Monte Carlo Event Generators

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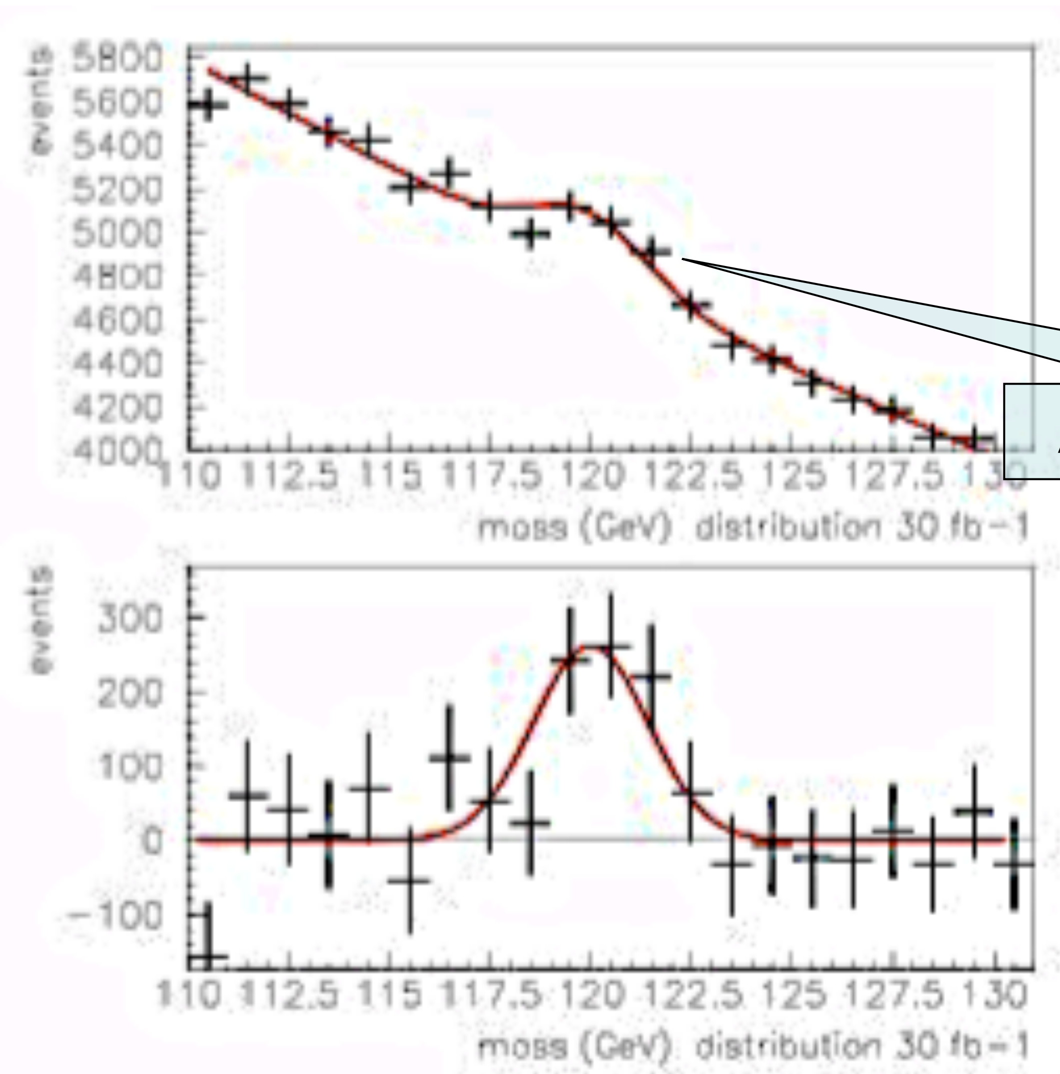
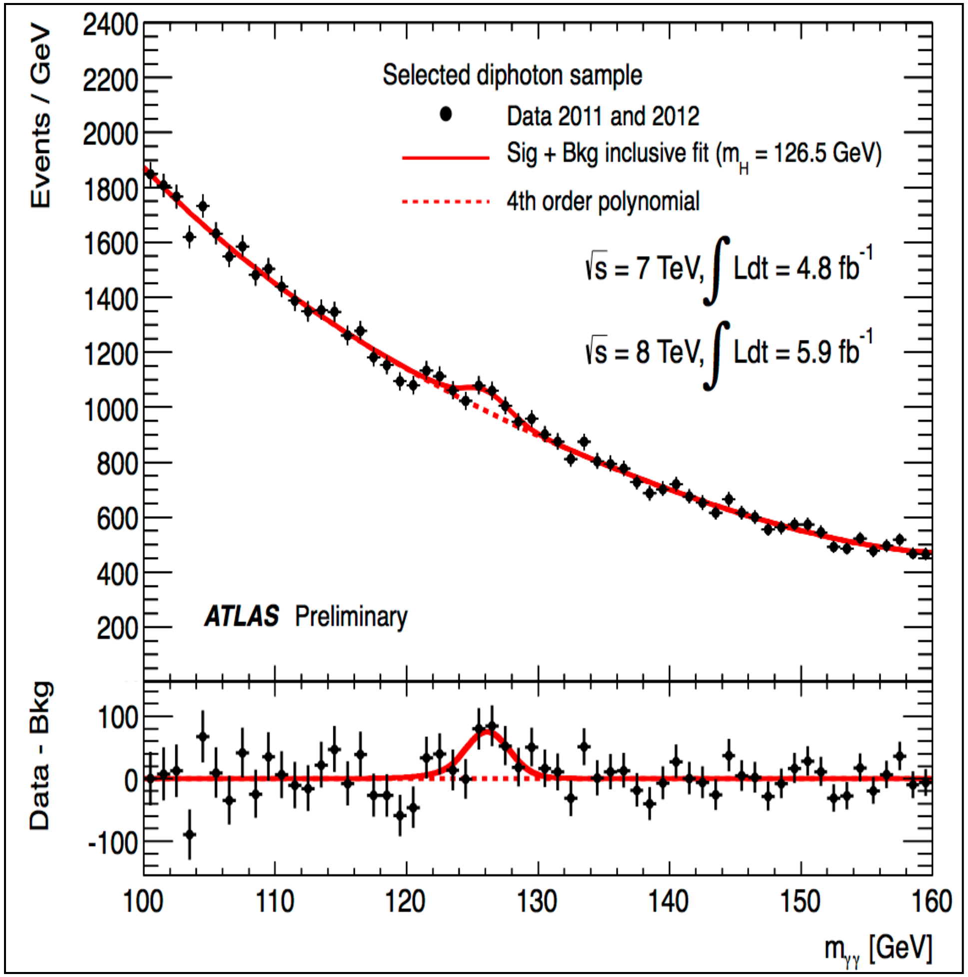
Overview and Motivation



ATLAS’ observation of $H \rightarrow \gamma\gamma$?

Overview and Motivation



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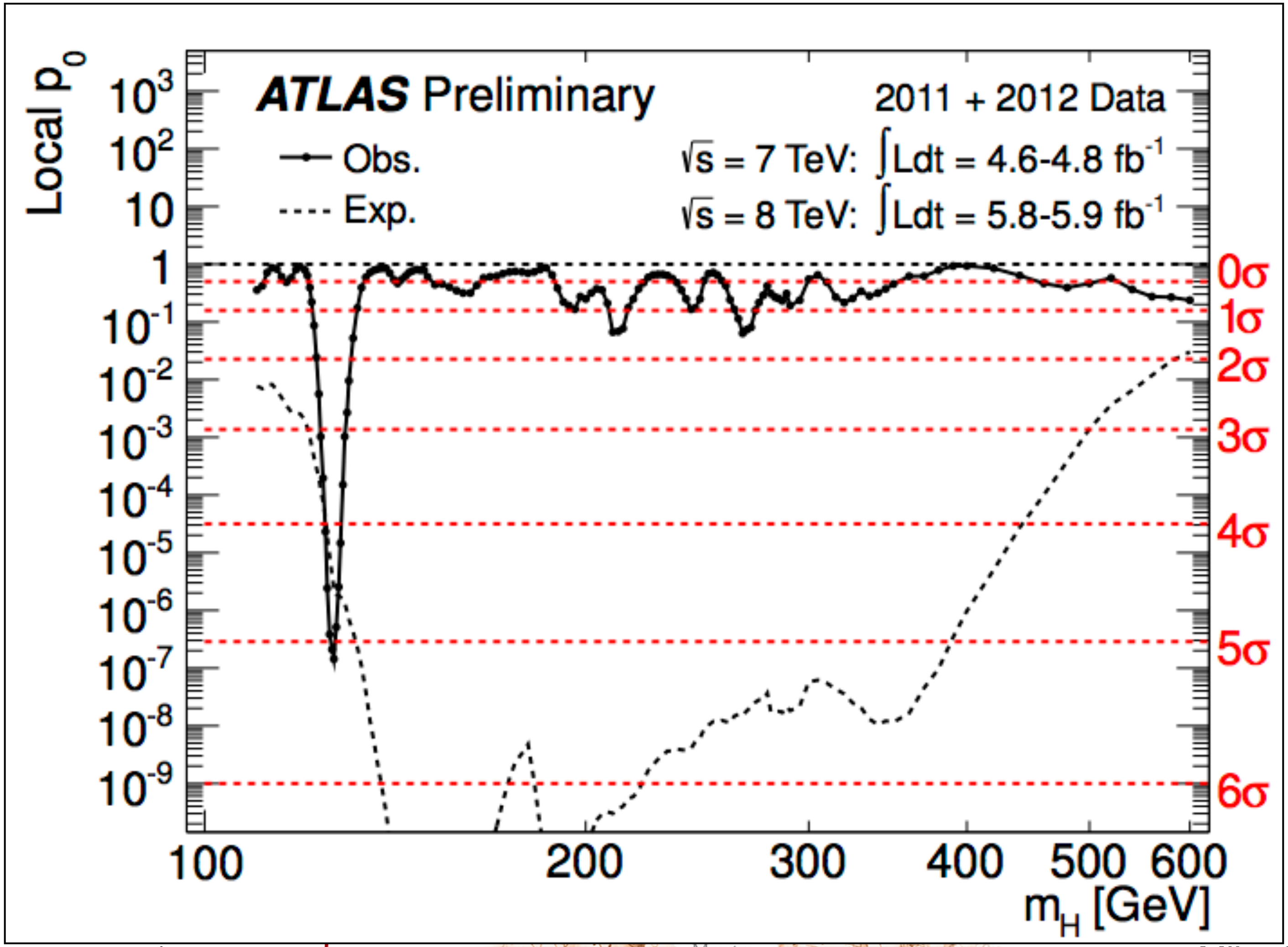
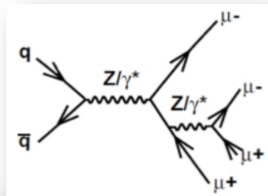
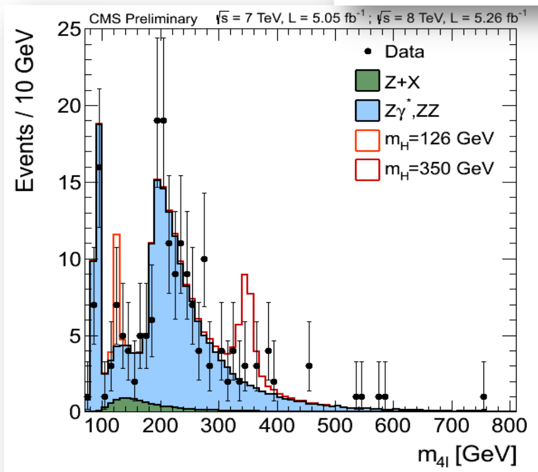
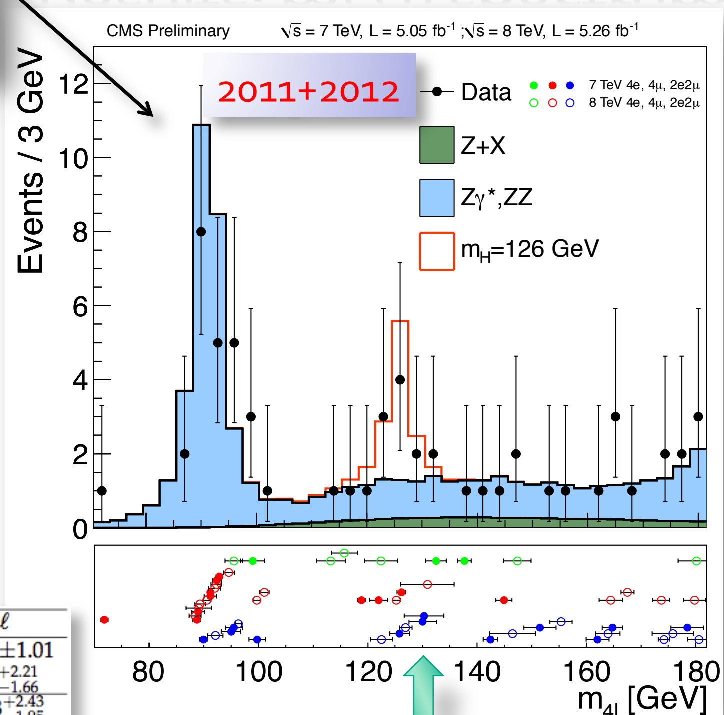


