Summing logarithms in QCD

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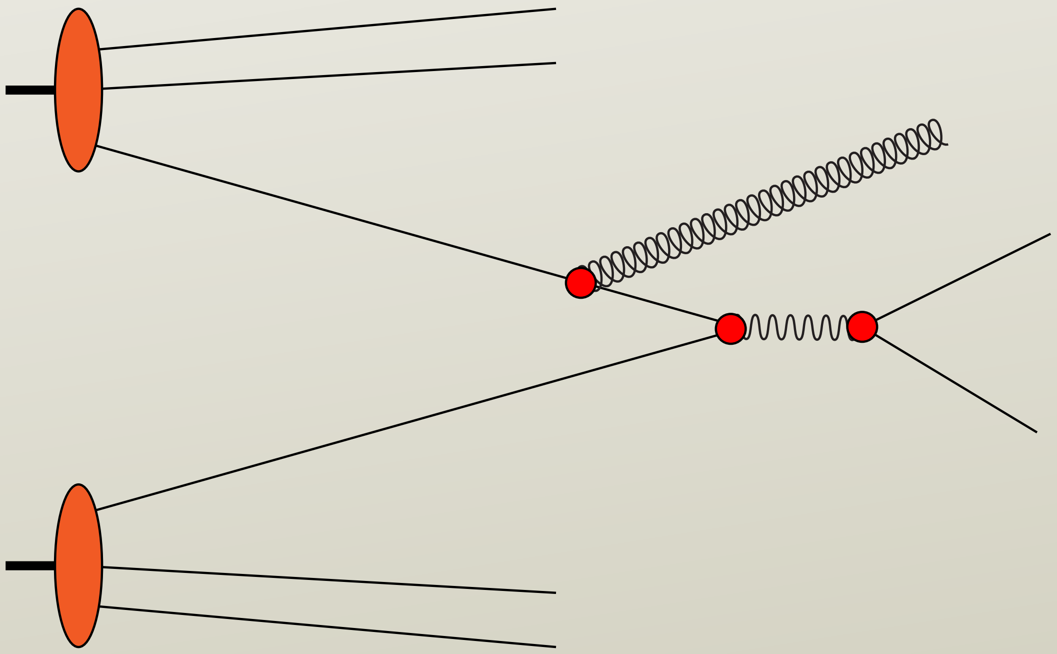
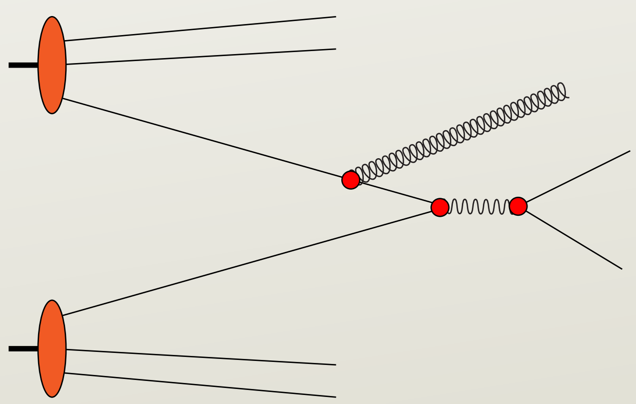
Factorization

- Perturbative QCD works with processes with a hard scale Q^2.
- *Eg.* $\mathrm{p} + \mathrm{p} \to \mu^+ + \mu^- + X$ with squared $\mu^+\mu^-$ mass Q^2 and rapidity Y.



- This is an "inclusive" process.
- We need $Q^2 \gg 1\ \mathrm{GeV}^2$.

Factorized form of the cross section

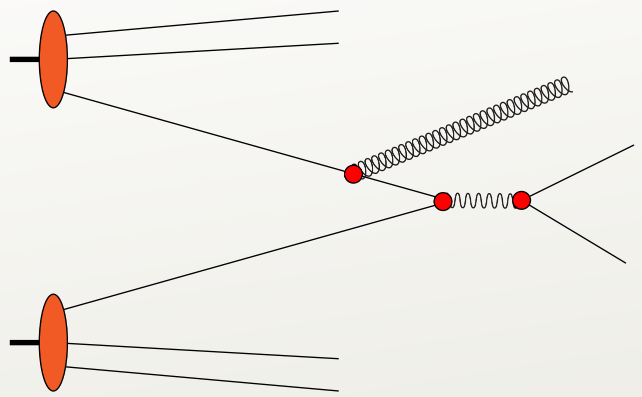


parton distribution functions

$$\frac{d\sigma}{dQ^2\,dY} = \sum_{a,b} \int d\eta_{\rm a}\; f_{a/A}(\eta_a,\mu_{\rm F}^2) \int d\eta_{\rm b}\; f_{b/B}(\eta_b,\mu_{\rm F}^2)$$

$$\times\ \frac{d\hat\sigma(a,b,\eta_a,\eta_b,Q^2,Y,\mu_{\rm F}^2)}{dQ^2\,dY}$$

hard scattering function



- The parton distribution functions are non-perturbative.
- The hard scattering function has a perturbative expansion.

$$
\begin{aligned}
\frac{d\hat{\sigma}(a,b,\eta_a,\eta_b,Q^2,Y,\mu_{\mathrm{F}}^2)}{dQ^2\,dY} &= h_0(a,b,\eta_a,\eta_b,Q^2,Y) \\
&\quad + \alpha_{\mathrm{s}}(\mu_{\mathrm{R}}^2)\,h_1(a,b,\eta_a,\eta_b,Q^2,Y,\mu_{\mathrm{F}}^2) \\
&\quad + \alpha_{\mathrm{s}}(\mu_{\mathrm{R}}^2)^2\,h_2(a,b,\eta_a,\eta_b,Q^2,Y,\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2) \\
&\quad + \cdots
\end{aligned}
$$

- We generally choose μ_{F}^2 and μ_{R}^2 of order Q^2.
- Contributions of order $1\ \mathrm{GeV}^2/Q^2$ are neglected.

