the photon rapidity)
- For direct photons, the factor of α_s and the scaling violations of the initial state PDFs cause the cross section to decrease slightly at fixed x_T and rapidity, so the power is slightly higher than 4
- Hadronic jet production has a somewhat higher exponent since there are two factors of α_s in lowest order
- Hadronic production of hadrons has an additional fragmentation function that raises the power even further (to almost 8 at fixed target energies)

Inclusive and scaled inclusive isolated photon cross sections





From d'Enterria and Rojo arXiv:1202.1762

Recap

- Thus far we have seen that
  - Photons are produced with the expected γ-jet angular distribution which is flatter than that for dijets
  - The cross section scales as expected once QCD effects have been accounted for
  - Photons appear to be accompanied by fewer hadrons than for purely hadronic triggers, although more than for W events
  - All of these items suggest that the basic mechanism for producing high-p_T photons is as expected from QCD
  - Is there any other way to probe the dynamics of photon-parton interactions that tests these same mechanisms?

Yes! Jet Photoproduction

Jet Photoproduction

- The same subprocesses are involved, but with the photon crossed to the initial state

$$qg \to \gamma q \quad \Rightarrow \quad \gamma q \to qg \ \text{ etc.}$$

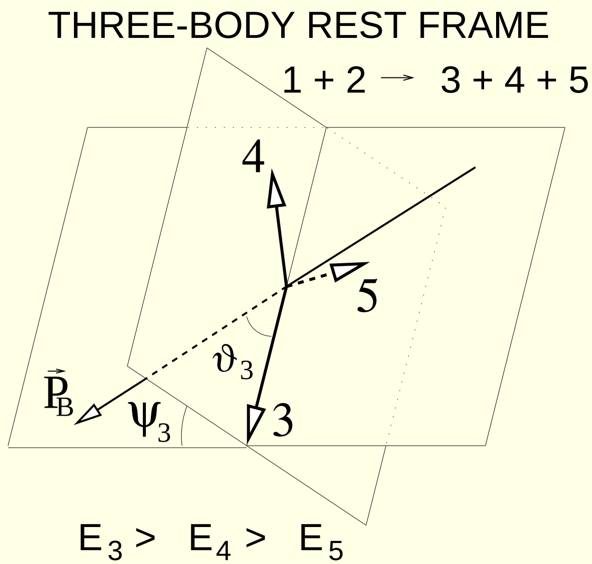
- Role of photon fragmentation is now in the initial state where it is treated using a photon PDF
- Details may be found in my CTEQ 2001 summer school lecture at www.cteq.org

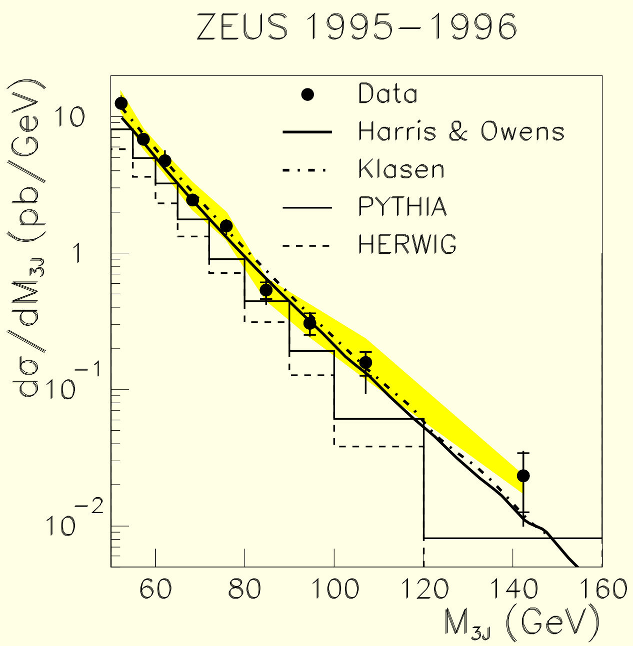
Examples

- Consider several different processes in order of increasing complexity concerning the interactions of the photons
- Photoproduction of three-jet final states
  - Calculation based on $\mathcal{O}(\alpha\alpha_s^2)$ subprocesses
    1. $\gamma q \to qgg$
    2. $\gamma g \to q\overline{q}g$
    3. $\gamma q \to qq'\overline{q}'$
  - Consider three identified high-p_T jets with high three-jet mass
  - Only looking at the pointlike interaction of the photon - no resolved contribution
  - Calculation at this level is lowest order (LO) and also leading-log (LL)
  - Compare to data from the ZEUS Collaboration (hep-ex/9810046)

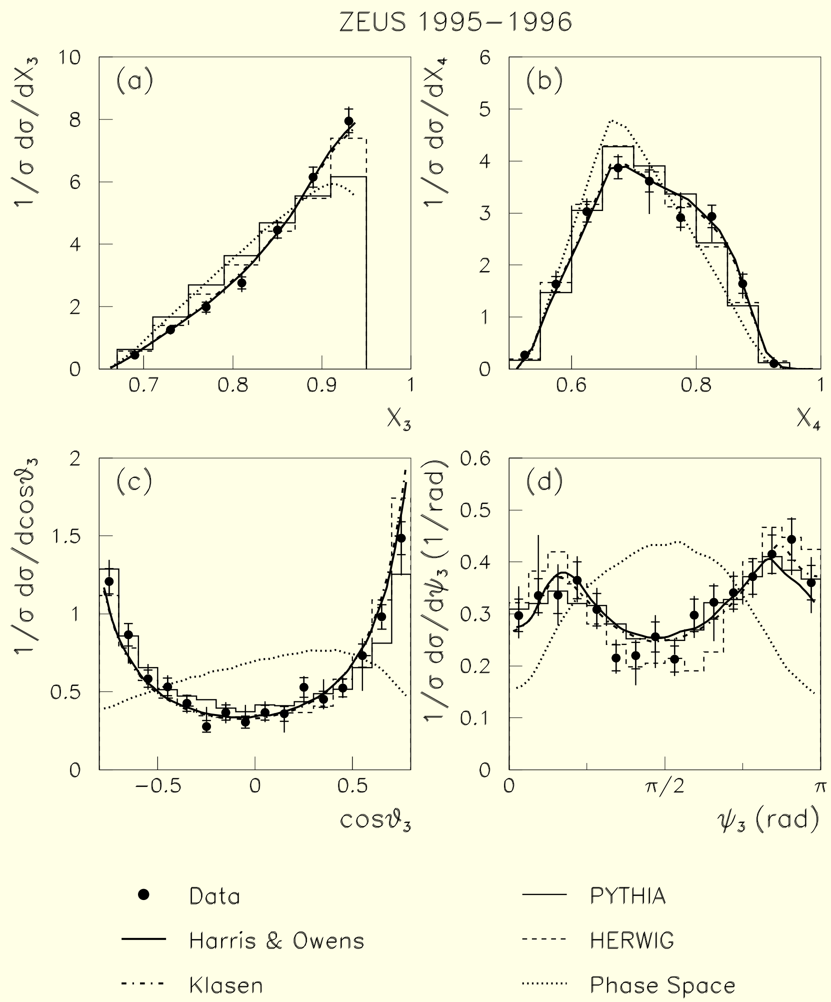
Examine the following

- three-jet mass M_{3j}
- energy fractions: jets ordered in energy with $E_3 > E_4 > E_5$ and energy fractions defined as $x_i = 2E_i/M_{3j}$
- jet angular distributions





- Energy fractions are ordered with $E_3 > E_4 > E_5$
- $\cos\theta_3, \phi_3$ describe the jet orientations



Observe good agreement between matrix element calculations and the data and also between shower Monte Carlo calculations and the data

Single jet and Dijet Photoproduction

- As noted earlier, there will be two components
  - direct or pointlike - photon contributes directly to the hard scattering
  - resolved - both photon and proton parton distributions convoluted with parton-parton scattering subprocesses
  - at $\mathcal{O}(\alpha\alpha_s^2)$ and beyond, specific diagrams can contribute to both. Need an experimental definition to distinguish between the two classes. For example the ZEUS Collaboration has used the following for dijet final states:

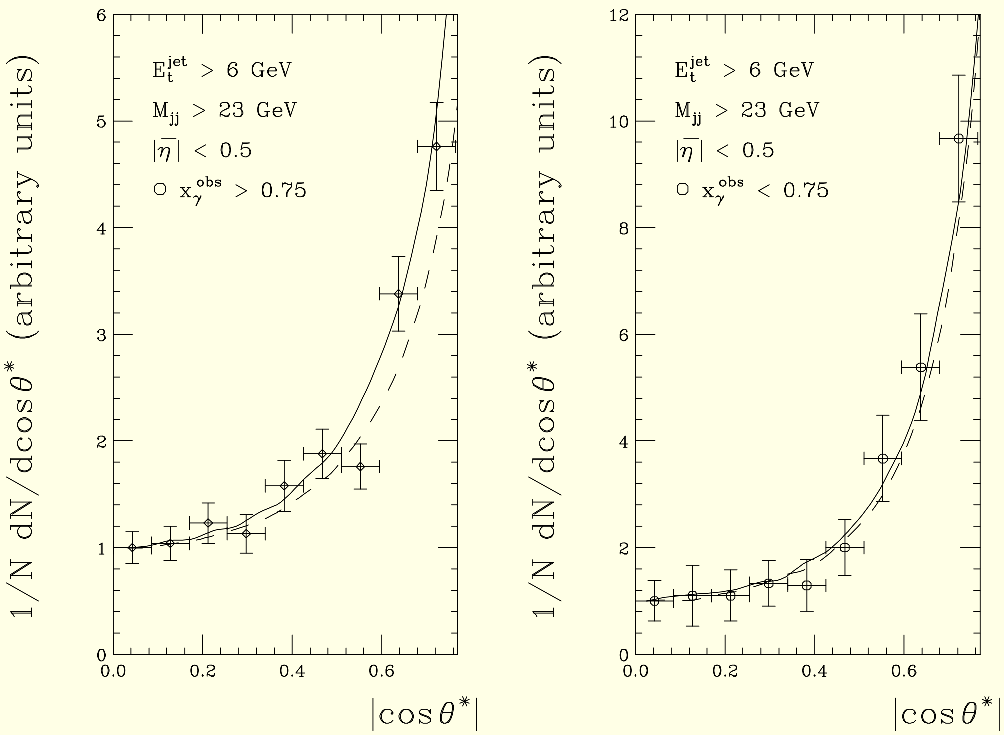
$$x_\gamma^{obs} = (E_T^{jet_1} e^{-\eta_{jet_1}} + E_T^{jet_2} e^{-\eta_{jet_2}})/2E_\gamma$$

  - positive η corresponds to the direction of the proton
  - consider two samples with $x_\gamma^{obs} > .75$ (mostly direct) and $x_\gamma^{obs} < .75$ (mostly resolved)

Dijet angular distribution

- $\gamma q \to qg$ and $\gamma g \to q\overline{q}$ are fermion exchange processes which have an angular distribution going as $1/(1 - \cos\theta)$ in the dijet center of mass system
- typical boson exchange processes in the resolved part $qq \to qq, qg \to qg, gg \to gg$ etc., behave as $1/(1 - \cos\theta)^2$ which is much steeper

Compare to data from the ZEUS Collaboration (M. Derrick *et al.*, Phys./ Lett. B**384**, 401 (1996).)
Curves from Harris and Owens, Phys. Rev. D**56**, 4007 (1997). Dashed lines are LO and solid are NLO.



Note the different scales on these normalized distributions - the dominantly resolved sample has a much steeper distribution

Aside

Why is it possible to separate the direct and resolved components in jet photoproduction, yet it is difficult to isolate the fragmentation or bremsstrahlung component in direct photon production?