Overview and Motivation

Results: m(4l) spectrum







Event-by-event errors

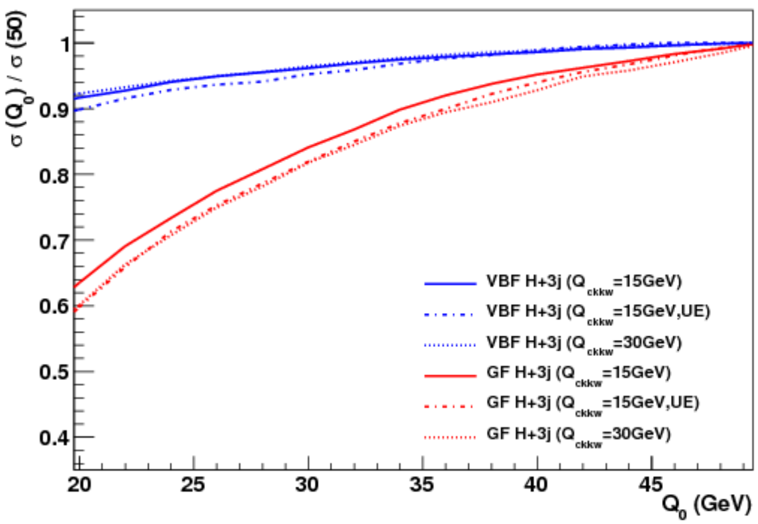
J. Incandela for the CMS COLLABORATION

Yields for m(4l)=110..160 GeV

Channel	4e	4μ	$2e2\mu$	4ℓ
ZZ background	2.65 ± 0.31	5.65 ± 0.59	7.17 ± 0.76	15.48 ± 1.01
Z+X	$1.20^{+1.08}_{-0.78}$	$0.92^{+0.65}_{-0.55}$	$2.29^{+1.81}_{-1.36}$	$4.41^{+2.21}_{-1.66}$
All backgrounds	$3.85^{+1.12}_{-0.84}$	$6.58^{+0.88}_{-0.81}$	$9.46^{+1.96}_{-1.56}$	$19.88^{+2.43}_{-1.95}$
$m_H = 126$ GeV	1.51 ± 0.48	2.99 ± 0.60	3.81 ± 0.89	8.31 ± 1.18

164 events expected in [100, 800 GeV]
172 events observed in [100, 800 GeV]

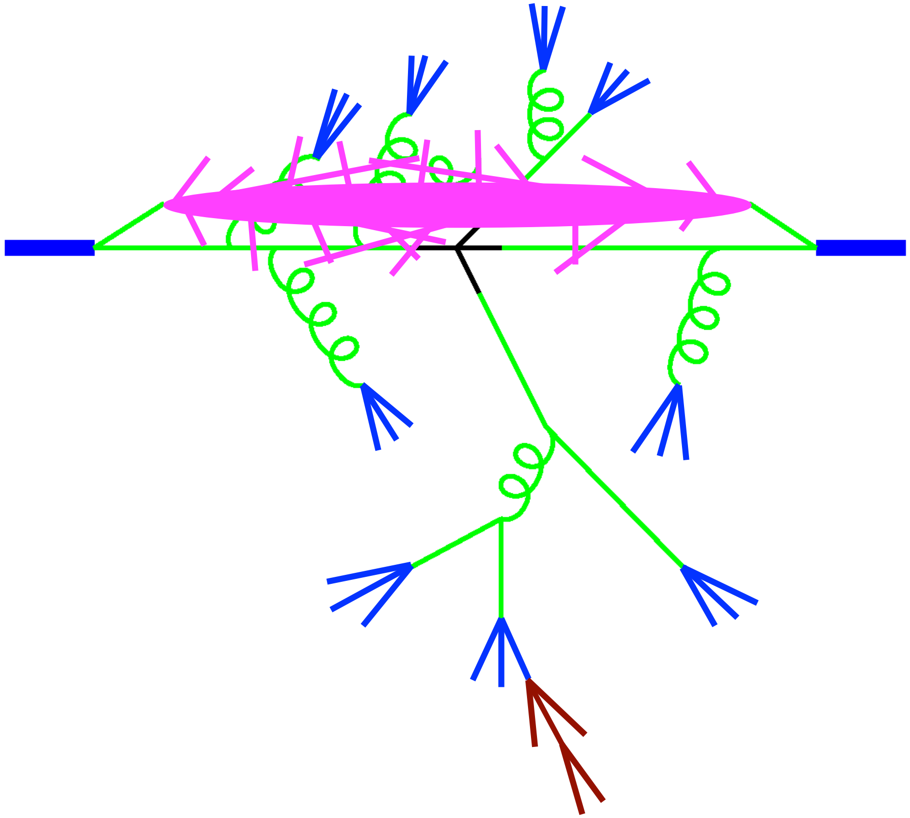
Overview and Motivation



- Beyond discovery:
  - measure Higgs couplings, e.g. separate gg→H from VBF→H using jet veto in central region
  - (B.E.Cox, J.R.Forshaw, A.D.Pilkington, Phys. Lett. B696 (2011) 87)
  - Needs accurate prediction of very detailed event properties

Structure of LHC Events

1. Hard process
2. Parton shower
3. Hadronization
4. Underlying event
5. Unstable particle decays



Intro to Monte Carlo Event Generators

1. Monte Carlo technique / hard process
2. Parton showers
3. Hadronization
4. Underlying Event / Soft Inclusive Models

Integrals as Averages

- Basis of all Monte Carlo methods: $$I = \int_{x_1}^{x_2} f(x)\, dx = (x_2 - x_1)\, \langle f(x) \rangle$$
- Draw N values from a uniform distribution: $$I \approx I_N \equiv (x_2 - x_1) \frac{1}{N} \sum_{i=1}^{N} f(x_i)$$
- Sum invariant under reordering: randomize
- Central limit theorem: $$I \approx I_N \pm \sqrt{V_N / N}$$

$$V = (x_2 - x_1) \int_{x_1}^{x_2} \left[f(x) \right]^2 dx - \left[\int_{x_1}^{x_2} f(x)\, dx \right]^2$$

Particle Decays

Simplest example

eg top quark decay:

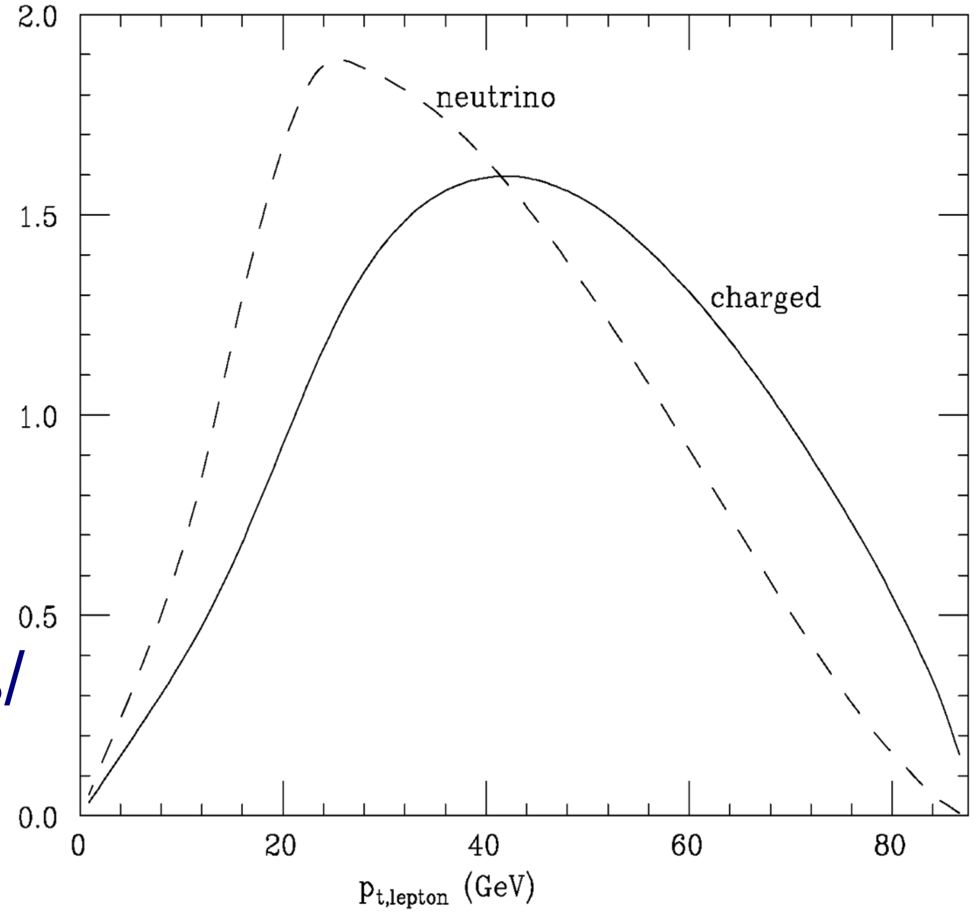
$$|\mathcal{M}|^2 = \frac{1}{2}\left(\frac{8\pi\alpha}{\sin^2\theta_w}\right)^2 \frac{p_t\cdot p_\nu \; p_b\cdot p_\ell}{(m_W^2 - M_W^2)^2 + \Gamma_W^2 M_W^2}$$

$$\Gamma = \frac{1}{2M}\,\frac{1}{128\pi^3}\int |\mathcal{M}|^2 dm_W^2 \left(1 - \frac{m_W^2}{M^2}\right)\frac{d\Omega}{4\pi}\,\frac{d\Omega_W}{4\pi}$$

Associated Distributions

- Big advantage of Monte Carlo integration:
- simply histogram any associated quantities.
- Almost any other technique requires new integration for each observable.
- Can apply arbitrary cuts/smearing.

eg lepton momentum in top decays:



Leading Order Monte Carlo Calculations

Now have everything we need to make leading order cross section calculations and distributions

Can be largely automated…

- MADGRAPH
- AMEGIC++/COMIX
- COMPHEP
- ALPGEN
- GRACE

But…

- Fixed parton/jet multiplicity
- No control of large logs
- Parton level

→ Need hadron level event generators

Structure of LHC Events

1. Hard process
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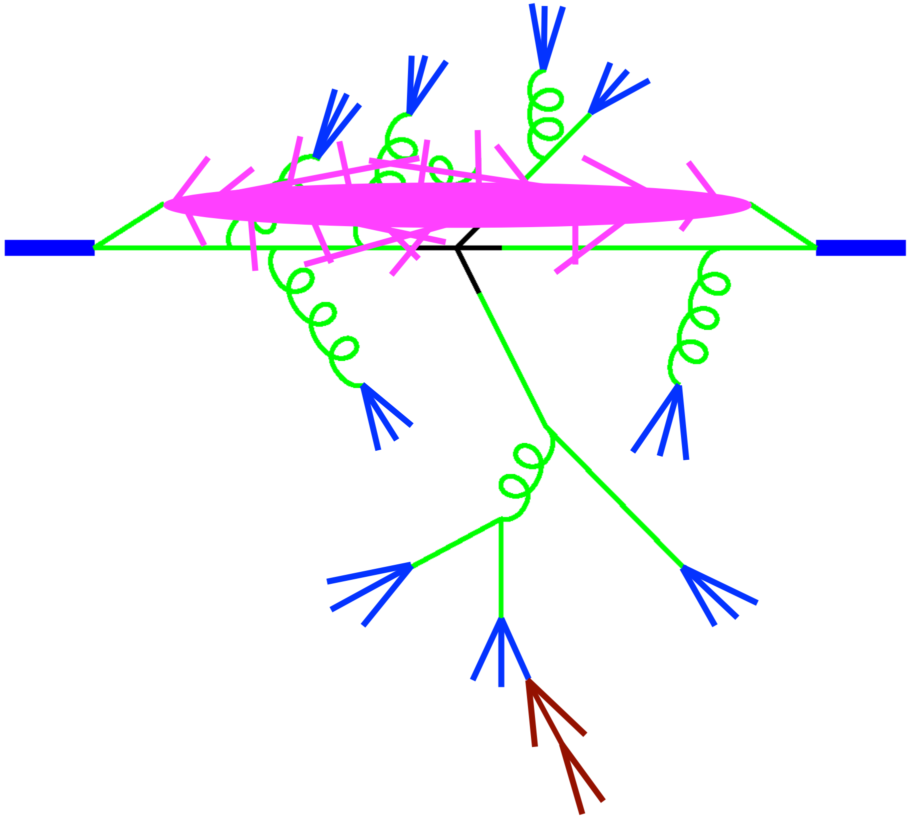


Parton Showers: Introduction

QED: accelerated charges radiate.

QCD identical: accelerated colours radiate.

gluons also charged.

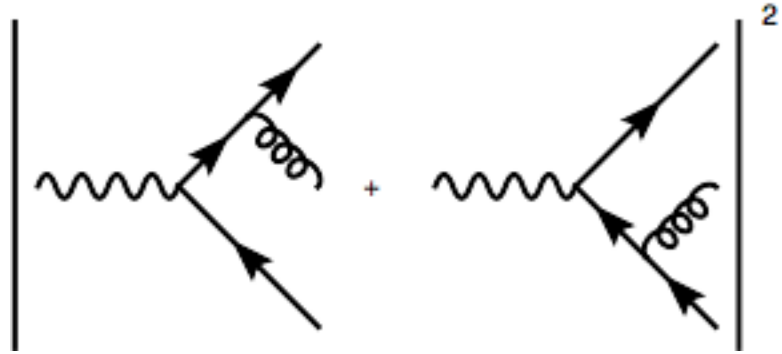
→ cascade of partons.

= parton shower.

1. e^+e^-annihilation to jets.
2. Universality of collinear emission.
3. Sudakov form factors.
4. Universality of soft emission.
5. Angular ordering.
6. Initial-state radiation.
7. Hard scattering.
8. Heavy quarks.
9. Dipole cascades.

QCD emission matrix elements diverge

e.g. $e^+e^- \to 3$ partons:



$$\frac{d\sigma}{d\cos\theta\, dz_g} \sim \sigma_0\, C_F\, \frac{\alpha_s}{2\pi}\, \frac{2}{\sin^2\theta}\, \frac{1+(1-z_g)^2}{z_g}$$

- dz_g: $E_g/E_{g,max}$
- σ_0: $e^+e^- \to 2$ partons
- C_F: "quark charge squared"
- α_s: QCD running coupling ~ 0.1

Divergent in collinear limit $\theta \to 0,\pi$ (for massless quarks)
and soft limit $z_g \to 0$

can separate into two independent jets:

$$\frac{2\,d\cos\theta}{\sin^2\theta} = \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\theta}{1+\cos\theta}$$
$$= \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\bar{\theta}}{1-\cos\bar{\theta}}$$
$$\approx \frac{d\theta^2}{\theta^2} + \frac{d\bar{\theta}^2}{\bar{\theta}^2}$$

jets evolve independently

$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}$$

Exactly same form for anything $\propto \theta^2$

eg transverse momentum: $k_\perp^2 = z^2(1-z)^2\,\theta^2\,E^2$

invariant mass: $q^2 = z(1-z)\,\theta^2\,E^2$

$$\frac{d\theta^2}{\theta^2} = \frac{dk_\perp^2}{k_\perp^2} = \frac{dq^2}{q^2}$$

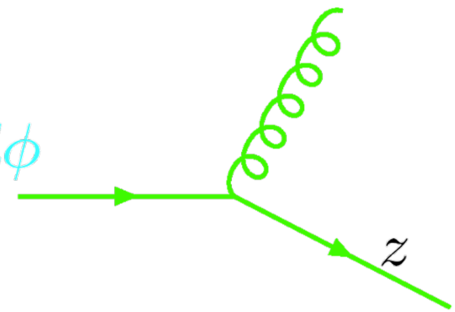
Collinear Limit

Universal:

$$d\sigma = \sigma_0 \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} dz\, P(z, \phi) d\phi$$

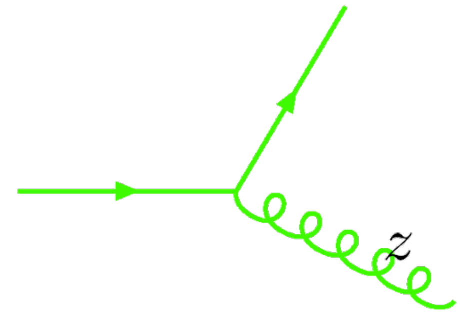
$P(z, \phi) =$

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting kernel: dependent on flavour and spin



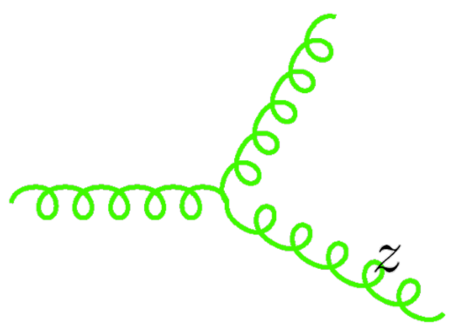
$q \to qg$

$C_F \frac{1+z^2}{1-z}$



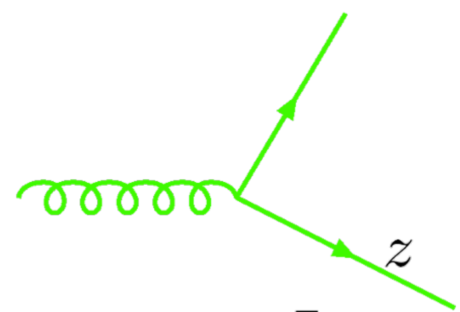
$q \to gq$

$C_F \frac{1+(1-z)^2}{z}$



$g \to gg$

$C_A \frac{z^4+1+(1-z)^4}{z(1-z)}$



$g \to q\bar{q}$

$T_R \left(z^2 + (1-z)^2\right)$

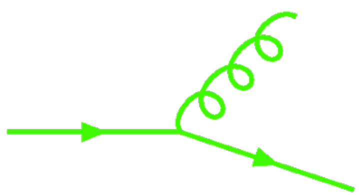
Resolvable partons

What is a parton?