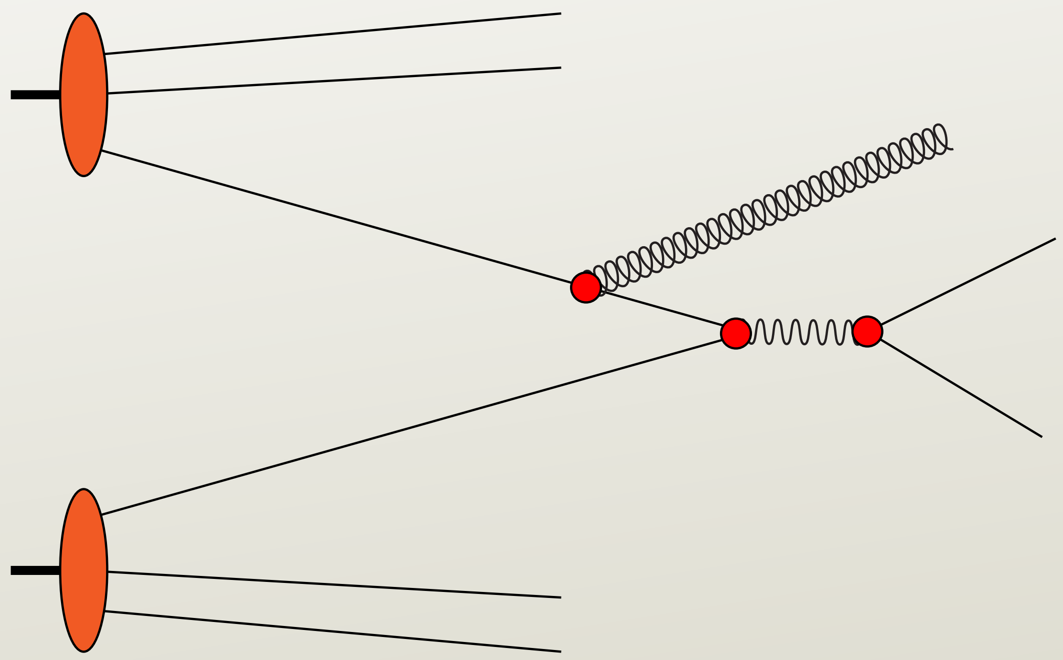
One scale

- Here there is only one large scale (assuming that we choose $\mu_\mathrm{F}^2 \sim \mu_\mathrm{R}^2 \sim Q^2$.)

$$
\begin{aligned}
\frac{d\hat\sigma(a,b,\eta_a,\eta_b,Q^2,Y,\mu_\mathrm{F}^2)}{dQ^2\,dY} &= h_0(a,b,\eta_a,\eta_b,Q^2,Y) \\
&\quad + \alpha_\mathrm{s}(\mu_\mathrm{R}^2)\, h_1(a,b,\eta_a,\eta_b,Q^2,Y,\mu_\mathrm{F}^2) \\
&\quad + \alpha_\mathrm{s}(\mu_\mathrm{R}^2)^2\, h_2(a,b,\eta_a,\eta_b,Q^2,Y,\mu_\mathrm{F}^2,\mu_\mathrm{R}^2) \\
&\quad + \cdots
\end{aligned}
$$

- The higher order coefficients h_n are not particularly large, about the same size as h_0.
- $\alpha_\mathrm{s}(\mu_\mathrm{R}^2)$ is small.
- So perturbation theory works well.

Two scales



- *Eg.* $\mathrm{p} + \mathrm{p} \to \mu^+ + \mu^- + X$ with squared $\mu^+\mu^-$ mass Q^2, rapidity Y, and squared transverse momentum $Q_\perp^2$.

$$\frac{d\sigma}{dQ^2\,dQ_\perp^2\,dY} = \sum_{a,b} \int d\eta_{\mathrm{a}}\; f_{a/A}(\eta_a, \mu_{\mathrm{F}}^2) \int d\eta_{\mathrm{b}}\; f_{b/B}(\eta_b, \mu_{\mathrm{F}}^2)$$

$$\times\ \frac{d\hat{\sigma}(a, b, \eta_a, \eta_b, Q^2, Q_\perp^2, Y, \mu_{\mathrm{F}}^2)}{dQ^2\,dQ_\perp^2\,dY}$$

Perturbative expansion

$$
\begin{aligned}
&\frac{d\hat{\sigma}(a,b,\eta_a,\eta_b,Q^2,Q_\perp^2,Y,\mu_{\rm F}^2)}{dQ^2\,dQ_\perp^2\,dY} \\
&\quad = h_0(a,b,\eta_a,\eta_b,Q^2,Q_\perp^2,Y) \\
&\qquad + \alpha_{\rm s}(\mu_{\rm R}^2)\,h_1(a,b,\eta_a,\eta_b,Q^2,Q_\perp^2,Y,\mu_{\rm F}^2) \\
&\qquad + \alpha_{\rm s}(\mu_{\rm R}^2)^2\,h_2(a,b,\eta_a,\eta_b,Q^2,Q_\perp^2,Y,\mu_{\rm F}^2,\mu_{\rm R}^2) \\
&\qquad + \cdots
\end{aligned}
$$

- Now the coefficients depend on $Q^2/Q_\perp^2$.
- They are proportional to $1/Q_\perp^2$ times logs of $Q^2/Q_\perp^2$.

$$
h_n \propto \frac{\log^{2n-1}(Q^2/Q_\perp^2)}{Q_\perp^2} + \cdots
$$

What if the logarithm is large?

$$\alpha_{\mathrm{s}}^n\, h_n \propto \alpha_{\mathrm{s}}^n\, \frac{\log^{2n-1}(Q^2/Q_\perp^2)}{Q_\perp^2} + \cdots$$

- If $Q^2/Q_\perp^2 \gg 1$, then $\alpha_{\mathrm{s}} \log^2(Q^2/Q_\perp^2)$ may not be small.
- The usefulness of perturbation theory may be destroyed.