- In jet photoproduction one has a sample with two identified jets in the final state. These can be used to reconstruct x_γ and x_p directly.
- Higher order effects and smearing due to beam fragments entering the jet cones smear out the direct component peak at $x_\gamma = 1$, but one can still define two samples of events which are dominantly resolved or dominantly direct.

- In contrast, for direct photon production
  - Some of the fragmentation contribution is removed by isolation cuts used in the trigger (see later)
  - For fragmentation events, one must reconstruct x_a, x_b, and the fragmentation variable z_c. There is not enough information.
- As shown earlier, direct photon events have accompanying hadronic energy that is indistinguishable from the minimum bias case.
- This means that one can not use the accompanying hadronic energy as a flag for fragmentation events.

Lessons Learned

- Photoproduction data support the view that photons interact with partons as expected in QCD
- Two-component picture of the dynamics
  - Direct (point-like) and resolved components in jet photoproduction - resolved piece uses a photon parton distribution to resum large logs coming from configurations where partons are produced collinear with the incoming photon
  - Direct (point-like) and fragmentation components in direct photon production - fragmentation piece uses a photon fragmentation function to resum large logs coming from configurations where the photon is a fragment of a jet and there are partons which are produced collinear with the outgoing photon.
- So, to this point it seems that the theory of direct photon production is supported by the data.

Then, what is all the fuss about?

Single Photon Inclusive Cross Section

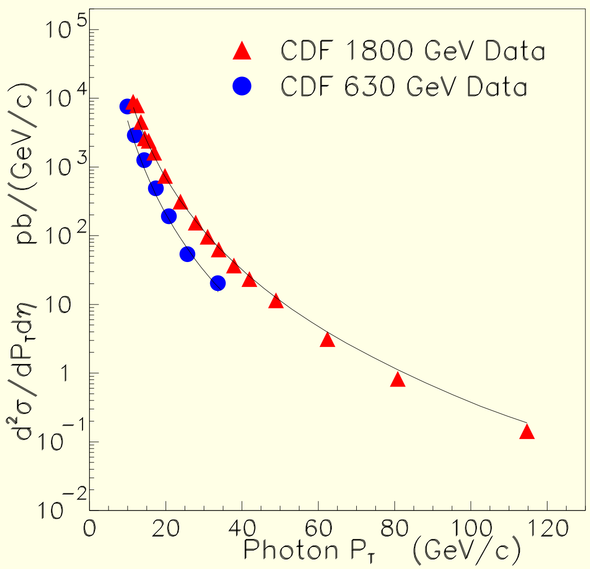
- Integrate over all partons, leaving only the photon as being observed
- In some sense this is the hardest measurement to interpret since if there is a disagreement between theory and experiment one has few clues as to the origin - all other details have been integrated out
- On the other hand, the inclusive nature of the observable makes the calculation easier - you don't have to model the distributions that are integrated over. Rather, you only have to get the integral over the distribution correct.
- Can measure - and calculate - the photon p_T and η distributions
- p_T spectrum is steeply falling so that it is especially susceptible to resolution smearing effects on the experimental side and approximations made in the theoretical calculations

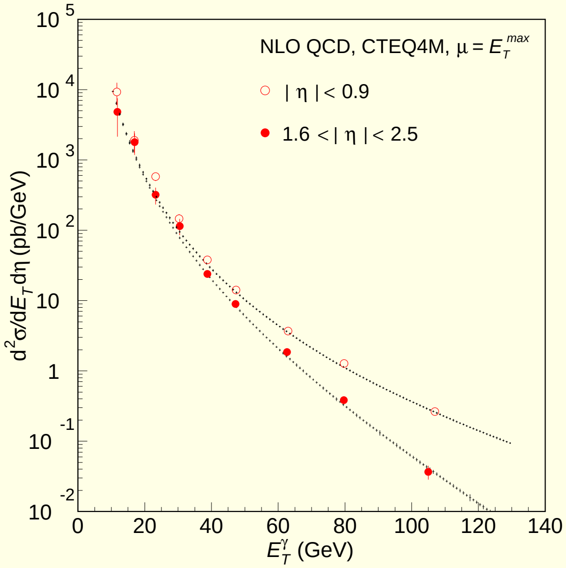
Calculations use the same techniques as described previously

- Analytic calculations result in faster running programs, since many of the integrations have already been done and only the convolutions with PDFs and fragmentation functions need to be done.
- Monte Carlo based programs are able to investigate the effects of the photon isolation cuts to be discussed below, although approximation techniques exist so that the analytic programs can also invoke these cuts.

Tevatron data

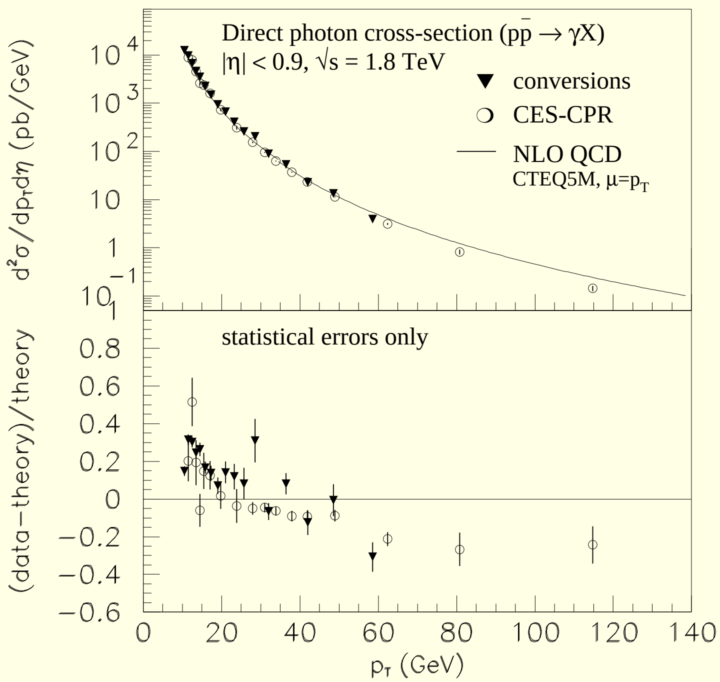
CDF: $\sqrt{s} = 1800, 630$ GeV, $|\eta| < 0.9$
DØ: $\sqrt{s} = 1800, 630$ GeV, $|\eta| < 0.9, 1.6 < |\eta| < 2.5$

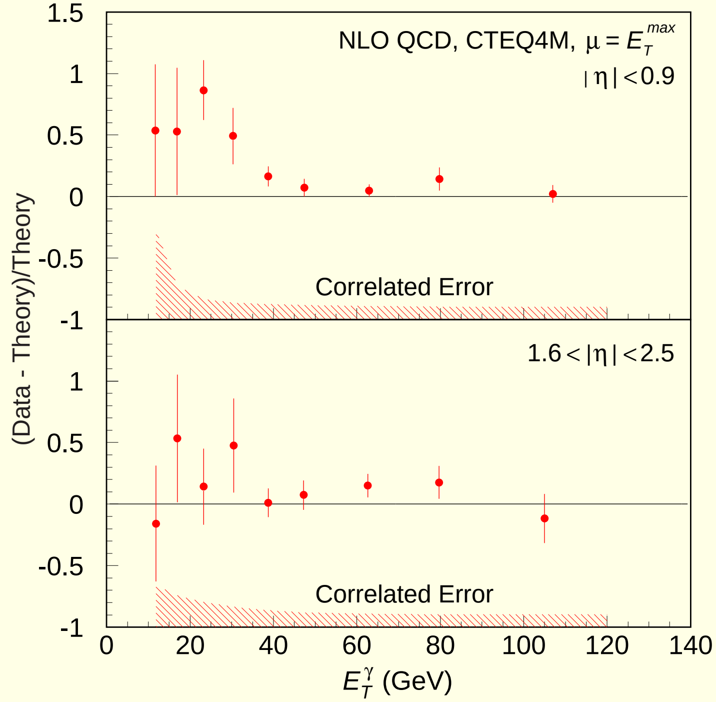




- Agreement looks good when plotted on a logarithmic scale
- Confirms expectation that the QCD description of direct photon production is correct
- But what if we look closer ...?

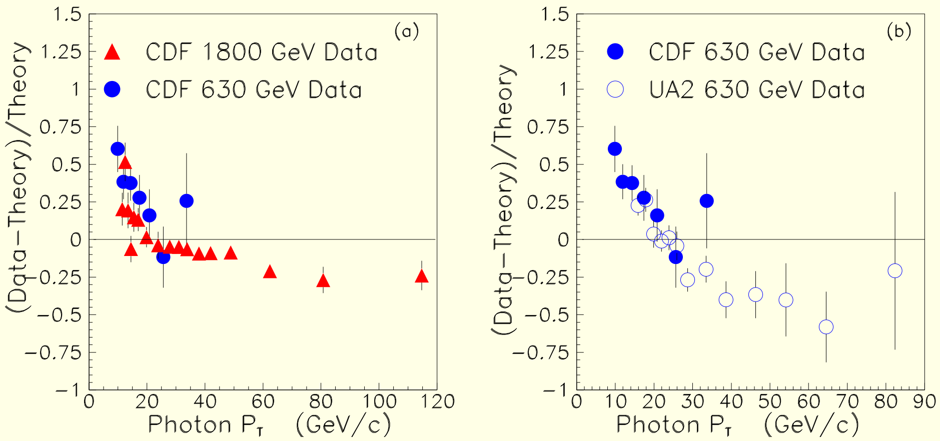
Look on a linear scale . . .



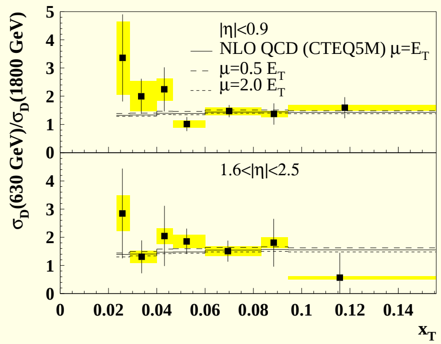


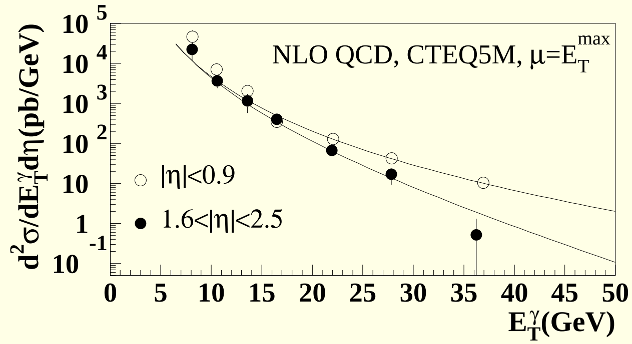
Both CDF and DØ see an excess at the low p_T end in the central region

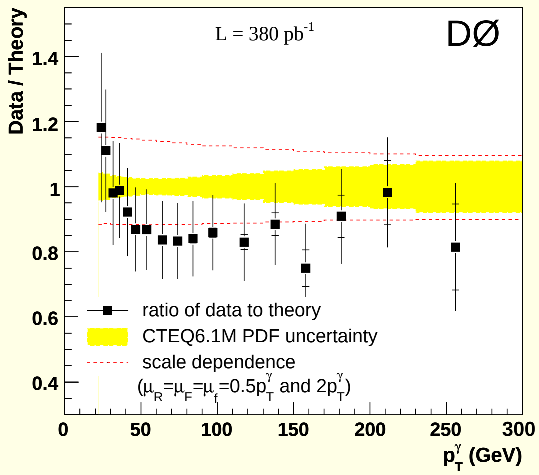
- Problem seen by CDF at both 1800 and 630 GeV
- Excess occurs at low p_T, not at fixed x_T, so the solution can not be a simple adjustment of the PDFs
- Effect also seen by UA-2