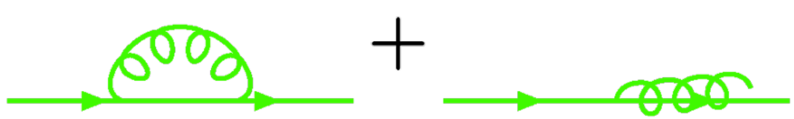
Collinear parton pair $\longleftrightarrow$ single parton

Introduce resolution criterion, eg $k_\perp > Q_0$.

Virtual corrections must be combined with unresolvable real emission



Resolvable emission
Finite



Virtual + Unresolvable emission
Finite

Unitarity: P(resolved) + P(unresolved) = 1

Sudakov form factor

Probability(emission between q^2 and $q^2 + dq^2$)

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\ P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

Define probability(no emission between Q^2 and q^2) to be $\Delta(Q^2, q^2)$. Gives evolution equation

$$\frac{d\Delta(Q^2,q^2)}{dq^2} = \Delta(Q^2,q^2)\frac{d\mathcal{P}}{dq^2}$$

$$\Rightarrow \Delta(Q^2,q^2) = \exp - \int_{q^2}^{Q^2}\frac{dk^2}{k^2}\bar{P}(k^2).$$

c.f. radioactive decay

atom has probability λ per unit time to decay.

Probability(no decay after time T) = $\exp - \int^T dt\,\lambda$

Sudakov form factor

Probability(emission between q^2 and $q^2 + dq^2$)

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\ P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2).$$

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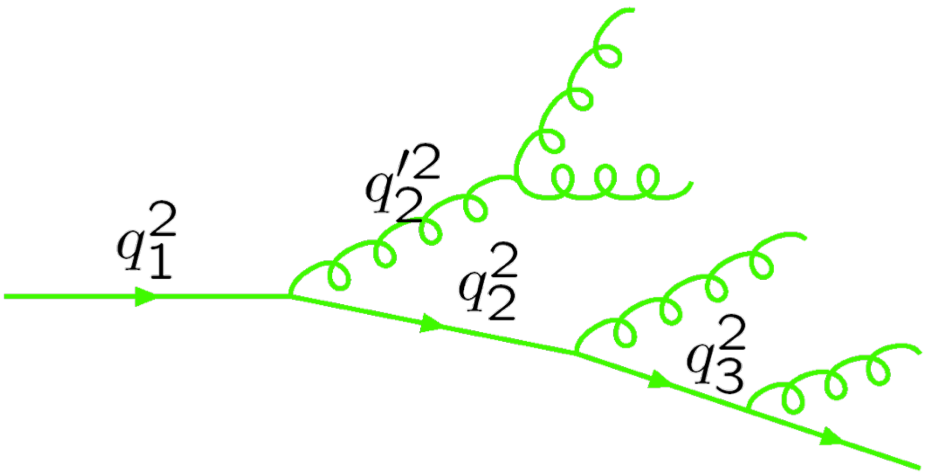
$$\Rightarrow \Delta(Q^2, q^2) = \exp - \int_{q^2}^{Q^2} \frac{dk^2}{k^2}\bar{P}(k^2).$$

$\Delta(Q^2, Q_0^2) \equiv \Delta(Q^2)$ Sudakov form factor

=Probability(emitting no resolvable radiation)

$$\Delta_q(Q^2) \sim \exp - C_F \frac{\alpha_s}{2\pi} \log^2 \frac{Q^2}{Q_0^2}$$

Multiple emission



$$q_1^2 > q_2^2 > q_3^2 > \dots$$
$$q_1^2 > q_2'^2 \dots$$

But initial condition? $q_1^2 < ???$

Process dependent

Monte Carlo implementation

Can generate branching according to

$$d\mathcal{P} = \frac{dq^2}{q^2}\bar{P}(q^2)\,\Delta(Q^2,q^2)$$

By choosing $0 < \rho < 1$ uniformly:

If $\rho < \Delta(Q^2)$ no resolvable radiation, evolution stops.

Otherwise, solve $\rho = \Delta(Q^2,q^2)$

for q^2 =emission scale

Considerable freedom:

Evolution scale: $q^2/k_\perp^2/\theta^2$?

z: Energy? Light-cone momentum?

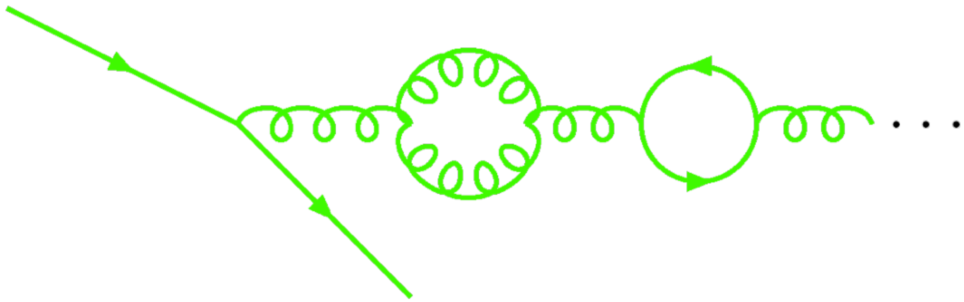
Massless partons become massive. How?

Upper limit for q^2?

All formally free choices, but can be very important numerically

Running coupling

Effect of summing up higher orders:



absorbed by replacing α_s by $\alpha_s(k_\perp^2)$.

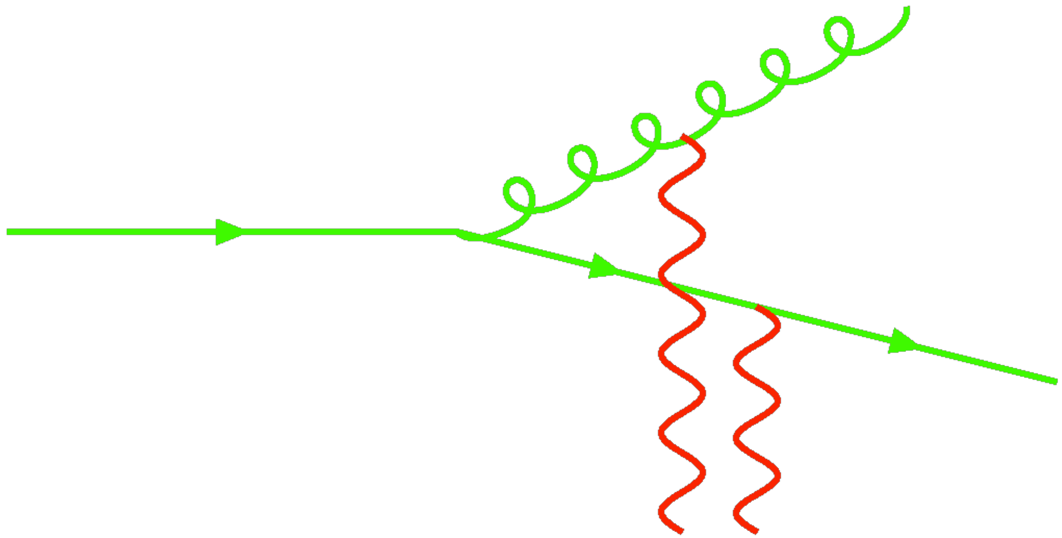
Much faster parton multiplication – phase space fills with soft gluons.

Must then avoid Landau pole: $k_\perp^2 \gg \Lambda^2$.

Q_0 now becomes physical parameter!

Soft limit

Also universal. But at amplitude level…



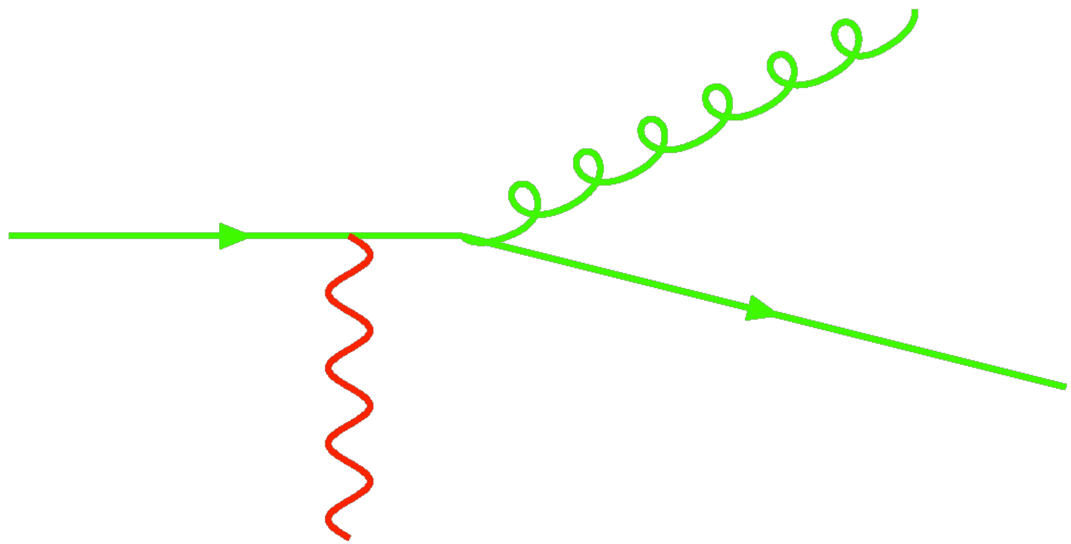
soft gluon comes from everywhere in event.

→ Quantum interference.

Spoils independent evolution picture?

Angular ordering

NO:



outside angular ordered cones, soft gluons sum coherently: only see colour charge of whole jet.