This is rather common

- The thrust distribution in e^+e^- annihilation for $1 - T \ll 1$.
- A jet cross section defined with a jet radius R with $R \ll 1$.
- The Higgs boson $Q_\perp^2$ distribution for $Q_\perp^2 \ll M_\mathrm{H}^2$.
- The cross section for a W plus one jet with $P_\perp \sim M_\mathrm{W}$ and no other jets with $P_\perp > Q_\perp$ where $Q_\perp \ll M_\mathrm{W}$.
- The evolution of the coupling and of the parton distributions with scale.

What should we do?

- We need to sum the large logarithms.

Example with one log per loop

- The running coupling at scale Q^2 is related to the coupling at scale M_Z^2 by

$$\alpha_\mathrm{s}(Q^2) = \alpha_\mathrm{s}(M_\mathrm{Z}^2) \sum_{n=0}^{\infty} \sum_{l=0}^{n} C(n,l)\, \alpha_\mathrm{s}^n(M_\mathrm{Z}^2) \log^l(Q^2/M_\mathrm{Z}^2)$$

- This series is not useful if $\log(Q^2/M_\mathrm{Z}^2)$ is large.

- We can try to sum the "leading logs": those with $l = n$.

- The leading logs are

$$\alpha_{\mathrm{s}}(Q^2) = \alpha_{\mathrm{s}}(M_{\mathrm{Z}}^2) \sum_{n=0}^{\infty} \left(-\frac{\beta_0}{4\pi} \right)^n \alpha_{\mathrm{s}}^n(M_{\mathrm{Z}}^2) \log^n(Q^2/M_{\mathrm{Z}}^2)$$

- These sum to

$$\alpha_{\mathrm{s}}(Q^2) = \frac{\alpha_{\mathrm{s}}(M_{\mathrm{Z}}^2)}{1 + \frac{\beta_0}{4\pi}\,\alpha_{\mathrm{s}}(M_{\mathrm{Z}}^2) \log(Q^2/M_{\mathrm{Z}}^2)}$$

- This is useful when $\alpha_{\mathrm{s}}(M_{\mathrm{Z}}^2) \log(Q^2/M_{\mathrm{Z}}^2) \lesssim 1$.

- We know more terms, so this can become very accurate.

Two logs per loop

- Return to the Drell-Yan cross section.

$$
\begin{aligned}
&\frac{d\hat{\sigma}(a,b,\eta_a,\eta_b,Q^2,Q_\perp^2,Y,\mu_{\mathrm{F}}^2)}{dQ^2\,dQ_\perp^2\,dY} \\
&\qquad = h_0(a,b,\eta_a,\eta_b,Q^2,Q_\perp^2,Y) \\
&\qquad\quad + \alpha_{\mathrm{s}}(\mu_{\mathrm{R}}^2)\,h_1(a,b,\eta_a,\eta_b,Q^2,Q_\perp^2,Y,\mu_{\mathrm{F}}^2) \\
&\qquad\quad + \alpha_{\mathrm{s}}(\mu_{\mathrm{R}}^2)^2\,h_2(a,b,\eta_a,\eta_b,Q^2,Q_\perp^2,Y,\mu_{\mathrm{F}}^2,\mu_{\mathrm{R}}^2) \\
&\qquad\quad + \cdots
\end{aligned}
$$

with

$$
\alpha_{\mathrm{s}}^n\,h_n \propto \alpha_{\mathrm{s}}^n\,\frac{\log^{2n-1}(Q^2/Q_\perp^2)}{Q_\perp^2} + \cdots
$$

- We need to sum the large logarithms.
- For example:

$$\frac{d\sigma}{dQ^2\,dY\,dQ_\perp^2} \approx \sigma_0\ \frac{1}{Q_\perp^2}\frac{\alpha_{\rm s}}{\pi}\,C_{\rm F}\log(Q^2/Q_\perp^2)$$
$$\times \exp\left(-\frac{\alpha_{\rm s}}{\pi}\,C_{\rm F}\frac{1}{2}\log^2(Q^2/Q_\perp^2)\right)$$

- This is adapted from Parisi and Petronzio (1979).
- Corrections,

$$\alpha_{\rm s}^n\,\Delta h_n \propto \alpha_{\rm s}^n\,\frac{\log^{2n-2}(Q^2/Q_\perp^2)}{Q_\perp^2} + \cdots$$

- The formula

$$\frac{d\sigma}{dQ^2\,dY\,dQ_\perp^2} \approx \sigma_0\ \frac{1}{Q_\perp^2}\,\frac{\alpha_{\rm s}}{\pi}\,C_{\rm F}\log(Q^2/Q_\perp^2)$$
$$\times\exp\!\left(-\frac{\alpha_{\rm s}}{\pi}\,C_{\rm F}\,\frac{1}{2}\log^2(Q^2/Q_\perp^2)\right)$$

sums the "leading logarithms"