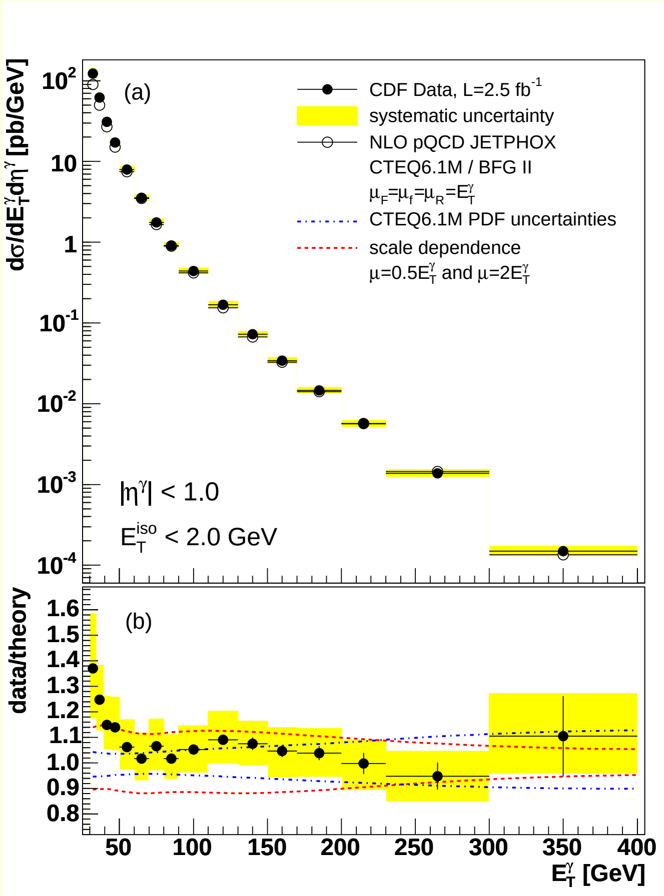
Also seen by DØ at 630 and 1960 GeV





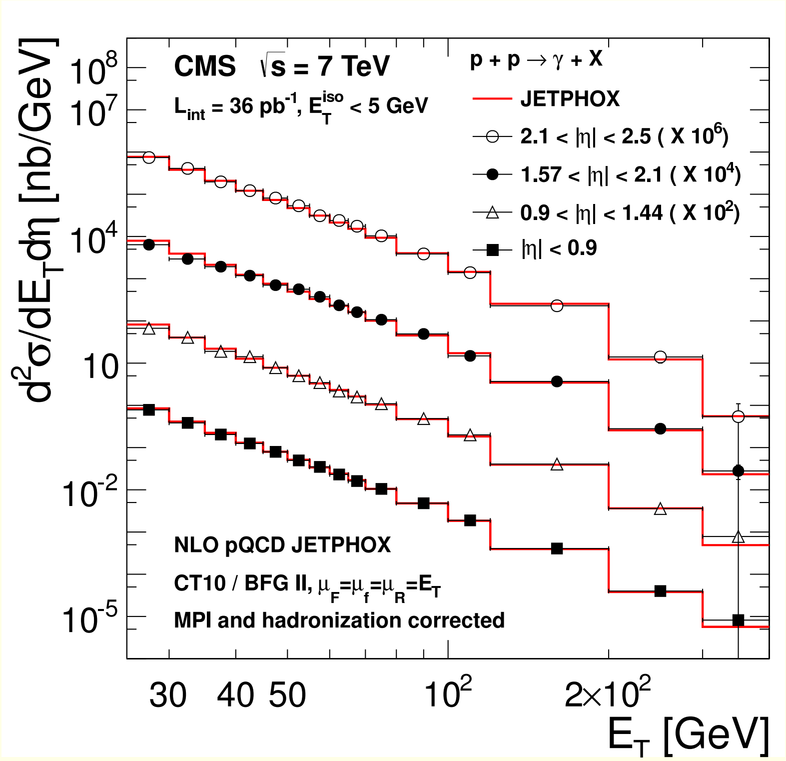


Latest Run II CDF data shows the same effect

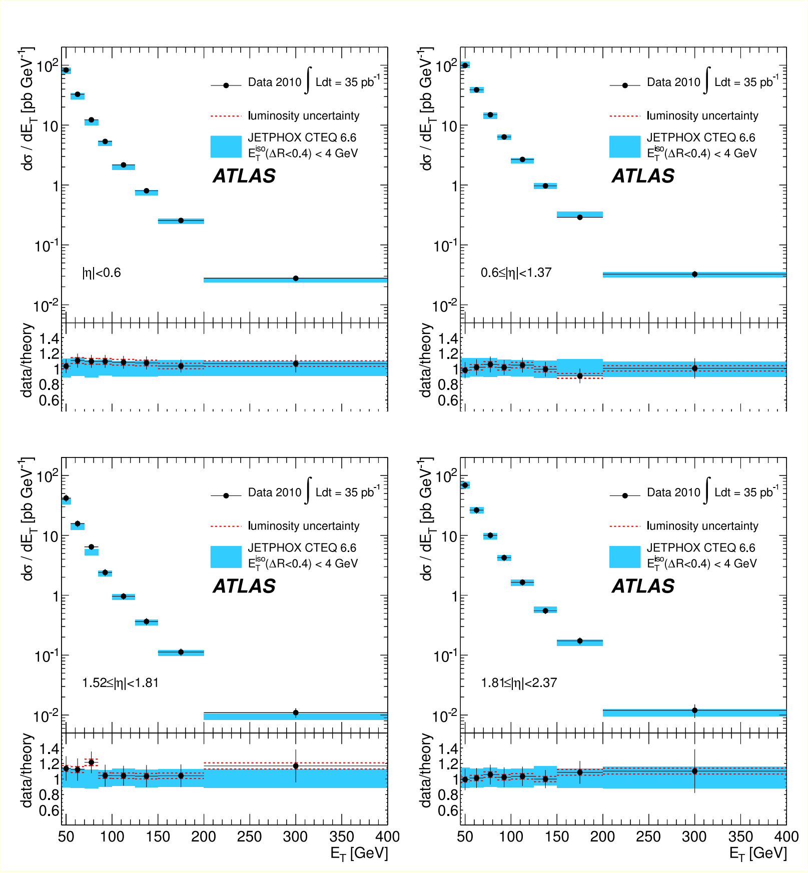


Effect appears below about 50 GeV transverse momentum - agreement is good above that

LHC results



- Results from CMS - arXiv:1108.2044[hep-ex]
- Good agreement for all rapidity and p_T bins
- No low P_T excess



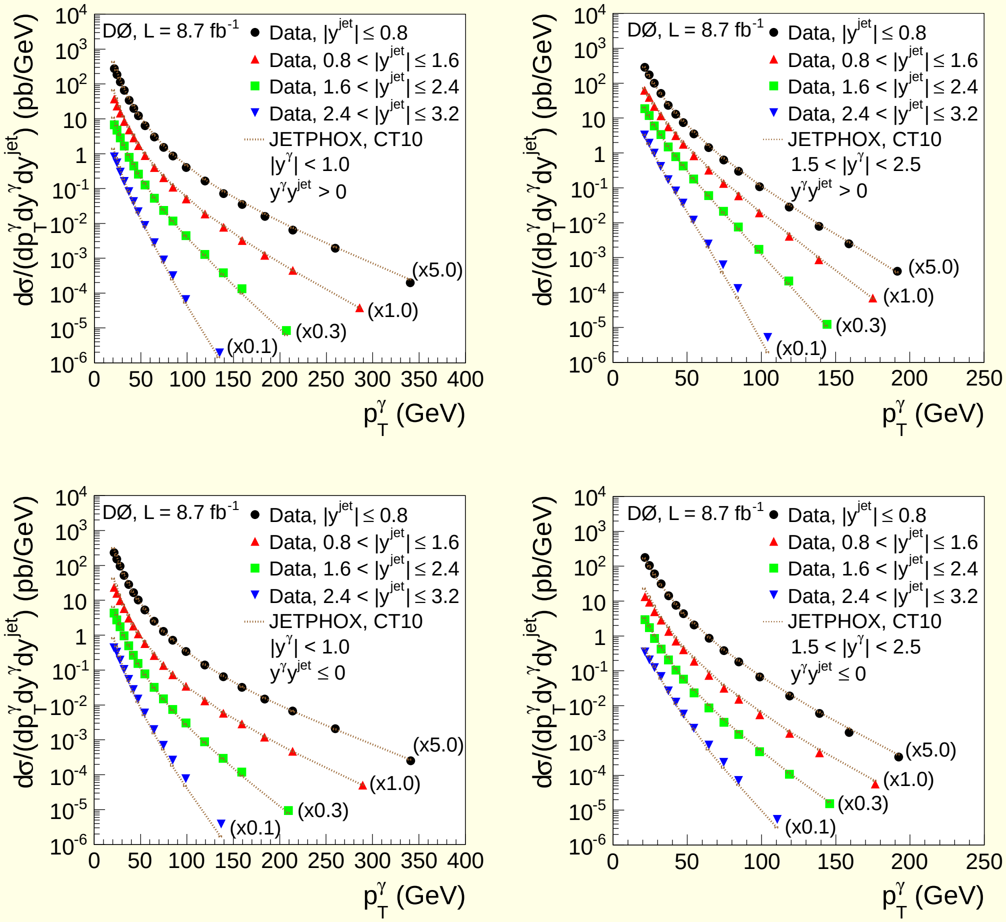
- ATLAS results: arXiv:1108.0253[hep-ex]
- Good agreement over the full kinematic range
- No low-p_T excess

Photon + jet cross sections

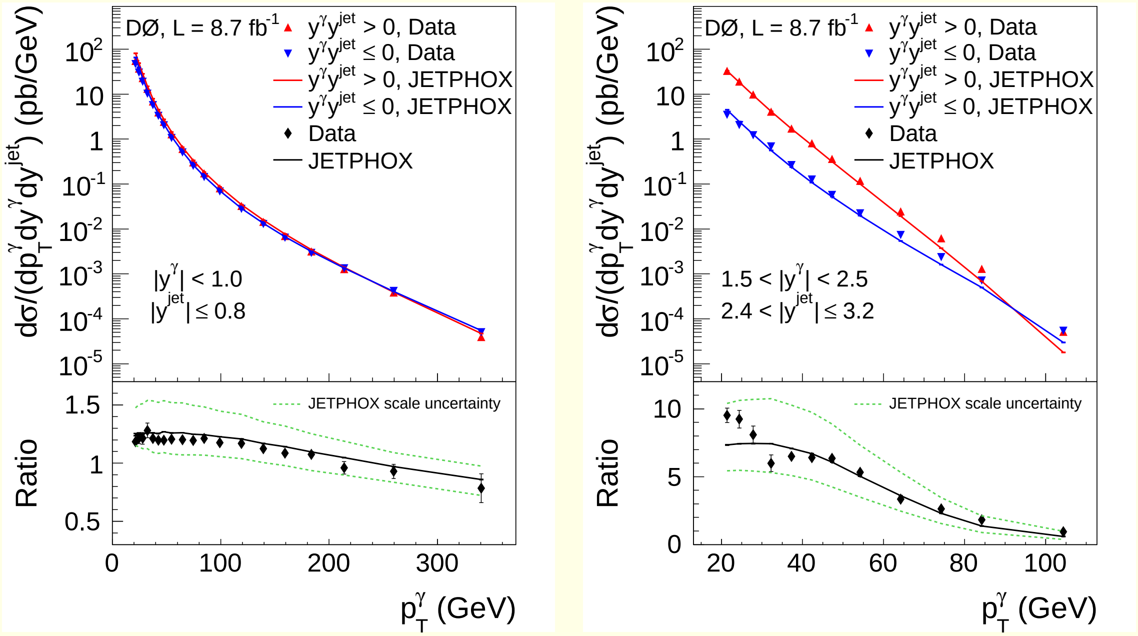
- Additional constraint from fixing the rapidity of the recoiling jet provides a more stringent test of the underlying dynamics
- $x_{\substack{a \\ b}} = \frac{x_T}{2}(e^{\pm\eta_\gamma} + e^{\pm\eta_{jet}})$
- $\cos\theta^* = \tanh(\frac{\eta_\gamma - \eta_{jet}}{2})$