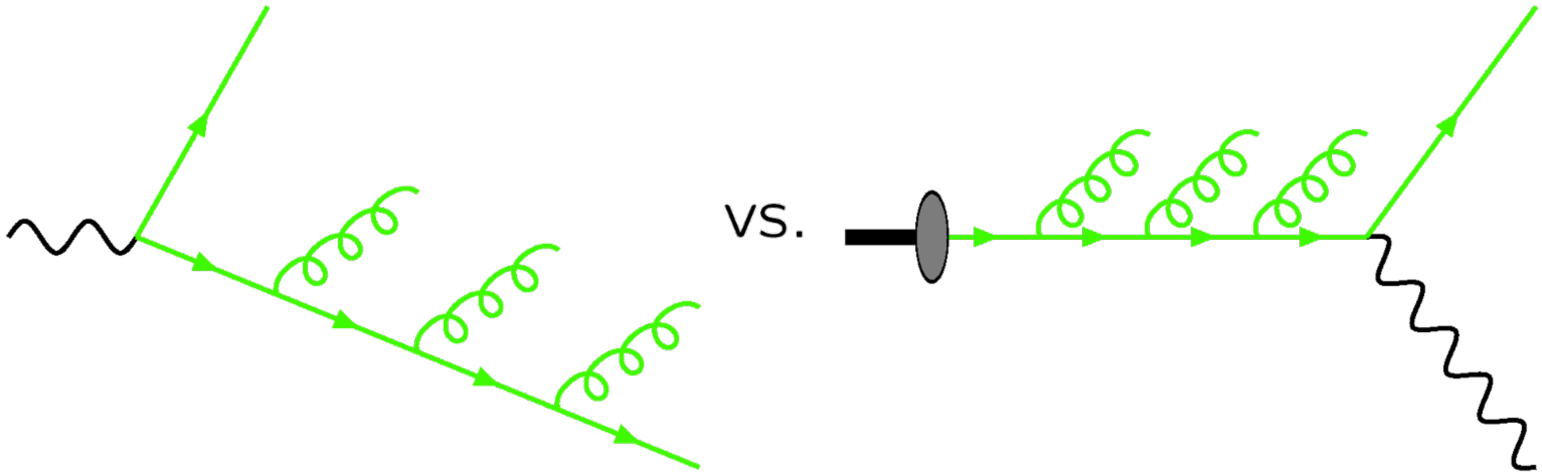
Soft gluon effects fully incorporated by using θ^2 as evolution variable: angular ordering

First gluon not necessarily hardest!

Initial state radiation

In principle identical to final state (for not too small x)

In practice different because both ends of evolution fixed:



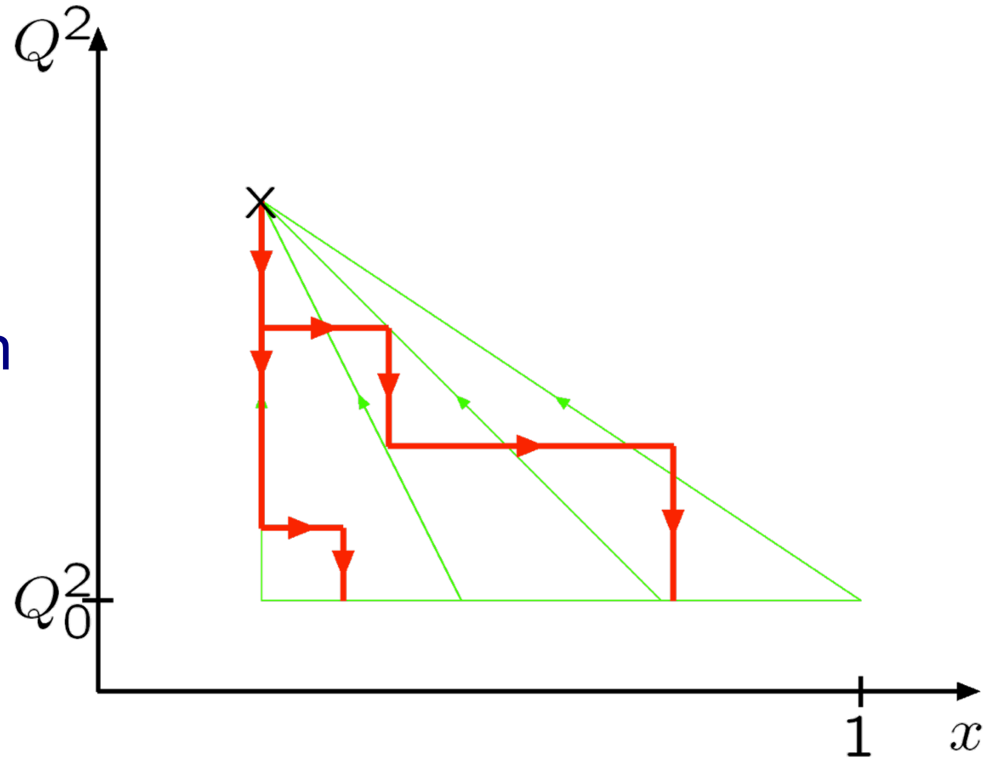
Use approach based on evolution equations…

Backward evolution

DGLAP evolution: pdfs at (x, Q^2) as function of pdfs at $(> x, Q_0^2)$:

Evolution paths sum over all possible events.

Formulate as backward evolution: start from hard scattering and work down in q^2, up in x towards incoming hadron.

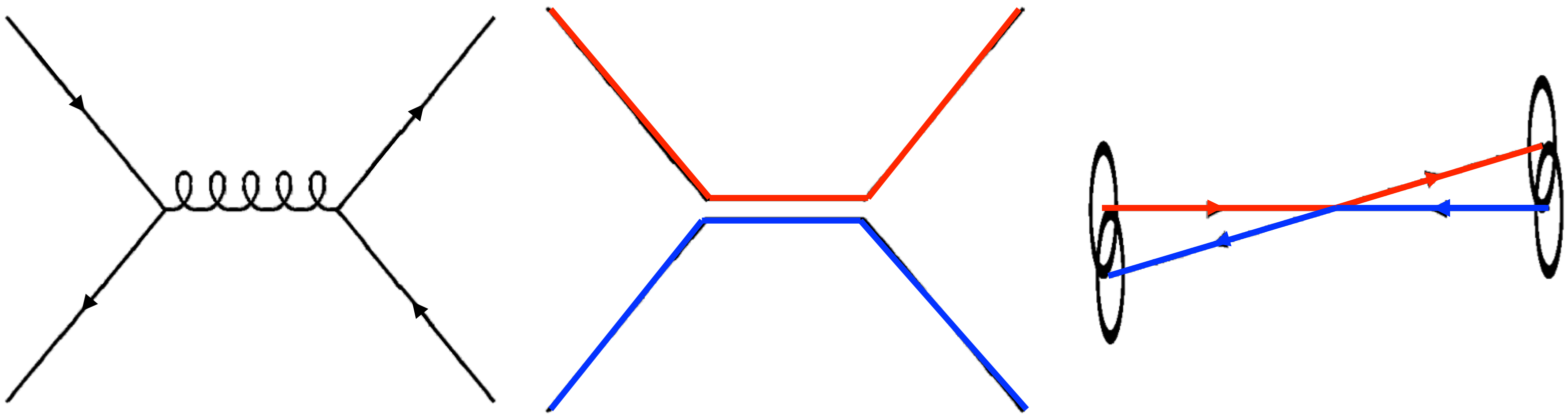


Algorithm identical to final state with $\Delta_i(Q^2, q^2)$ replaced by $\Delta_i(Q^2, q^2)/f_i(x, q^2)$.

Hard Scattering

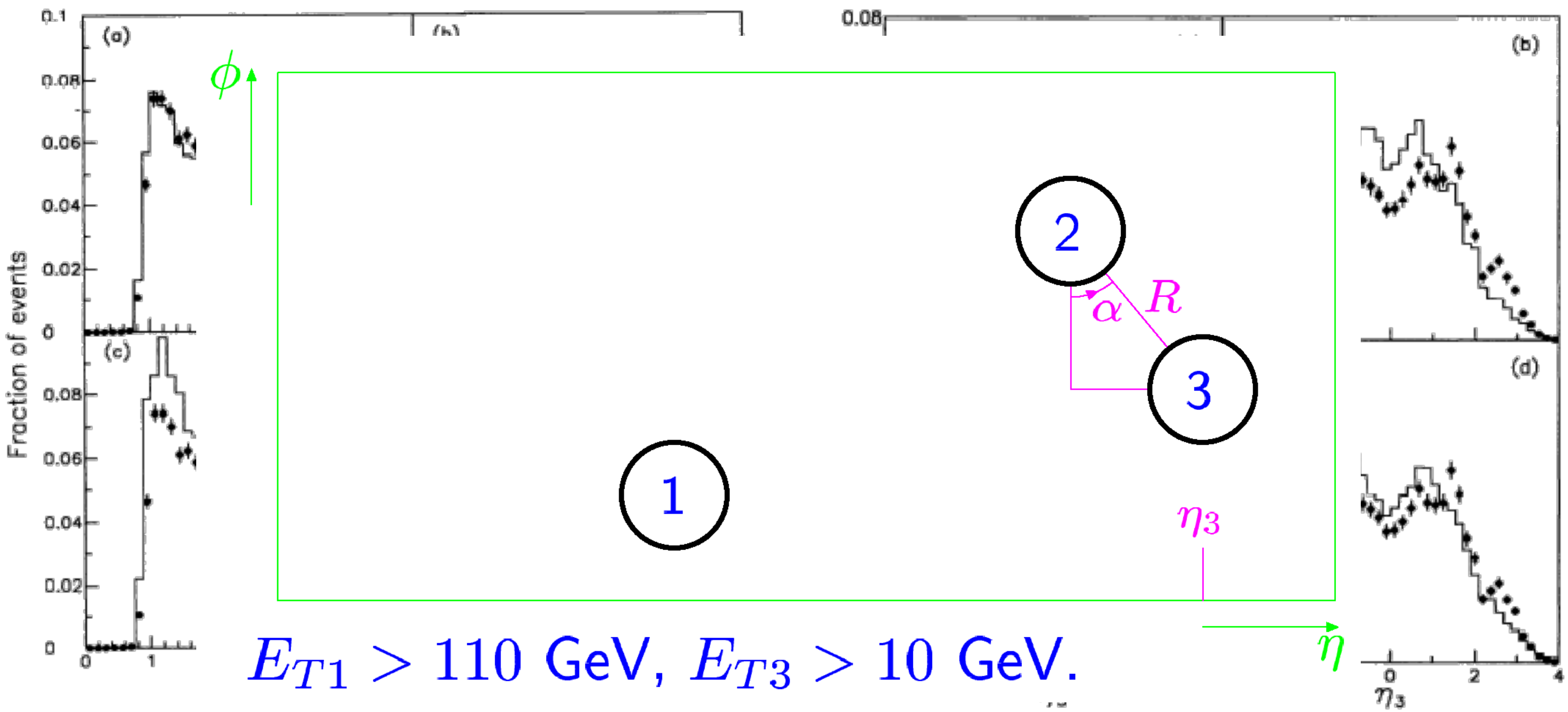
Sets up initial conditions for parton showers.