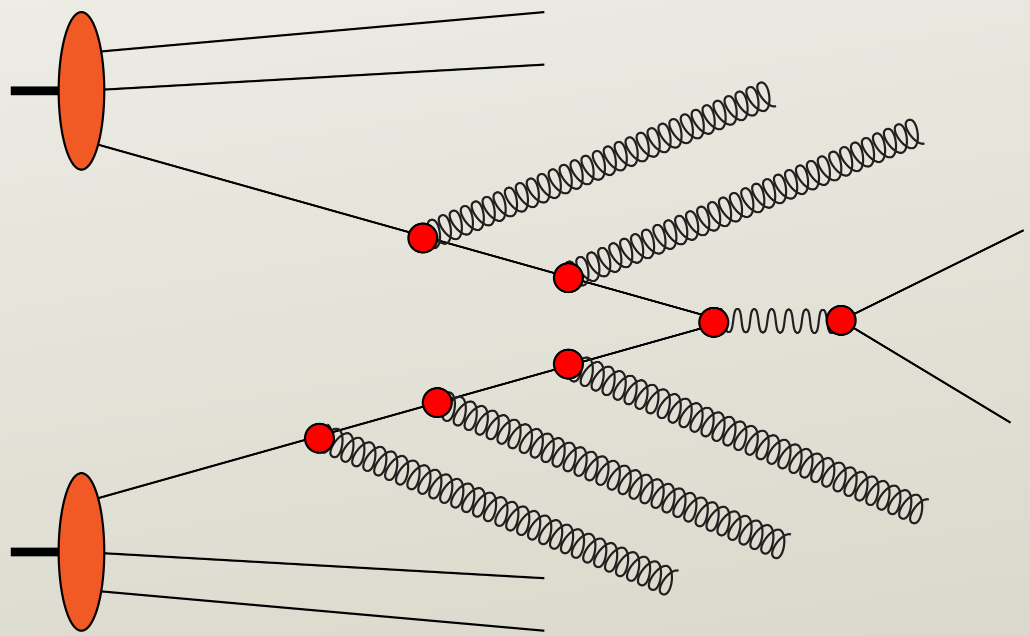
$$\alpha_{\rm s}^n\,h_n^{\rm LL} = \alpha_{\rm s}^n\,C_n\,\frac{\log^{2n-1}(Q^2/Q_\perp^2)}{Q_\perp^2}$$

- This should work if $\alpha_{\rm s}\log^2(Q^2/Q_\perp^2) \lesssim 1$.

- But that is a pretty limited range.

How do do better

- The muon pair gets $\boldsymbol{Q}_\perp$ by recoiling against soft gluons.



- This gives

$$\int d\boldsymbol{q}_1\; d\boldsymbol{q}_2 \cdots d\boldsymbol{q}_n\; \delta\Big(\boldsymbol{Q}_\perp - \sum \boldsymbol{q}_j\Big)\cdots$$

- The delta function analysis difficult.

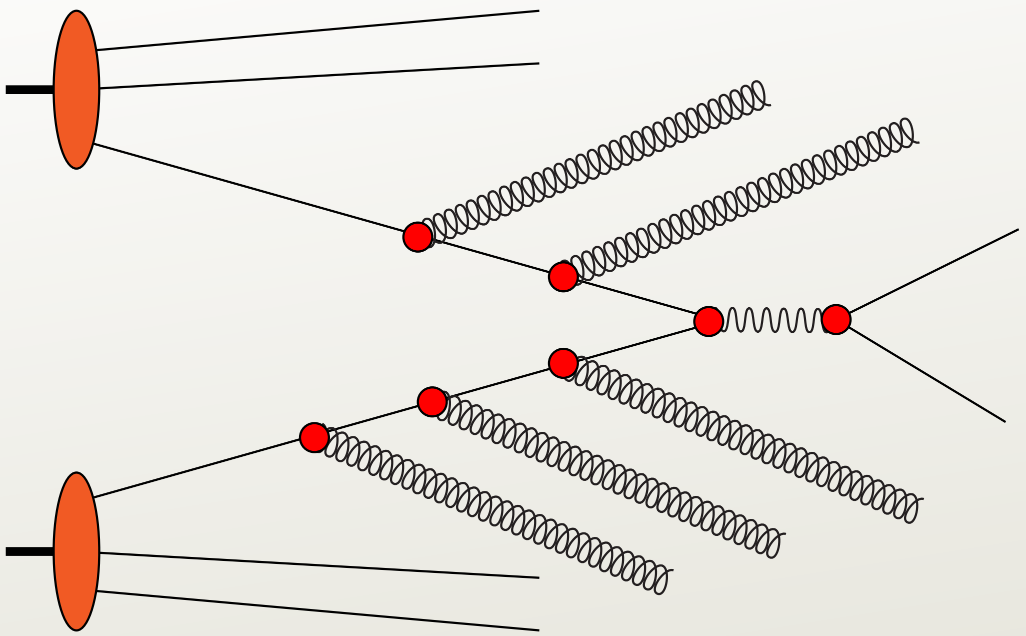
- Parisi and Petronzio (1979) suggested using a Fourier transform:

$$\int d\boldsymbol{Q}_\perp\; e^{-\mathrm{i}\boldsymbol{Q}_\perp\cdot\boldsymbol{b}} \int d\boldsymbol{q}_1 \int d\boldsymbol{q}_2 \cdots \int d\boldsymbol{q}_n\; \delta\Big(\boldsymbol{Q}_\perp - \sum \boldsymbol{q}_j\Big)\cdots$$
$$= \int d\boldsymbol{q}_1\; e^{-\mathrm{i}\boldsymbol{q}_1\cdot\boldsymbol{b}} \int d\boldsymbol{q}_2\; e^{-\mathrm{i}\boldsymbol{q}_2\cdot\boldsymbol{b}} \ldots \int d\boldsymbol{q}_n\; e^{-\mathrm{i}\boldsymbol{q}_n\cdot\boldsymbol{b}} \ldots$$

- Take Fourier transforms so that each emission is in $\boldsymbol{b}$ space.

- Multiply.

- Fourier transform back at the end.



- Parisi and Petronzio argued that one can treat all of the emissions as independent.