Basic kinematics - two regions often used for comparisons

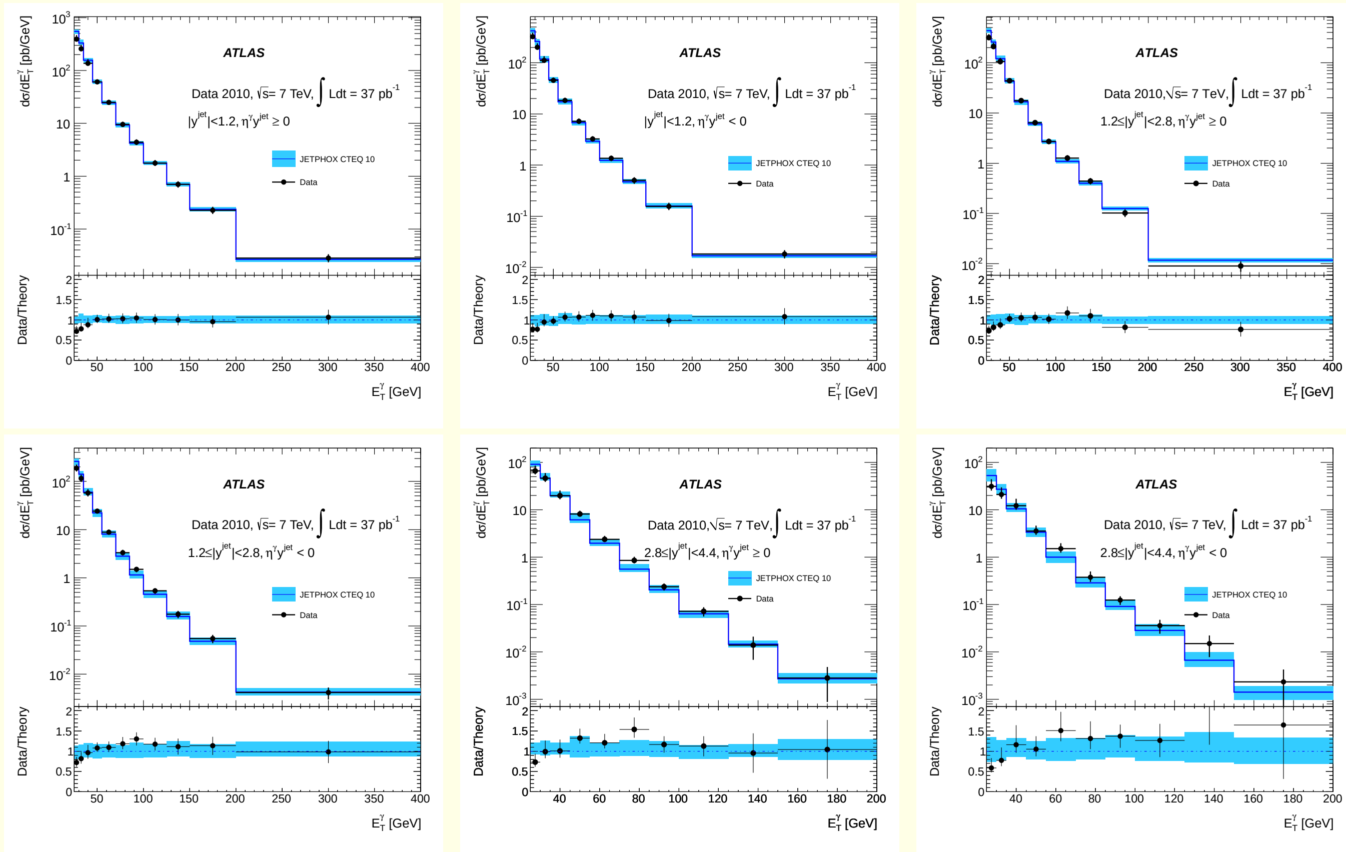
- Both photon and jet rapidities the same - subprocess scattering angle is near 90 degrees
- Photon and jet rapidities have opposite signs - subprocess scattering angle is smaller



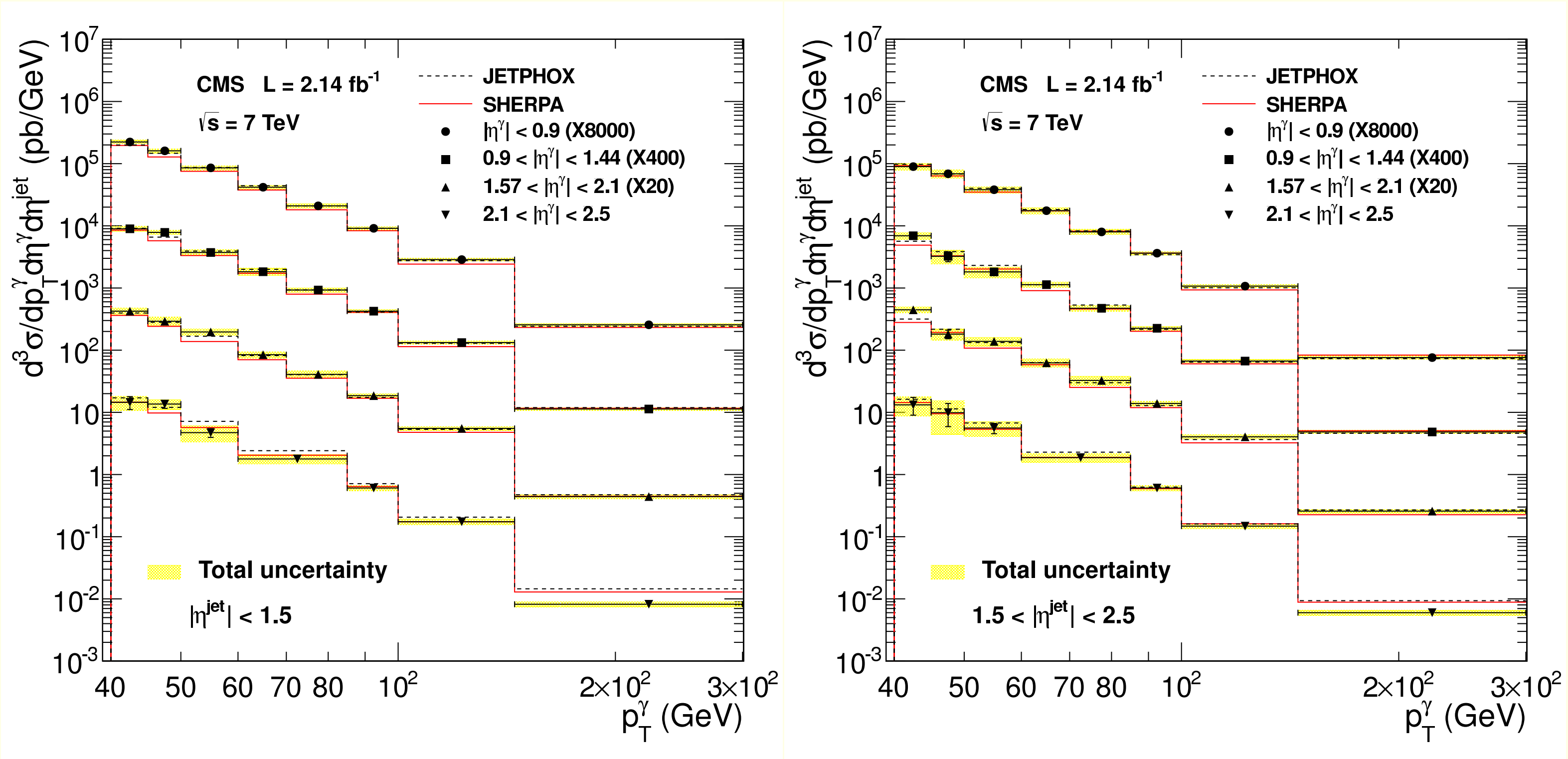
- D0 data from arXiv:1308.2708[hep-ex]
- Good agreement seen between theory and experiment



- Good agreement between theory and experiment for cross section ratios in different rapidity bins
- Suggests that the underlying parton angular distributions are well described



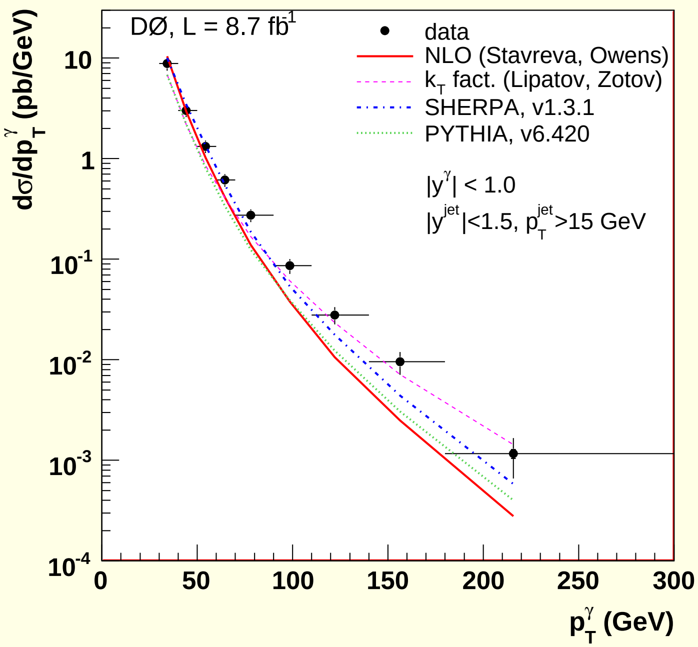
- ATLAS Collaboration from arXiv:1203.3161[hep-ex]
- $|\eta^{\gamma}| < 1.37$
- Good agreement seen between theory and experiment

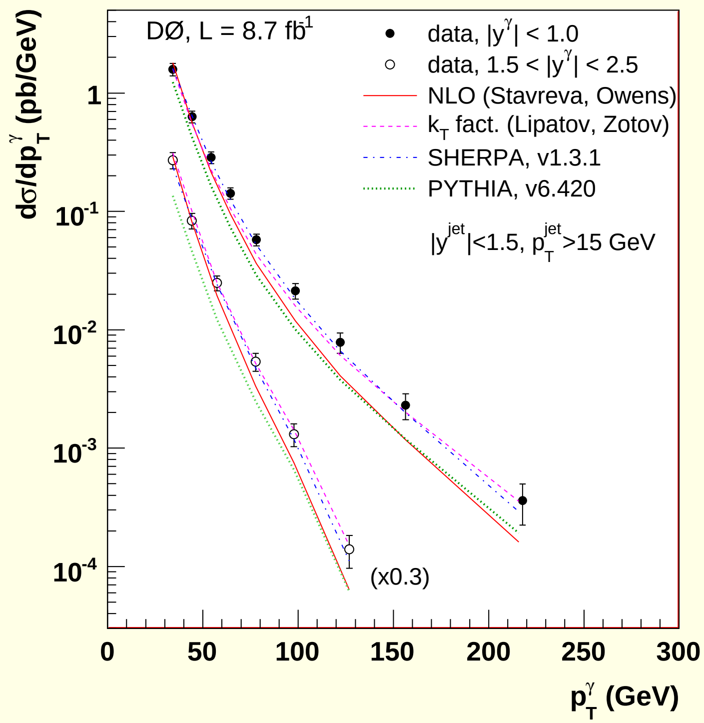


- CMS Collaboration from arXiv:1311.6141[hep-ex]
- Two jet rapidity bins, four photon rapidity bins
- Good agreement between theory and experiment