Colour coherence important here too.



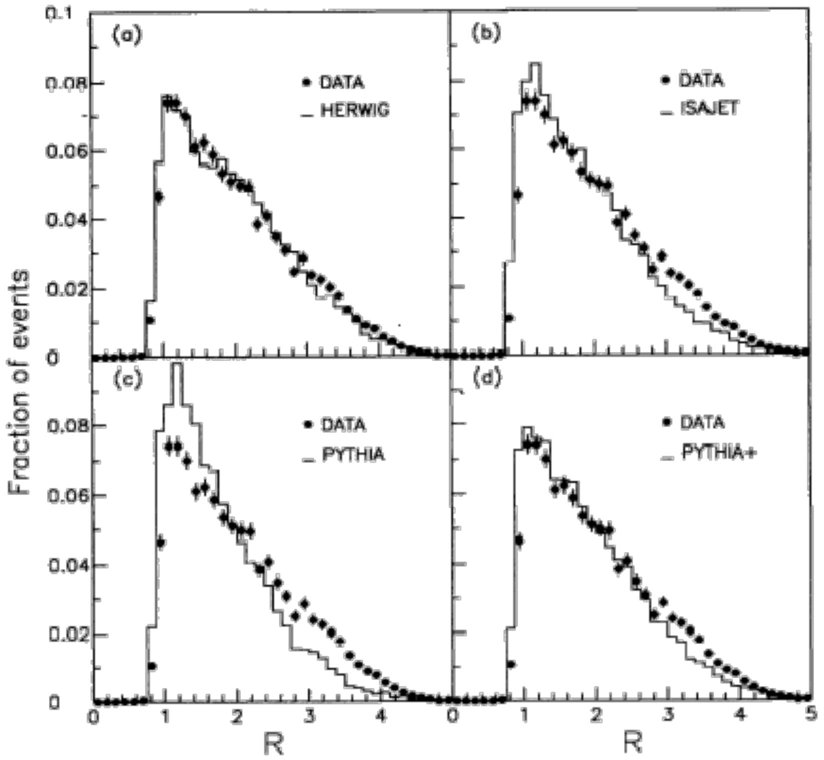
Emission from each parton confined to cone stretching to its colour partner

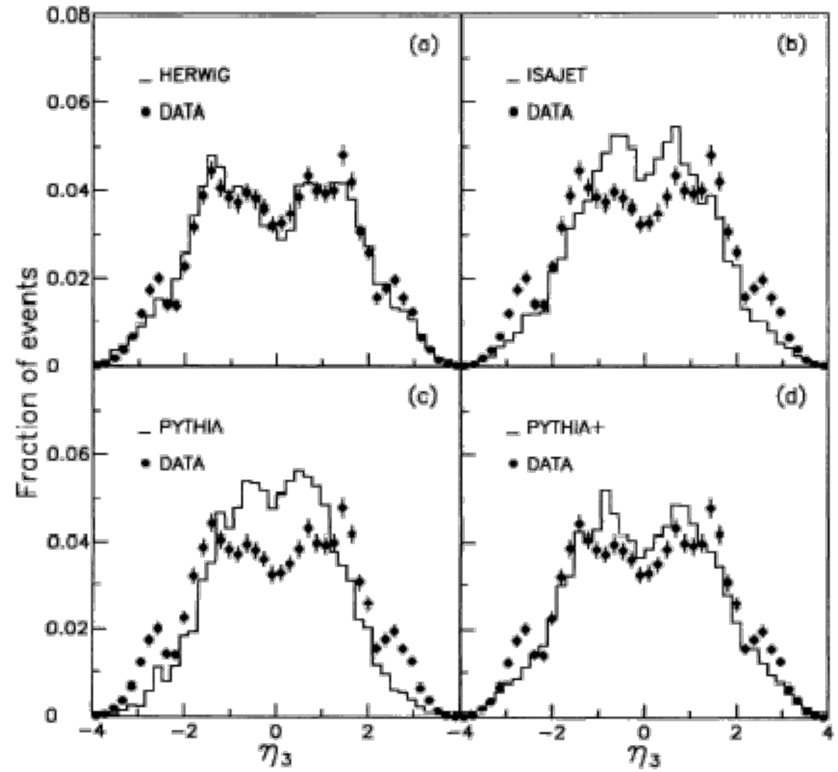
Essential to fit data…



$E_{T1} > 110$ GeV, $E_{T3} > 10$ GeV.

Distributions of third-hardest jet in multi-jet events



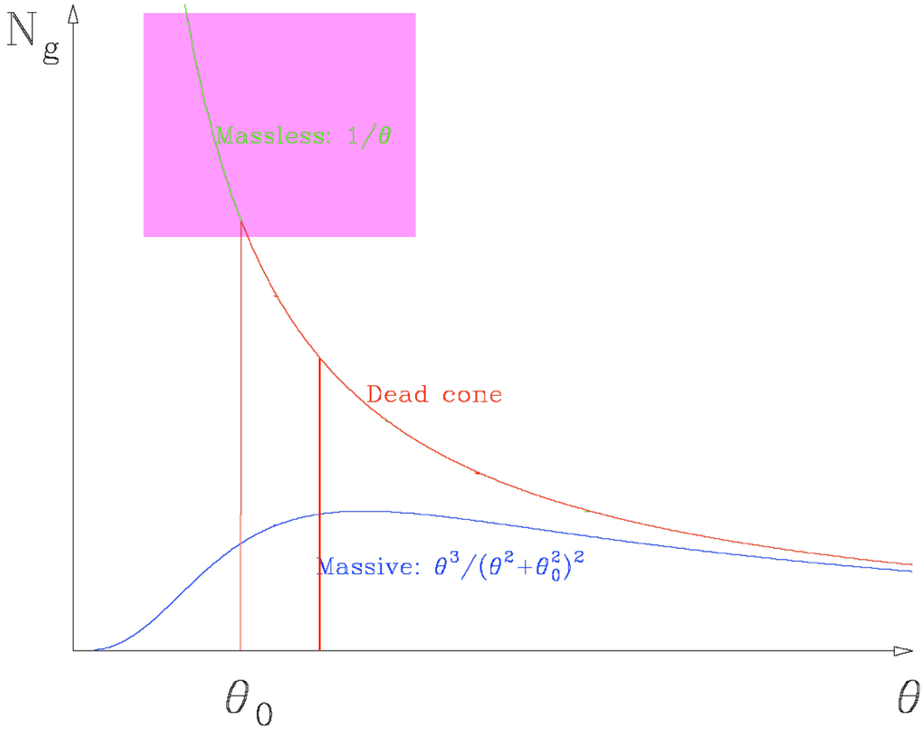


Distributions of third-hardest jet in multi-jet events

HERWIG has complete treatment of colour coherence, PYTHIA+ has partial

Heavy Quarks/Spartons

look like light quarks at large angles, sterile at small angles:



approximated as energy-dependent cutoff: $\theta > \theta_0 = \dfrac{m_q}{E_q}$.

The ‘dead cone’. Too extreme?

Heavy Quarks/Spartons

More properly treated using quasi-collinear splitting:

$$\mathrm{d}\mathcal{P}_{\widetilde{ij}\to ij} = \frac{\alpha_S}{2\pi}\,\frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2}\,\mathrm{d}z\,P_{\widetilde{ij}\to ij}\left(z,\tilde{q}\right),$$

$$\begin{aligned}
P_{q\to qg} &= \frac{C_F}{1-z}\left[1+z^2-\frac{2m_q^2}{z\tilde{q}^2}\right], \\
P_{g\to gg} &= C_A\left[\frac{z}{1-z}+\frac{1-z}{z}+z\left(1-z\right)\right], \\
P_{g\to q\bar{q}} &= T_R\left[1-2z\left(1-z\right)+\frac{2m_q^2}{z\left(1-z\right)\tilde{q}^2}\right], \\
P_{\tilde{g}\to\tilde{g}g} &= \frac{C_A}{1-z}\left[1+z^2-\frac{2m_{\tilde{g}}^2}{z\tilde{q}^2}\right], \\
P_{\tilde{q}\to\tilde{q}g} &= \frac{2C_F}{1-z}\left[z-\frac{m_{\tilde{q}}}{z\tilde{q}^2}\right],
\end{aligned}$$

$\rightarrow$ smooth suppression in forward region

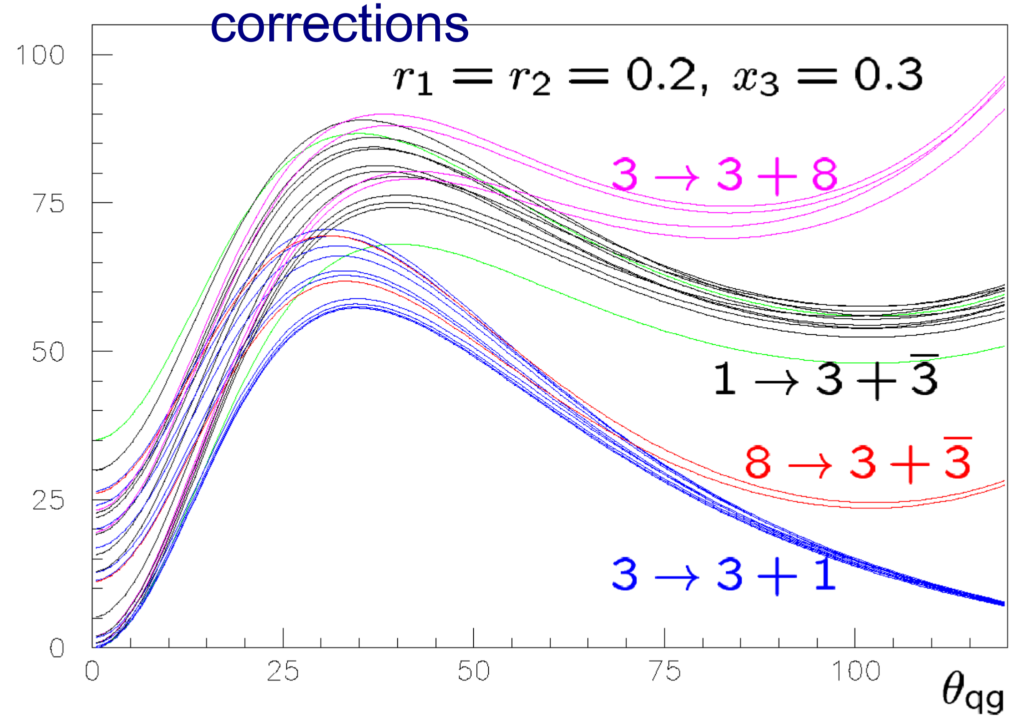
Heavy Quarks/Spartons

- Dead cone only exact for
  - emission from spin-0 particle, or
  - infinitely soft emitted gluon