- Then

$$\frac{d\sigma}{dQ^2\,dY\,dQ_\perp^2} \approx \sigma_0\ (2\pi)^{-2}\int d\boldsymbol{b}\ e^{\mathrm{i}\boldsymbol{Q}_\perp\cdot\boldsymbol{b}} \\ \times\exp\left(-\frac{\alpha_{\mathrm{s}}}{\pi}\,C_{\mathrm{F}}\,\frac{1}{2}\,\log^2(Q^2\boldsymbol{b}^2)\right)$$

- The $\boldsymbol{b}$-space formula

$$\frac{d\sigma}{dQ^2\,dY\,dQ_\perp^2} \approx \sigma_0\ (2\pi)^{-2} \int d\boldsymbol{b}\ e^{\mathrm{i}\boldsymbol{Q}_\perp\cdot\boldsymbol{b}} \times \exp\left(-\frac{\alpha_{\mathrm{s}}}{\pi}\,C_{\mathrm{F}}\,\frac{1}{2}\,\log^2(Q^2\boldsymbol{b}^2)\right)$$

reproduces

$$\frac{d\sigma}{dQ^2\,dY\,dQ_\perp^2} \approx \sigma_0\ \frac{1}{Q_\perp^2}\,\frac{\alpha_{\mathrm{s}}}{\pi}\,C_{\mathrm{F}}\log(Q^2/Q_\perp^2) \times \exp\left(-\frac{\alpha_{\mathrm{s}}}{\pi}\,C_{\mathrm{F}}\,\frac{1}{2}\log^2(Q^2/Q_\perp^2)\right)$$

if we keep only the leading logs, $\log(Q^2/Q_\perp^2)$.

- The $\boldsymbol{b}$-space formula gives a finite cross section for $\boldsymbol{Q}_\perp \to 0$.
- The $\boldsymbol{Q}_\perp$-space formula gives zero for $\boldsymbol{Q}_\perp \to 0$.
- It seems plausible that the $\boldsymbol{b}$-space formula is better.
- It is not obvious how much better the $\boldsymbol{b}$-space formula is.

A more exact analysis

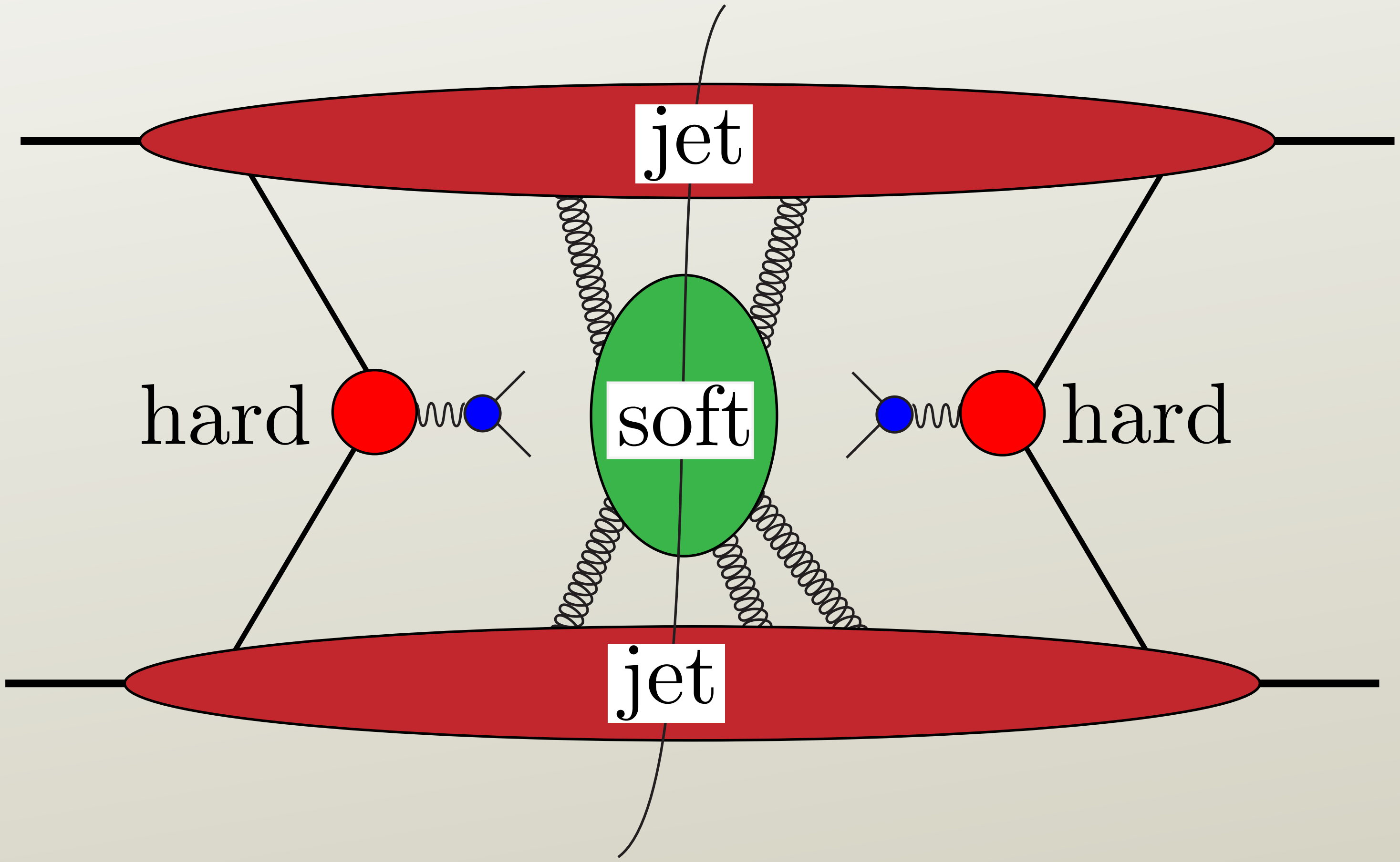
Collins, Soper, Sterman (1985)

- Use the $\boldsymbol{b}$-space idea.

- Work at all orders in QCD.

- Use a physical gauge. (A better treatment uses Feynman gauge; see J.C. Collins, *Foundations of Perturbative QCD*.)

- Begin with an analysis of what integration regions for parton momenta are important.



- Soft-collinear effective theory (SCET) starts this way.
- Develop differential equations for the pieces.