γ + heavy quark jet

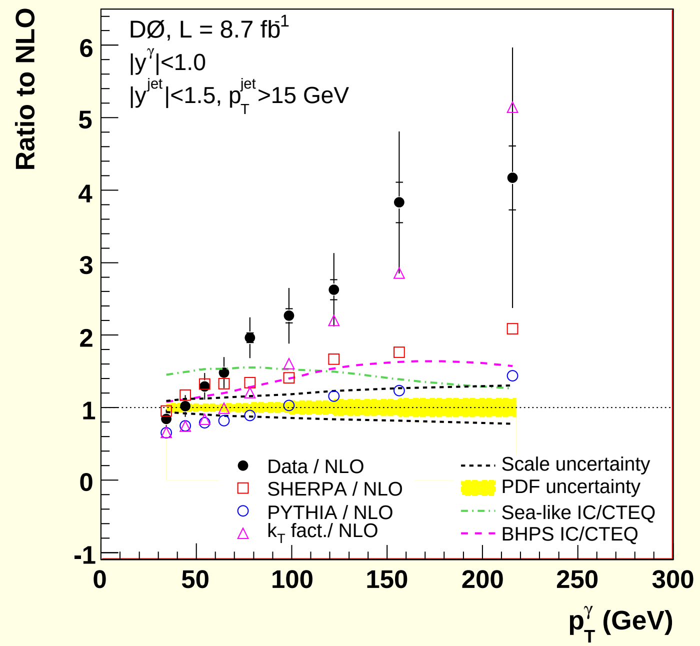
- Use the photon to study properties of heavy quark PDFs, production mechanisms
- Treat heavy quarks in the initial state as having their own (perturbatively calculated) PDFs
- Lowest order is $Qg \to \gamma Q$
- Photon couples to charge, so one expects larger charm than bottom cross sections in addition to the relative sizes of the PDFs
- NLO calculation available, as well: Stavreva and Owens, arXiv:0901.3791[hep-ph]
- Data available from the D0 Collaboration: arXiv:1210.5033[hep-ex] (charm) and arXiv:1203.5865[hep-ex](bottom)





- Good agreement at the lower p_T end for both c(left) and b(right) jets
- Theory falls below the data at the high-p_T end - more so for charm than for bottom

Intrinsic Charm



- Intrinsic charm could add a nonperturbative contribution to the perturbatively generated charm PDF
- Could increase the cross section at high p_T values
- Several candidate models do not yield a sufficient increase

Other Possibilities

- NLO may be insufficient - yet higher orders may provide more ways of getting heavy quarks in jets
- Example - in NLO one encounters the subprocess $q\bar{q} \rightarrow \gamma Q\bar{Q}$ which essentially generates heavy quarks in a gluon jet
- Contribution is large and one must got to NNLO to get the $\mathcal{O}(\alpha_f)$ corrections to this subprocess
- There is room for more work here...

Inclusive versus isolated cross sections

- In order to define a photon signal, certain cuts are required to limit the hadronic energy in a region near the candidate electromagnetic signal
- Algorithms are different for each experiment but there are similarities amongst the various treatments

$\Rightarrow$ fragmentation component is reduced

- Expect fragmentation to be most important at the low p_T end of the distribution
- Fragmentation provides only a portion of the available energy to the photon. The pointlike or direct subprocesses are more efficient and so dominate at high values of p_T

Specific Example - CDF cone algorithm

- Require that there be less than 1 GeV of hadronic transverse energy in a cone of radius
$$\mathcal{R} = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} < 0.4$$
about the direction of the photon.

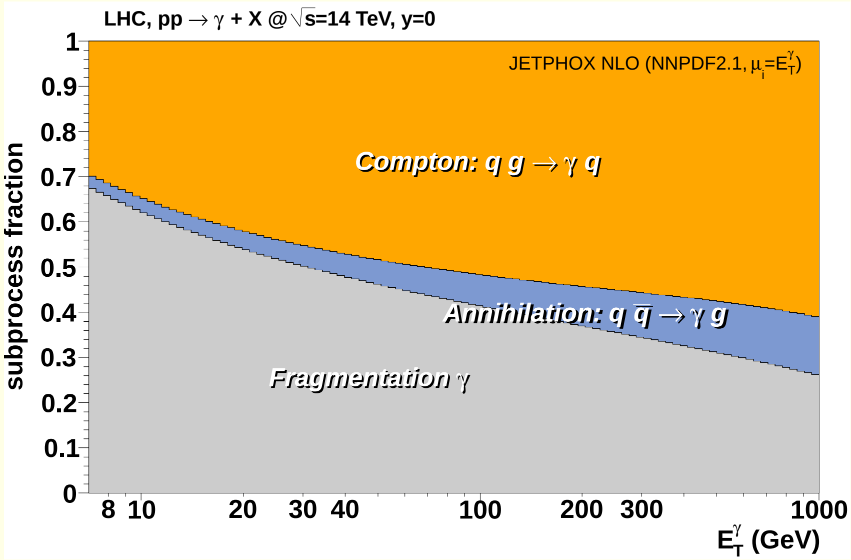
Theoretical modeling (phase space slicing method)

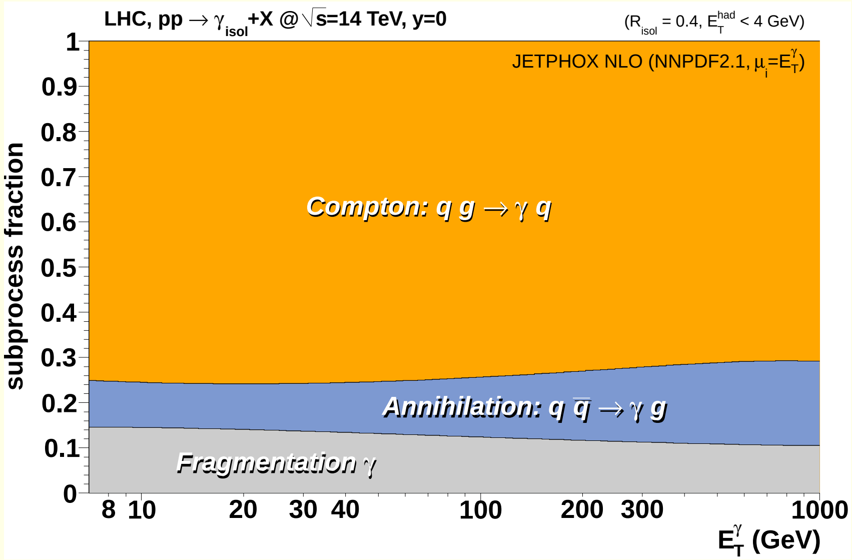
- Must treat the two- and three-body contributions separately
- For the $2 \to 3$ pointlike subprocesses, one can explicitly enforce the isolation condition on an event-by-event basis in the Monte Carlo at the parton level.

- For the two-body fragmentation component there is no dependence on $\mathcal{R}$ since the fragmentation functions are inclusive quantities.
  - Work in the collinear approximation (all emitted partons or photons are collinear with the parent parton)
  - parent parton transverse momentum is p_{Tpart}
  - photon transverse momentum is $p_{T\gamma} = zp_{Tpart}$
  - hadronic E_T is $(1-z)p_{Tpart} = (1-z)p_{t\gamma}/z$.
- Requiring that the hadronic E_T is less than E_{Tcut} results in

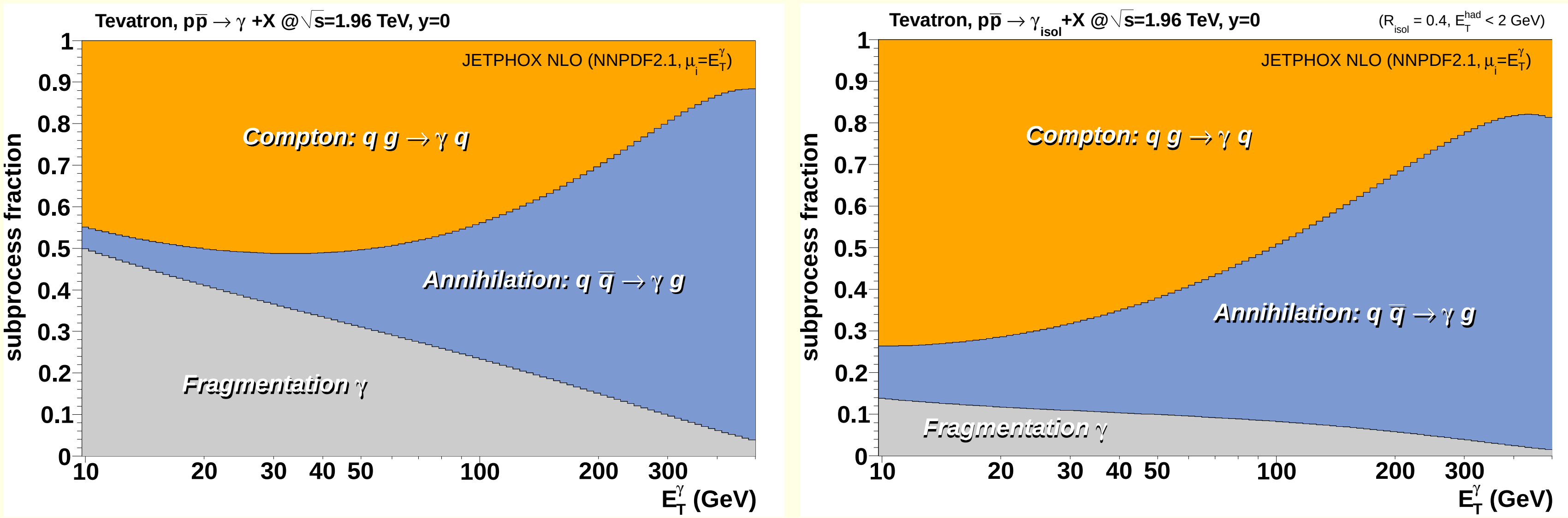
$$z > \frac{1}{1 + E_{Tcut}/p_{T\gamma}}.$$

- One can also enforce a similar isolation condition on the $2 \to 3$ fragmentation component