- In general, depends on
  - energy of gluon
  - colours and spins of emitting particle and colour partner

→ process-dependent mass corrections

colour	spin	γ_5	example
$1 \to 3 + \bar{3}$	—	—	(eikonal)
$1 \to 3 + \bar{3}$	$1 \to \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$Z^0 \to q\bar{q}$
$3 \to 3 + 1$	$\frac{1}{2} \to \frac{1}{2} + 1$	$1, \gamma_5, 1 \pm \gamma_5$	$t \to bW^+$
$1 \to 3 + \bar{3}$	$0 \to \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$H^0 \to q\bar{q}$
$3 \to 3 + 1$	$\frac{1}{2} \to \frac{1}{2} + 0$	$1, \gamma_5, 1 \pm \gamma_5$	$t \to bH^+$
$1 \to 3 + \bar{3}$	$1 \to 0 + 0$	1	$Z^0 \to \tilde{q}\bar{\tilde{q}}$
$3 \to 3 + 1$	$0 \to 0 + 1$	1	$\tilde{q} \to \tilde{q}'W^+$
$1 \to 3 + \bar{3}$	$0 \to 0 + 0$	1	$H^0 \to \tilde{q}\bar{\tilde{q}}$
$3 \to 3 + 1$	$0 \to 0 + 0$	1	$\tilde{q} \to \tilde{q}'H^+$
$1 \to 3 + \bar{3}$	$\frac{1}{2} \to \frac{1}{2} + 0$	$1, \gamma_5, 1 \pm \gamma_5$	$\chi \to q\bar{\tilde{q}}$
$3 \to 3 + 1$	$0 \to \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$\tilde{q} \to q\chi$
$3 \to 3 + 1$	$\frac{1}{2} \to 0 + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$t \to \tilde{t}\chi$
$8 \to 3 + \bar{3}$	$\frac{1}{2} \to \frac{1}{2} + 0$	$1, \gamma_5, 1 \pm \gamma_5$	$\tilde{g} \to q\bar{\tilde{q}}$
$3 \to 3 + 8$	$0 \to \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$\tilde{q} \to q\tilde{g}$
$3 \to 3 + 8$	$\frac{1}{2} \to 0 + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$t \to \tilde{t}\tilde{g}$



The Colour Dipole Model

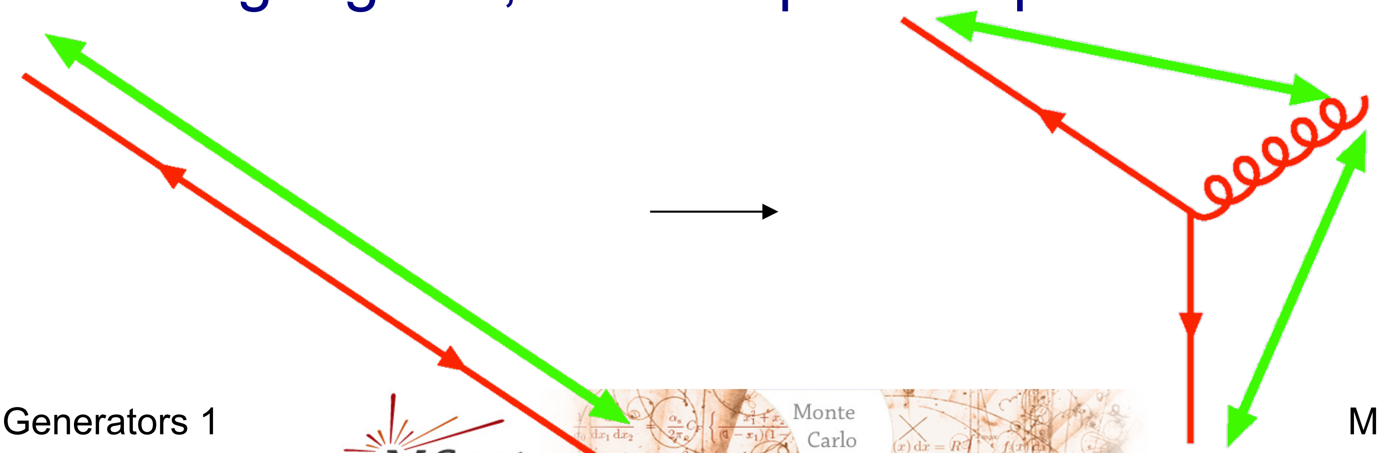
Conventional parton showers: start from collinear limit, modify to incorporate soft gluon coherence

Colour Dipole Model: start from soft limit

Emission of soft gluons from colour-anticolour dipole universal (and classical):

$$d\sigma \approx \sigma_0 \, \tfrac{1}{2} C_A \frac{\alpha_s(k_\perp)}{2\pi} \frac{dk_\perp^2}{k_\perp^2} \, dy, \qquad y = \text{rapidity} = \log \tan \theta/2$$

After emitting a gluon, colour dipole is split:

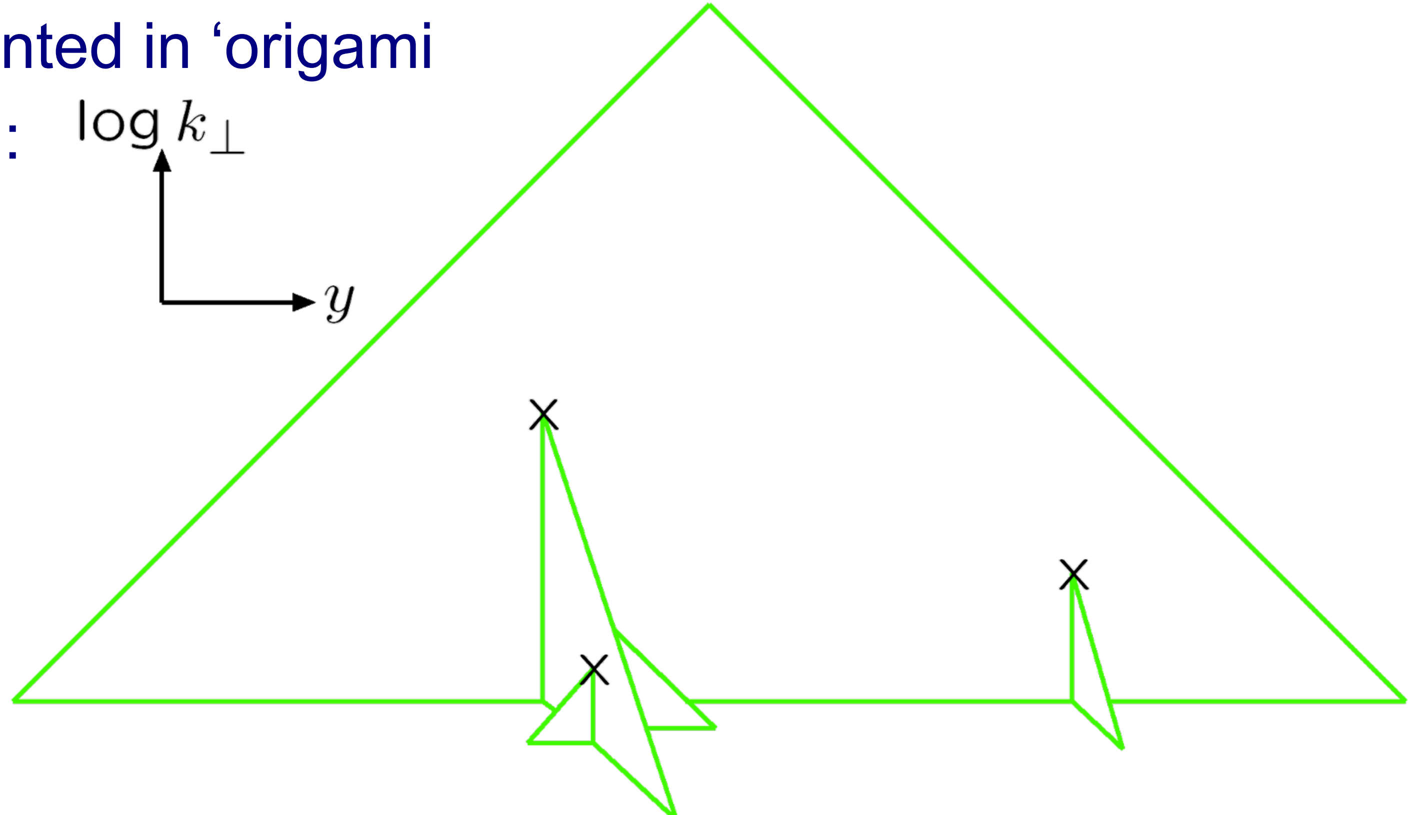


Subsequent dipoles continue to cascade

c.f. parton shower: one parton → two

CDM: one dipole → two = two partons → three

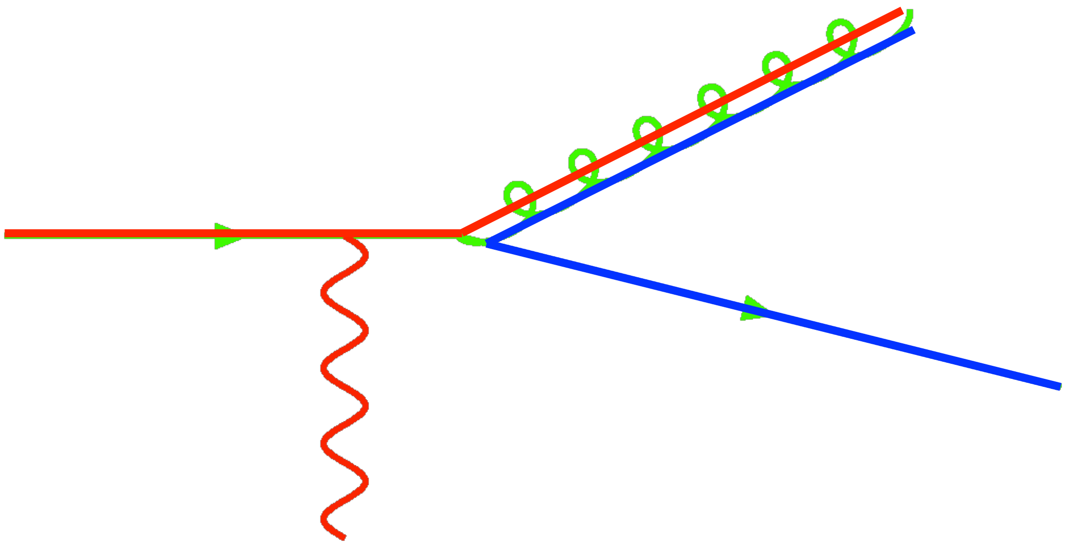
Represented in 'origami diagram':