- The QCD result for $Q_\perp^2 \ll Q^2$.

$$
\begin{aligned}
&\frac{d\sigma}{dQ^2\,dY\,dQ_\perp^2} \approx \\
&\quad \frac{1}{4\pi}\int d\boldsymbol{b}\; e^{\mathrm{i}\boldsymbol{Q}_\perp\cdot\boldsymbol{b}} \\
&\qquad \times \sum_{a,b}\int_{x_\mathrm{a}}^{1}\frac{d\eta_\mathrm{a}}{\eta_\mathrm{a}}\int_{x_\mathrm{a}}^{1}\frac{d\eta_\mathrm{b}}{\eta_\mathrm{b}}\; f_{a/A}(\eta_\mathrm{a},C^2/\boldsymbol{b}^2)\, f_{b/B}(\eta_\mathrm{b},C^2/\boldsymbol{b}^2) \\
&\qquad \times \exp\left(-\int_{C^2/\boldsymbol{b}^2}^{Q^2}\frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\left[A(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2))\log\left(\frac{Q^2}{\boldsymbol{k}_\perp^2}\right)+B(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2))\right]\right) \\
&\qquad \times \sum_{a'b'} H^{(0)}_{a'b'}\; C_{a'a}\left(\frac{x_\mathrm{a}}{\eta_\mathrm{a}},\alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_\mathrm{b}}{\eta_\mathrm{b}},\alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right)
\end{aligned}
$$

- A, B, and C have perturbative expansions.
- Need some nonperturbative input for very large $\boldsymbol{b}^2$.

- Look at

$$\exp\left(-\int_{C^2/\boldsymbol{b}^2}^{Q^2}\frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\left[A(\alpha_s(\boldsymbol{k}_\perp^2))\log\left(\frac{Q^2}{\boldsymbol{k}_\perp^2}\right)+B(\alpha_s(\boldsymbol{k}_\perp^2))\right]\right)$$

$$\begin{aligned} A(\alpha_s(\boldsymbol{k}_\perp^2)) &= A^{(1)}\,\frac{\alpha_s(\boldsymbol{k}_\perp^2)}{2\pi}+\cdots \\ &= A^{(1)}\,\frac{\alpha_s(Q^2)}{2\pi}+\cdots \end{aligned}$$

- $A^{(1)} = 2C_F$.
- This gives

$$\exp\left(-A^{(1)}\,\frac{\alpha_s(Q^2)}{2\pi}\,\frac{1}{2}\log^2(Q^2\boldsymbol{b}^2/C^2)+\cdots\right)$$

- This agrees with Parisi and Petronzio.

- Look some more.

$$\exp\left(-\int_{C^2/\boldsymbol{b}^2}^{Q^2}\frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\left[A(\alpha_s(\boldsymbol{k}_\perp^2))\log\left(\frac{Q^2}{\boldsymbol{k}_\perp^2}\right)+B(\alpha_s(\boldsymbol{k}_\perp^2))\right]\right)$$

$$A(\alpha_s) = A^{(1)}\,\frac{\alpha_s}{2\pi} + A^{(2)}\left(\frac{\alpha_s}{2\pi}\right)^2 + \cdots$$

$$B(\alpha_s) = B^{(1)}\,\frac{\alpha_s}{2\pi} + \cdots$$

- We get

$$\exp\left(-\sum_{n=1}^{\infty}\alpha_s^n(Q^2)\sum_{l=0}^{n+1}\log^l(Q^2\boldsymbol{b}^2/C^2)\right)$$

- Only one power of log for each new power of α_s.

Accuracy of approximation

$$\exp\left(-\int_{C^2/\boldsymbol{b}^2}^{Q^2}\frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\left[A(\alpha_s(\boldsymbol{k}_\perp^2))\log\left(\frac{Q^2}{\boldsymbol{k}_\perp^2}\right)+B(\alpha_s(\boldsymbol{k}_\perp^2))\right]\right)$$

- If we know $A^{(1)}$, $A^{(2)}$, and $B^{(1)}$, then unknown terms are of order $\alpha_s(Q^2)\,[\alpha_s(Q^2)\log(Q^2\boldsymbol{b}^2/C^2)]^n$.
- Unknown terms are small as long as $\alpha_s(Q^2)\log(Q^2\boldsymbol{b}^2/C^2) \lesssim 1$.
- We need the exponent for $\boldsymbol{b}^2 < b_{\max}^2$ where $\alpha_s(Q^2)\log^2(Q^2\boldsymbol{b}^2/C^2) = \text{few}$.
- So for large enough Q^2, we have a result that is accurate over the entire needed range of $\boldsymbol{b}^2$.

$$
\begin{aligned}
&\frac{d\sigma}{dQ^2\, dY\, dQ_\perp^2} \approx \\
&\quad \frac{1}{4\pi} \int d\boldsymbol{b}\; e^{\mathrm{i}\boldsymbol{Q}_\perp \cdot \boldsymbol{b}} \\
&\qquad \times \sum_{a,b} \int_{x_\mathrm{a}}^1 \frac{d\eta_\mathrm{a}}{\eta_\mathrm{a}} \int_{x_\mathrm{a}}^1 \frac{d\eta_\mathrm{b}}{\eta_\mathrm{b}}\; f_{a/A}(\eta_\mathrm{a}, C^2/\boldsymbol{b}^2)\, f_{b/B}(\eta_\mathrm{b}, C^2/\boldsymbol{b}^2) \\
&\qquad \times \exp\left(- \int_{C^2/\boldsymbol{b}^2}^{Q^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2} \left[A(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \log\left(\frac{Q^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2)) \right] \right) \\
&\qquad \times \sum_{a'b'} H^{(0)}_{a'b'}\; C_{a'a}\left(\frac{x_\mathrm{a}}{\eta_\mathrm{a}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_\mathrm{b}}{\eta_\mathrm{b}}, \alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right)
\end{aligned}
$$

- Improvements:
  - Match to a non-perturbative model at large $\boldsymbol{b}^2$.
  - Match to fixed order perturbation theory at large $Q_\perp^2$.