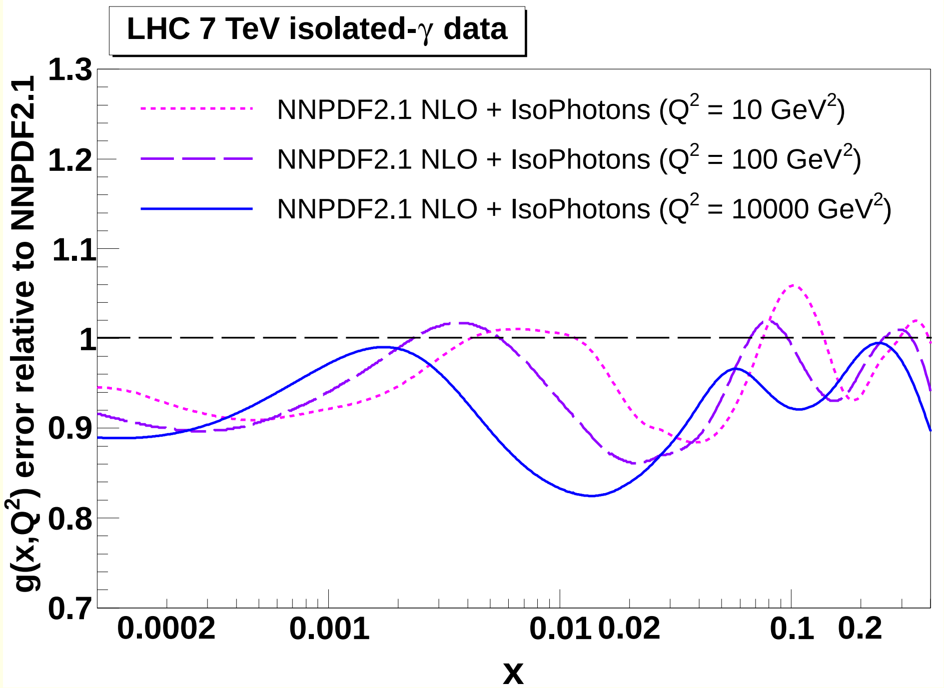
- From d'Enterria and Rojo, arXiv:1202.1762[hep-ph]
- pp at $\sqrt{s} = 14$ TeV
- Inclusive decomposition on left, isolated decomposition on right
- Note significant reduction in the fragmentation contribution to the isolated result
- Increases relative importance of the gluon initiated Compton subprocess



- Same as previous plots, but for the Tevatron
- $\bar{p}p$ at $\sqrt{s} = 1.96$ TeV
- See the same reduction
- Note the relative increase in the role of the annihilation subprocess due to the antiproton beam

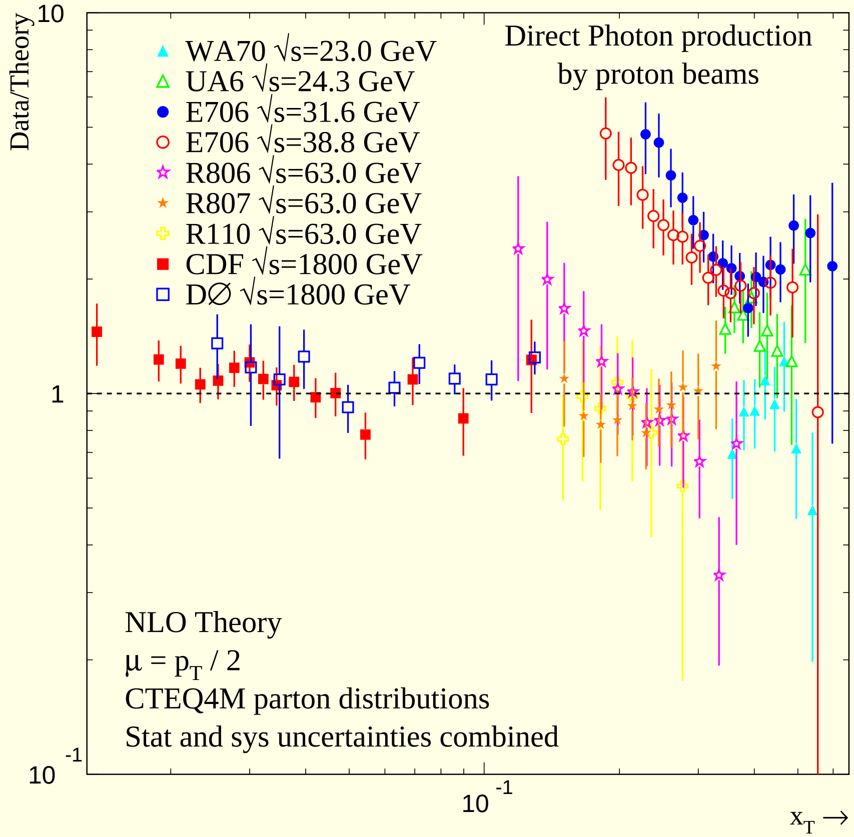
Constraints on the gluon PDF

- d'Enterria and Rojo studied the improvement in the uncertainty of the gluon PDF coming from using isolated photon data from colliders (arXiv: 1202.1762[hep-ph]
- Concluded that there could be a decrease in the uncertainty of up to 20% in some kinematic regions



Fixed Target and Lower Energy Collider Data

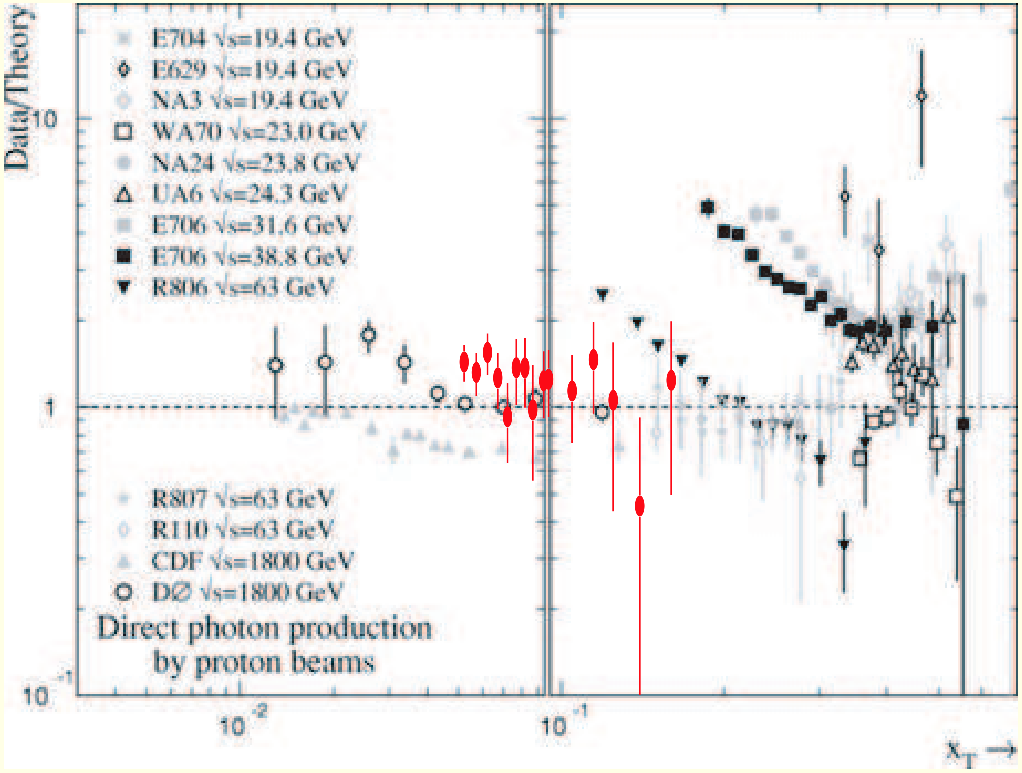
It is now time to consider the situation for the inclusive cross section at lower energies



Comments

1. Data/Theory plotted versus x_T
2. Data plotted at same x_T but different $\sqrt{s}$ correspond to different p_T's
3. E706 higher than theory, UA6 somewhat above theory, WA70 and theory agree
   - Likely that there is some experimental inconsistency here since the range in $\sqrt{s}$ is relatively small and it would take a significant modification of the theory to explain all three sets simultaneously.
4. See some shape disagreements among the ISR experiments
5. Plotted on this scale, the previously noted deviations of the theory from the CDF and DØ data look pretty darned small!

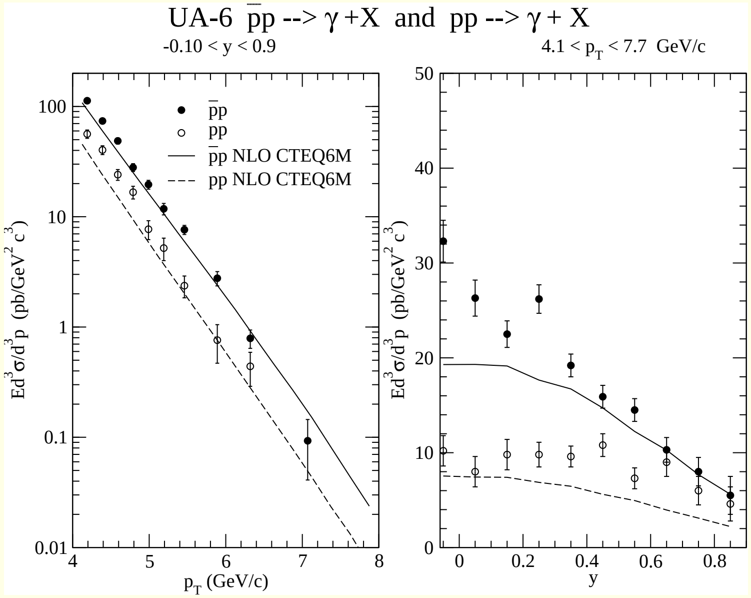
- Similar plot showing PHENIX data (nucl-ex/0504013)



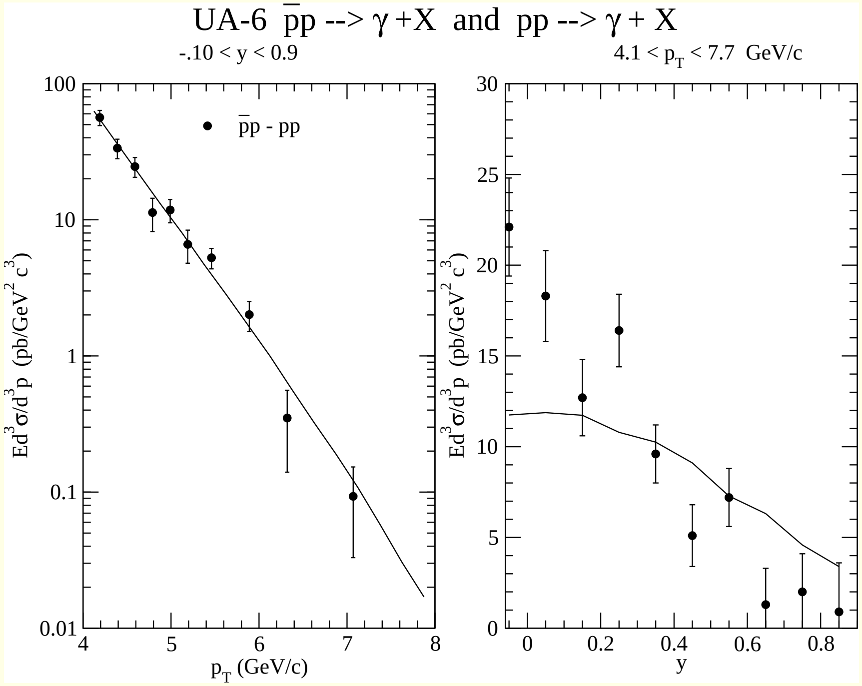
- Agreement is comparable to that for other collider experiments and better than for the fixed target regime

Example - UA-6

- Measured both pp and $\bar{p}p$ at $\sqrt{s} = 24.3$ GeV/c
- Initial state gluon and gluon fragmentation contributions cancel in the $\bar{p}p - pp$ difference



- Theory below the data at the lower end of the p_T range
- Rapidity theory curves are flatter than the data



- Cross section difference cancels contributions from gluons
- p_T difference is well described
- Rapidity theory curve is somewhat flatter than the data
- So, the situation is mixed - the $\bar{p}p$ and pp curves are individually below the data, the p_T difference is well described, while the rapidity difference curve is a bit too flat