Similar to angular-ordered parton shower for e^+e^- annihilation

Dipole cascades and colour coherence

Recall:



soft wide angle gluon sees the colour of the whole jet

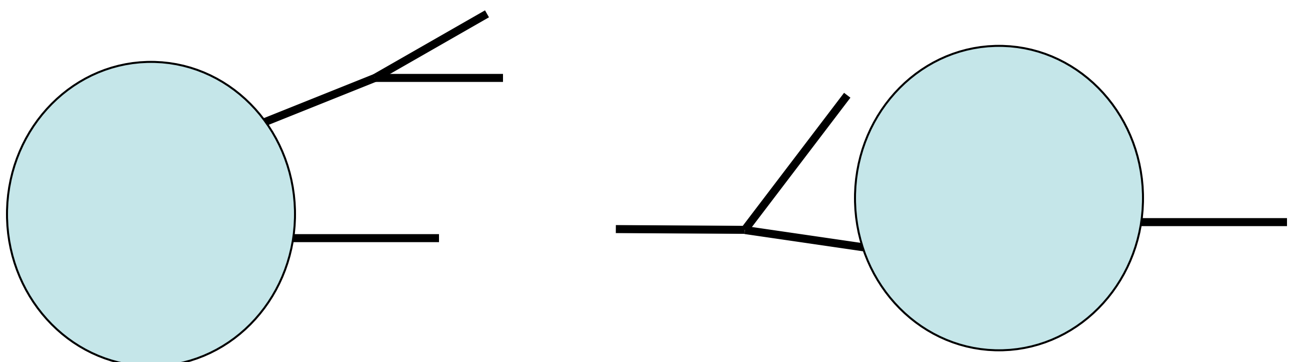
⇒ emitted first in parton shower language

but colour of whole jet is carried by emitted gluon

⇒ soft gluon emitted by hard gluon’s dipole is emitted by the whole jet

Dipole Cascades

- Most new implementations based on dipole picture:
  - Catani & MHS (1997)
  - Kosower (1998)
  - Nagy & Soper (May 2007) DEDUCTOR
  - Giele, Kosower & Skands (July 2007) VINCIA
  - Dinsdale, Ternick & Weinzierl (Sept 2007)
  - Schumann & Krauss (Sept 2007) SHERPA
  - Winter & Krauss (Dec 2007) SHERPA
  - Plätzer & Gieseke (Sept 2009) Herwig++ / Matchbox



Matrix Element Matching

Parton shower built on approximations to QCD matrix elements valid in **collinear** and **soft** approximations

→ describe bulk of radiation well → hadronic final state

→ but …

- searches for new physics
- top mass measurement
- *n* jet cross sections
- …

→ hard, well-separated jets

- described better by fixed ("leading") order matrix element
- would also like next-to-leading order normalization

→ need matrix element matching

Supported Programs

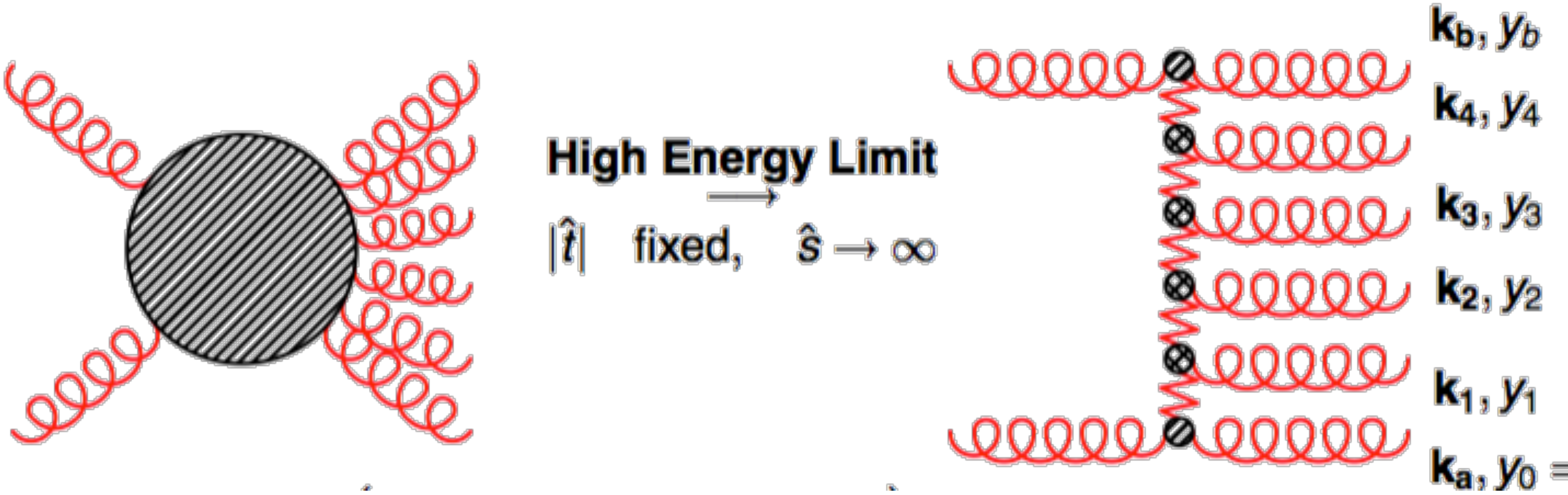
- PYTHIA 6.3: p_T-ordered parton showers, interleaved with multi-parton interactions; dipole-style recoil; matrix element for first emission in many processes.
- PYTHIA 8: new program with many of the same features as PYTHIA 6.3, many ‘obsolete’ features removed.
- SHERPA: new program built from scratch; p_T-ordered dipole showers; multi-jet and NLO matching schemes built in.
- Herwig++: new program with angular ordered parton shower (like HERWIG) plus quasi-collinear limit and recoil strategy based on colour flow; spin correlations. Coming soon: new dipole shower, with multi-jet and NLO matching schemes built in (Matchbox).

Other Programs

- There are also many specialised parton showers that have been developed, but not as part of hadron-level event generators
  - GENEVA
  - DEDUCTOR
  - VINCIA
  - Ariadne
- and also approaches based on high energy QCD evolution
  - Cascade
  - HEJ

New approaches

- HEJ (Andersson & Smillie) resums rapidity-enhanced (i.e. small-x) terms
- Can be combined with dipole shower (+Lönnblad)



New approaches

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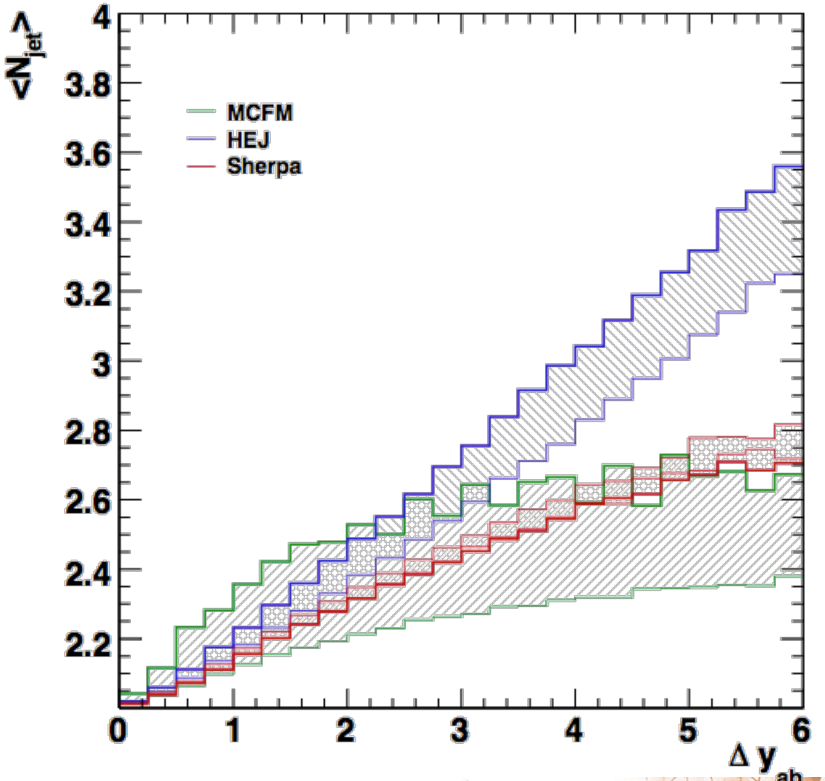

- important for Higgs production [Andersen, Campbell & Höche, arXiv:1003.1241]
- mean no. of jets as a function of rapidity distribution between most forward and most backward
  - c.f. VBF cuts/rapidity veto

Summary

- Accelerated colour charges radiate gluons.
  Gluons are also charged → cascade.

- Probabilistic language derived from factorization theorems of full gauge theory.
  Colour coherence is a fact of life: do not trust those who ignore it!
  - but corrections beyond leading colour are non-probabilistic!

- Modern parton shower models are very sophisticated implementations of perturbative QCD, but would be useless without hadronization models…