Resbos

- These formulas are implemented in ResBos by C.P. Yuan, Csaba Balazs, and Pavel Nadolsky.
- See `http://hep.pa.msu.edu/resum/`

Coordinated Theoretical-Experimental study on Quantum chromodynamics

Q_T resummation portal

at Michigan State University

A collection of resources on transverse momentum resummation

Online plotter of resummed cross sections

- Home • Theory overview • Computer programs and usage policy • Particle processes • Our publications • Bibliography

NEW CTEQ6.6 grids for W, Z, H, and di-boson pair productions; ResBos with PDF reweighting and output into ROOT ntuples using FROOT Some sample input files for various processes can be found here.

Online Plotter of ResBos

Download the latest resummation code (Fortran)

- ResBos (C, P , CP versions)
- ResBos-A
- RhicBos
- ResBos for SIDIS

Why different versions?

Processes

- $p\overset{(-)}{p} \to W^{\pm}X$
- $p\overset{(-)}{p} \to Z^0 X$
- $p\overset{(-)}{p} \to \gamma^* X$

Transverse momentum (or Q_T) resummation is a powerful method to predict differential distributions of elementary particles in quantum chromodynamics. Its main features and differences from Monte-Carlo showering methods are discussed in the brief **overview of resummation theory**. Our group is actively involved in the development of transverse momentum resummation methods in essential collider processes. This page collects various resources for computation of resummed cross sections, including publicly distributed **computer codes**, **references to journal papers** published by our group, and relevant **bibliography**.

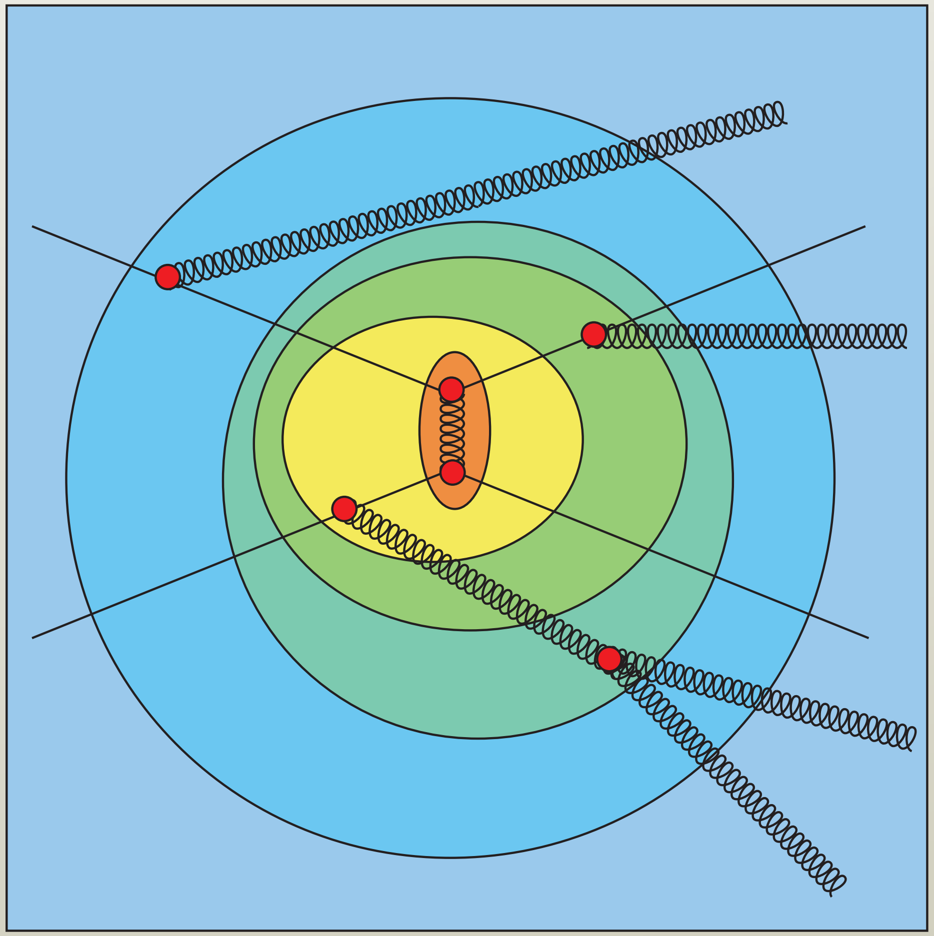
Computer programs

A quick plot of the resummed Q_T distribution for a given invariant mass and rapidity can be made with the help of **the online plotter of resummed cross sections**, which provides an intuitive user interface and produces figures in Postscript and GIF formats. For more detailed studies of resummed cross sections, a ResBos family of Fortran programs is publicly available.