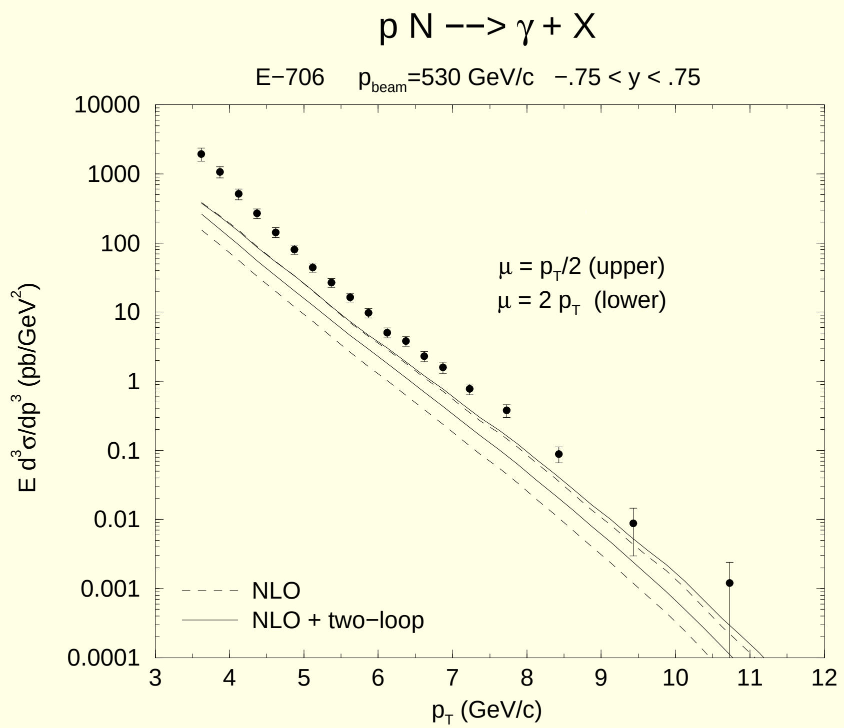
Threshold Resummation

Basic Physics -

- For inclusive calculations singularities from soft real gluon emission cancel against infrared singularities from virtual gluon emission
- Limitations on real gluon emission imposed by phase space constraints can upset this cancellation
- Singular terms still cancel, but there can be large logarithmic remainders
- Classic example is thrust distribution in $e^+e^- \rightarrow jets$
- For hadronic reactions with PDFs and FFs the collinear factors actually conspire to enhance the partonic cross sections (See my previous lecture at this school)

What about direct photons? Can threshold resummation help?

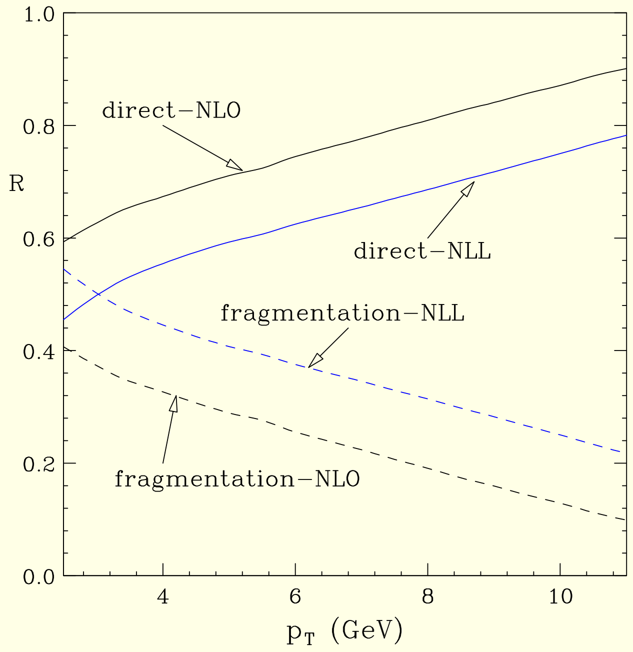
- Example application to the fixed target data - N. Kidonakis and J.F. Owens, Phys. Rev. D61, 094004, 2000; hep-ph/9912388
- Fixed target region dominated by annihilation and Compton subprocesses
- Fragmentation doesn't play as large a role as at higher energies since it costs extra energy to have a photon produced by fragmentation
- No significant enhancement to the annihilation and Compton terms
- Reduced scale dependence observed

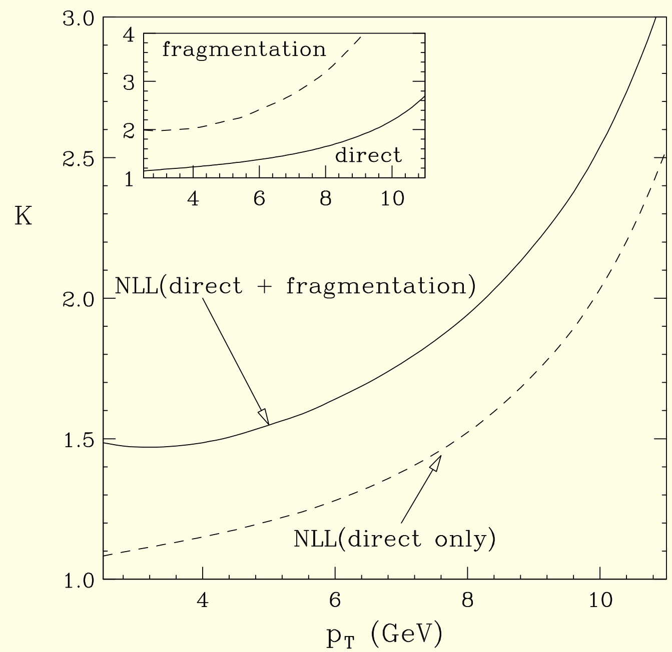


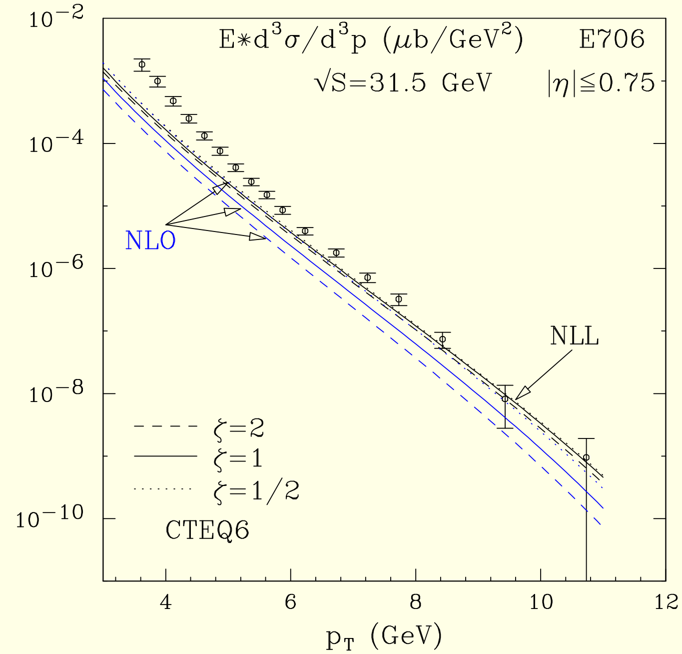
p N --> γ + X
E−706 p_beam=530 GeV/c −.75 < y < .75
μ = p_T/2 (upper)
μ = 2 p_T (lower)
NLO
NLO + two−loop
E d³σ/dp³ (pb/GeV²)
p_T (GeV/c)
10000
1000
100
10
1
0.1
0.01
0.001
0.0001
3
4
5
6
7
8
9
10
11
12

But...

- The fragmentation contribution is *not* zero at fixed target energies
- Vogelsang and de Florian had previously shown that the fragmentation contribution in *hadro*production was significantly enhanced by threshold resummation
- They subsequently applied the formalism to direct photons in hep-ph/0506150
- Relative contribution of fragmentation versus direct *is* enhanced

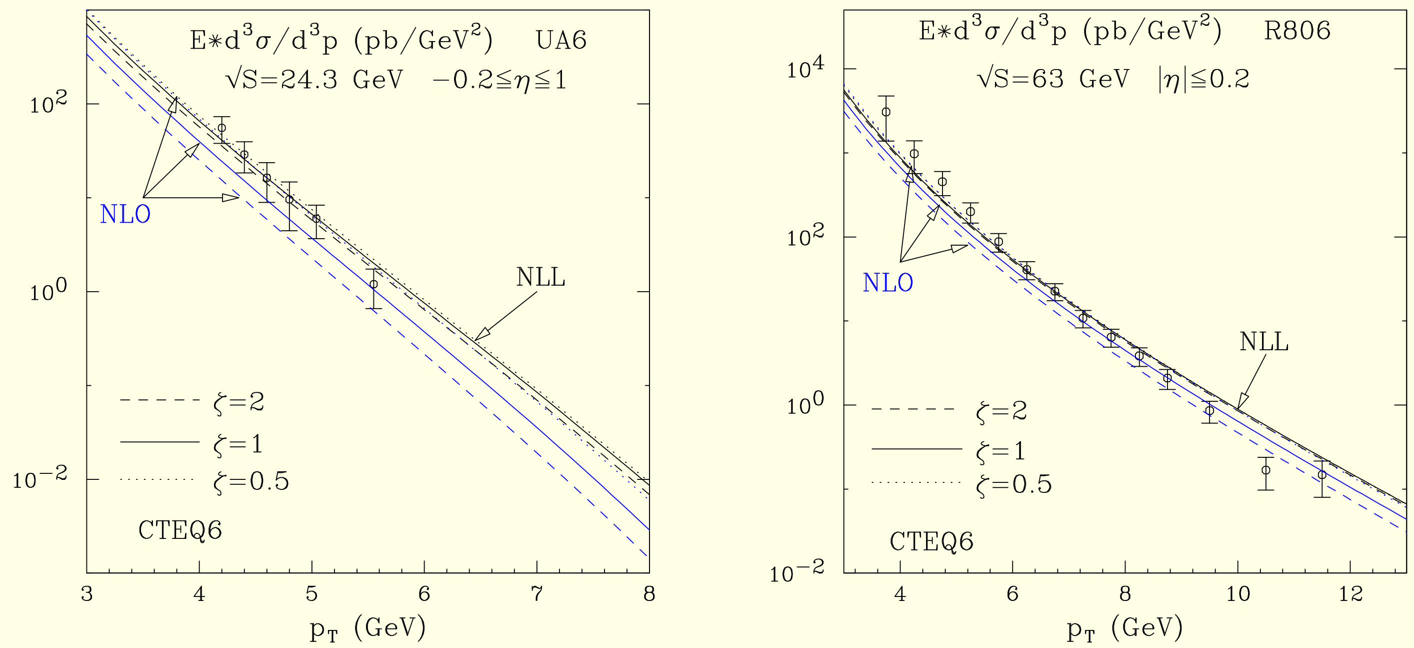






- Resumming the fragmentation component results in a larger increase than if just the direct component is resummed
- Still isn't enough to describe the E-706 results

Resummation *can* result in a good description of the UA-6 pp data



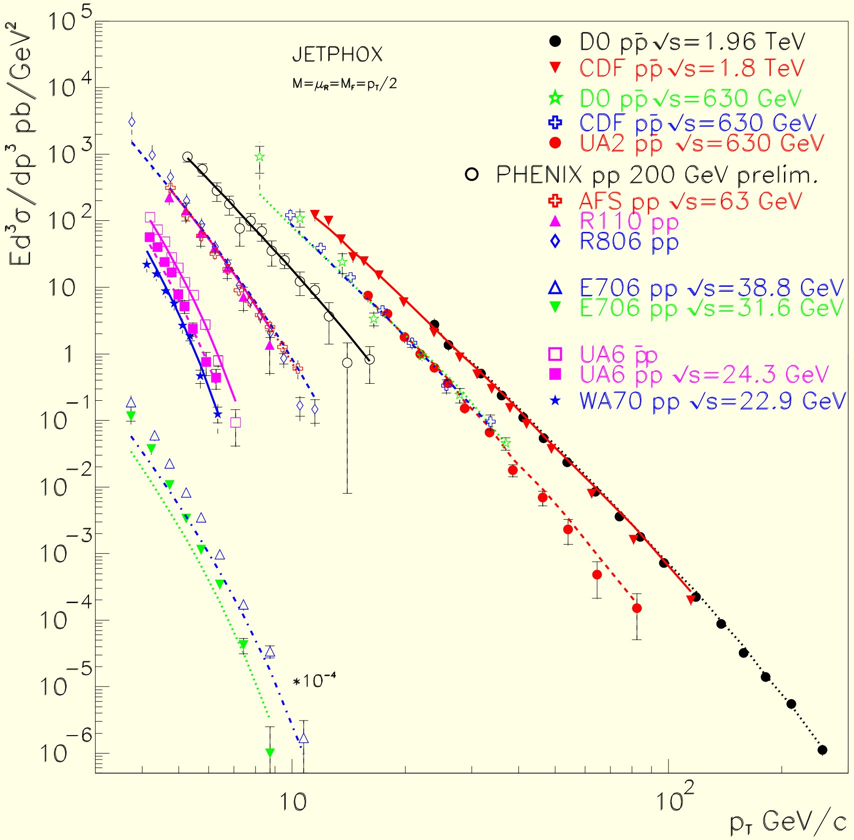
- Fragmentation component largely cancels in the $\overline{p}p - pp$ difference, so the previous good agreement for this is retained
- Enhancement decreases rapidly with energy as in the hadroproduction case so that agreement with higher energy data is retained

Bottom line on threshold resummation

- Provides reduced scale dependence
- Provides an enhancement in the fixed target regime, but the effect is much smaller at higher energies
- Can improve the agreement with some fixed target experiments without adversely affecting the agreement at higher energies

So where do we stand?

- Generally good agreement between theory and experiment for isolated direct photon production from colliders
- Existing data can provide some reduction in the uncertainty on the gluon PDF
- Threshold resummation affects the fragmentation component most strongly
- Including threshold resummation can help the theoretical description in the fixed target regime
- It is possible to further reduce the uncertainty on the gluon PDF using fixed target data if resummation effects are included (N. Sato, Ph.D. dissertation, publication in process)



This is a summary of the world's data on direct photon production. There is a reasonably consistent picture covering 9 orders of magnitude

Summary and Conclusions

1. Examining γ-jet observables suggests that high-p_T photons are produced in accordance with the expectations based on QCD
2. There is broad agreement between the theory and most experimental results for the photon p_T distribution
3. Photoproduction observables confirm that QCD matrix elements give a good description of photon interactions there, as well
4. Threshold resummation has been shown to play an important role in hadroproduction at fixed target energies and can offer some improvement for direct photons
   - Enhances the fragmentation contribution more than was previously anticipated in the fixed target regime
   - Effects are reduced at collider energies as the since smaller values of x_T are probed

While there are a few remaining issues, the overall description of the data over nine orders of magnitude is encouraging

Appendix: some miscellaneous and hopefully useful stuff

1. Check out the CTEQ web page at www.cteq.org
   - information on past summer schools, including transparencies of many of the lectures
   - information and links for parton distributions
   - CTEQ List of Challenges in Perturbative QCD
   - CTEQ Pedagogical Page
   - CTEQ Handbook of Perturbative QCD
2. Four-vectors and rapidity
   - rapidity is defined as $y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z}$. For massless particles this reduces to the pseudorapidity which is defined as $\eta = \ln \cot \theta/2$.
   - the four-vector for a massless particle with transverse momentum p_T and rapidity y may be conveniently expressed as

$$p^\mu = (p_T \cosh y, p_T, 0, p_T \sinh y)$$

3. Mandelstam variables

- For a two-body process $p_1 + p_2 \to p_3 + p_4$ the three Mandelstam variables are defined as

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ t &= (p_1 - p_3)^2 \\ u &= (p_1 - p_4)^2 \end{aligned}$$

- For processes with more particles one sometimes encounters variables such as $s_{ij} = (p_i + p_j)^2$ which is just the squared invariant mass of particles i and j and $t_{ij} = (p_i - p_j)^2$ which is the squared four-momentum transfer between particles i and j.

4. Another example: direct photon production $qg \to \gamma + q$

- four-vectors in the hadron-hadron center-of-mass frame

$$\begin{aligned} p_q &= \frac{\sqrt{s}}{2} x_a (1, 0, 0, 1) \\ p_g &= \frac{\sqrt{s}}{2} x_b (1, 0, 0 - 1) \\ p_\gamma &= p_T (\cosh y, 1, 0, \sinh y) \end{aligned}$$

- Substituting these four-vectors into the expressions for the Mandelstam variables above yields

$$\begin{aligned} \hat{s} &= x_a x_b \; s \\ \hat{t} &= -x_a \; p_T \sqrt{s} \; e^{-y} \\ \hat{u} &= -x_b \; p_T \sqrt{s} \; e^{y} \end{aligned}$$

- A ˆ is often used to denote a variable at the parton level.

5. Convolutions

- The symbol $\otimes$ is sometimes used to denote a convolution:

$$f \otimes g = \int_0^1 dy \int_0^1 dz\ f(y)\ g(z)\ \delta(x - yz)$$
$$= \int_x^1 \frac{dz}{z}\ f(x/z)\ g(z)$$

6. Subprocesses and angular distributions

The two lowest order subprocesses for direct photon production are (in units of $\pi\alpha\alpha_s/\hat{s}^2$)

$$\begin{aligned}
\frac{d\sigma}{d\hat{t}}(gq \to \gamma q) &= -\frac{e_q^2}{3}\left[\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}}\right] \\
\frac{d\sigma}{d\hat{t}}(q\overline{q} \to \gamma g) &= \frac{8}{9}e_q^2\left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}}\right]
\end{aligned}$$

The dominant parton-parton scattering subprocesses for hadroproduction are (in units of $\pi\alpha_s^2/\hat{s}^2$)

$$\frac{d\sigma}{d\hat{t}}(qq' \to qq') = \frac{4}{9}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]$$

$$\frac{d\sigma}{d\hat{t}}(qg \to qg) = -\frac{4}{9}\left[\frac{\hat{s}}{\hat{u}}+\frac{\hat{u}}{\hat{s}}\right]+\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

$$\frac{d\sigma}{d\hat{t}}(gg \to gg) = \frac{9}{2}\left[3-\frac{\hat{t}\hat{u}}{\hat{s}^2}-\frac{\hat{s}\hat{u}}{\hat{t}^2}-\frac{\hat{s}\hat{t}}{\hat{u}^2}\right]$$

- In the parton-parton center-of-mass frame, one can write

$$\begin{aligned} \hat{t} &= -\frac{\hat{s}}{2}\,(1 - \cos(\theta^*)) \\ \hat{u} &= -\frac{\hat{s}}{2}\,(1 + \cos(\theta^*)) \end{aligned}$$

- Therefore, as $\cos(\theta^*) \to 1(-1), \hat{t}\,(\hat{u}) \to 0$. Hence, in this limit, the first two subprocesses on the preceding pages behave as $(1 - |\cos(\theta^*)|)^{-1}$ while the next three behave as $(1 - |\cos(\theta^*)|)^{-2}$.

7. Center of mass scattering angle

Start in the parton-parton center of mass frame where one has

$$\begin{aligned} p_1 &= \frac{\sqrt{\hat{s}}}{2}(1,0,0,1) \quad p_2 = \frac{\sqrt{\hat{s}}}{2}(1,0,0-1) \\ p_3 &= \frac{\sqrt{\hat{s}}}{2}(1,\sin\theta^*,0,\cos\theta^*) \quad p_4 = \frac{\sqrt{\hat{s}}}{2}(1,-\sin\theta^*,0,-\cos\theta^*) \end{aligned}$$

from which one can derive

$$\hat{t} = -\frac{\hat{s}}{2}\left(1-\cos(\theta^*)\right) \quad \hat{u} = -\frac{\hat{s}}{2}\left(1+\cos(\theta^*)\right).$$

Next, write the parton four-vectors in the hadron-hadron frame as

$$
\begin{aligned}
p_1 &= \frac{\sqrt{s}}{2} x_a (1, 0, 0, 1) \\
p_2 &= \frac{\sqrt{s}}{2} x_b (1, 0, 0 - 1) \\
p_3 &= p_T (\cosh y_3, 1, 0, \sinh y_3) \\
p_4 &= p_T (\cosh y_4, -1, 0, \sinh y_4)
\end{aligned}
$$

which can be used to derive

$$
\begin{aligned}
\hat{t} &= -\sqrt{s} x_a p_T e^{-y_3} \\
\hat{u} &= -\sqrt{s} x_a p_T e^{-y_4}.
\end{aligned}
$$

From these two sets of expressions one can obtain

$$\frac{\hat{t}}{\hat{u}} = e^{-(y_3 - y_4)} = \frac{1 - \cos\theta^*}{1 + \cos\theta^*}.$$

It then follows that

$$\cos\theta^* = \tanh\frac{y_3 - y_4}{2}.$$

8. Some comments on the asymptotic solution of the evolution equations for parton distributions in a photon

- Rewrite the evolution equations by taking moments of both sides using the following definitions:

$$
\begin{aligned}
M_q^n &= \int_0^1 dx\ x^{n-1}\ G_{q/\gamma}(x) \\
M_g^n &= \int_0^1 dx\ x^{n-1}\ G_{g/\gamma}(x) \\
A_{ij}^n &= \frac{1}{2\pi b}\int_0^1 dx\ x^{n-1}\ P_{ij}(x) \\
a^n &= \frac{\alpha}{2\pi}\int_0^1 dx\ x^{n-1}\ P_{q\gamma} \\
\alpha_s(t) &= \frac{1}{bt}
\end{aligned}
$$

  where $t = \ln(Q^2/\Lambda^2)$.

- The evolution equations can now be written as

$$\begin{aligned} \frac{dM_q^n}{dt} &= e_q^2\, a^n + \frac{1}{t}\left[A_{qq}^n\, M_q^n + A_{qg}^n\, M_g^n\right] \\ \frac{dM_g^n}{dt} &= \frac{1}{t}\left[\sum_q A_{gq}^n M_q^n + A_{gg}^n M_g^n\right] \end{aligned}$$

- If each of the moments is proportional to t, the t dependence drops out of the equations and they may be solved algebraically

- The asymptotic solution is

$$
\begin{aligned}
M_q^n &= a^n \left(\frac{e_q^2 - 5/18}{1 - A_{qq}^n} + \frac{5}{18} \frac{1 - A_{gg}^n}{F^n} \right) t \\
M_g^n &= \frac{5f}{9} a^n \frac{A_{gg}^n}{F^n} t \\
F^n &= 1 - A_{qq}^N - A_{gg}^n + A_{qq}^n A_{gg}^n - 2f A_{qg}^n A_{gq}^n
\end{aligned}
$$

where f is the number of flavors

- Note how the moments are each proportional to t
- Compare to the case where $P_{q\gamma} = 0$ where the moments are of the form

$$
M^n(t_0) \left(\frac{t}{t_0} \right)^{A^n}
$$

- Note that one can add any solution of the homogeneous evolution equations to this asymptotic solution