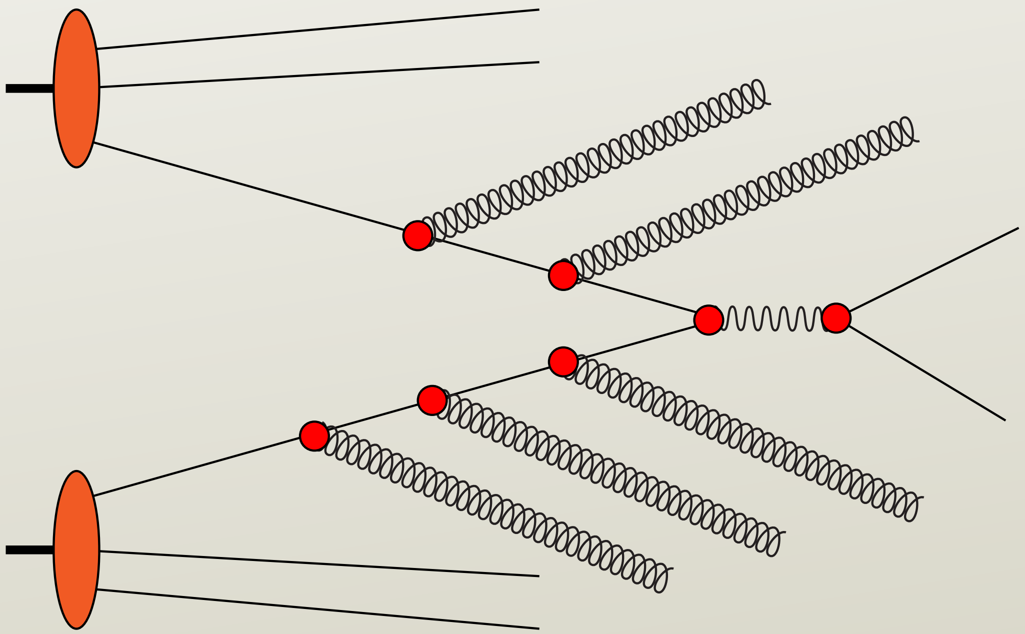
- **ResBos** -- calculation of resummed initial-state contributions in unpolarized Drell-Yan-like processes at hadron-hadron colliders. At present, two branches of the ResBos code are supported. They are mostly compatible with one another, but optimized for different tasks:
  - **branch C** -- original ResBos version, supported by **Csaba Balazs** (**old versions**);
  - **branch P** -- the ResBos version adapted for various CTEQ studies, supported by **Pavel Nadolsky**.
  - **branch CP** -- the ResBos version adapted for various CTEQ studies, supported by **C.-P. Yuan**. The needed grid files are here**.**
- **ResBos-A** -- a program spawned by ResBos that includes final-state NLO electromagnetic contributions in W boson production, supported by **C.-P. Yuan**. The inputs for this program are not compatible with ResBos inputs and can be downloaded **here**. More grid files are available below.
- **RhicBos** -- ResBos optimized for polarized hadron-hadron collisions at the Relativistic Heavy Ion Collider; supported by **Pavel Nadolsky**.
- **ResBos-DIS** -- a program for computation of resummed hadronic distributions in semi-inclusive deep inelastic scattering at lepton-hadron colliders; supported by **Pavel Nadolsky**.

Parton shower approach

- We will study parton shower event generators in this school.
- These typically start at a hard interaction.
- The participating partons are really jets. But their structure is unresolved.
- Then they move to successively softer interactions, resolving more.

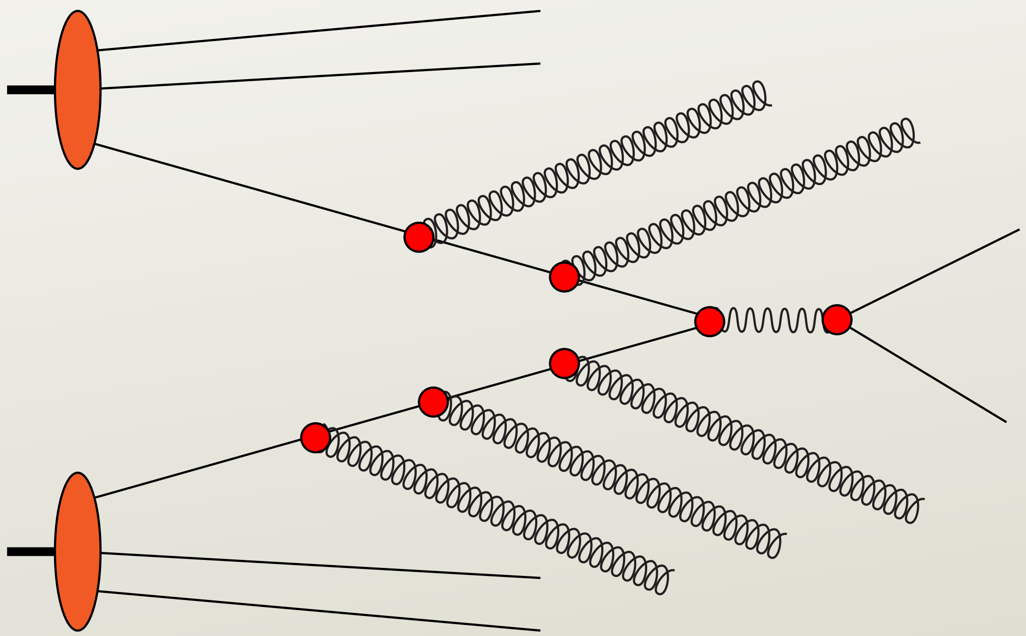


- Parton shower event generators contain the matrix elements for soft gluon emissions.



- So maybe they can correctly sum logs of $Q_\perp$ in the Drell-Yan process.

- Zoltan Nagy and I investigated this.



- We considered a virtuality ordered parton shower.

- We developed differential equations for what the shower does and solved the equations analytically.

- Result for the general structure:

$$\frac{d\sigma}{dQ^2\,dY\,dQ_\perp^2} \approx$$

✓ Exponentiation

$$\frac{1}{4\pi}\int d\boldsymbol{b}\; e^{\mathrm{i}\boldsymbol{Q}_\perp\cdot\boldsymbol{b}}$$

$$\times \sum_{a,b}\int_{x_\mathrm{a}}^{1}\frac{d\eta_\mathrm{a}}{\eta_\mathrm{a}}\int_{x_\mathrm{a}}^{1}\frac{d\eta_\mathrm{b}}{\eta_\mathrm{b}}\; f_{a/A}(\eta_\mathrm{a},C^2/\boldsymbol{b}^2)\; f_{b/B}(\eta_\mathrm{b},C^2/\boldsymbol{b}^2)$$

$$\times \exp\left(-\int_{C^2/\boldsymbol{b}^2}^{Q^2}\frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\left[A(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2))\log\left(\frac{Q^2}{\boldsymbol{k}_\perp^2}\right)+B(\alpha_\mathrm{s}(\boldsymbol{k}_\perp^2))\right]\right)$$

$$\times \sum_{a'b'}H_{a'b'}^{(0)}\; C_{a'a}\left(\frac{x_\mathrm{a}}{\eta_\mathrm{a}},\alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_\mathrm{b}}{\eta_\mathrm{b}},\alpha_\mathrm{s}\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right)$$

- Note: seemingly minor details of the shower algorithm matter for this.

- Result for the coefficients:

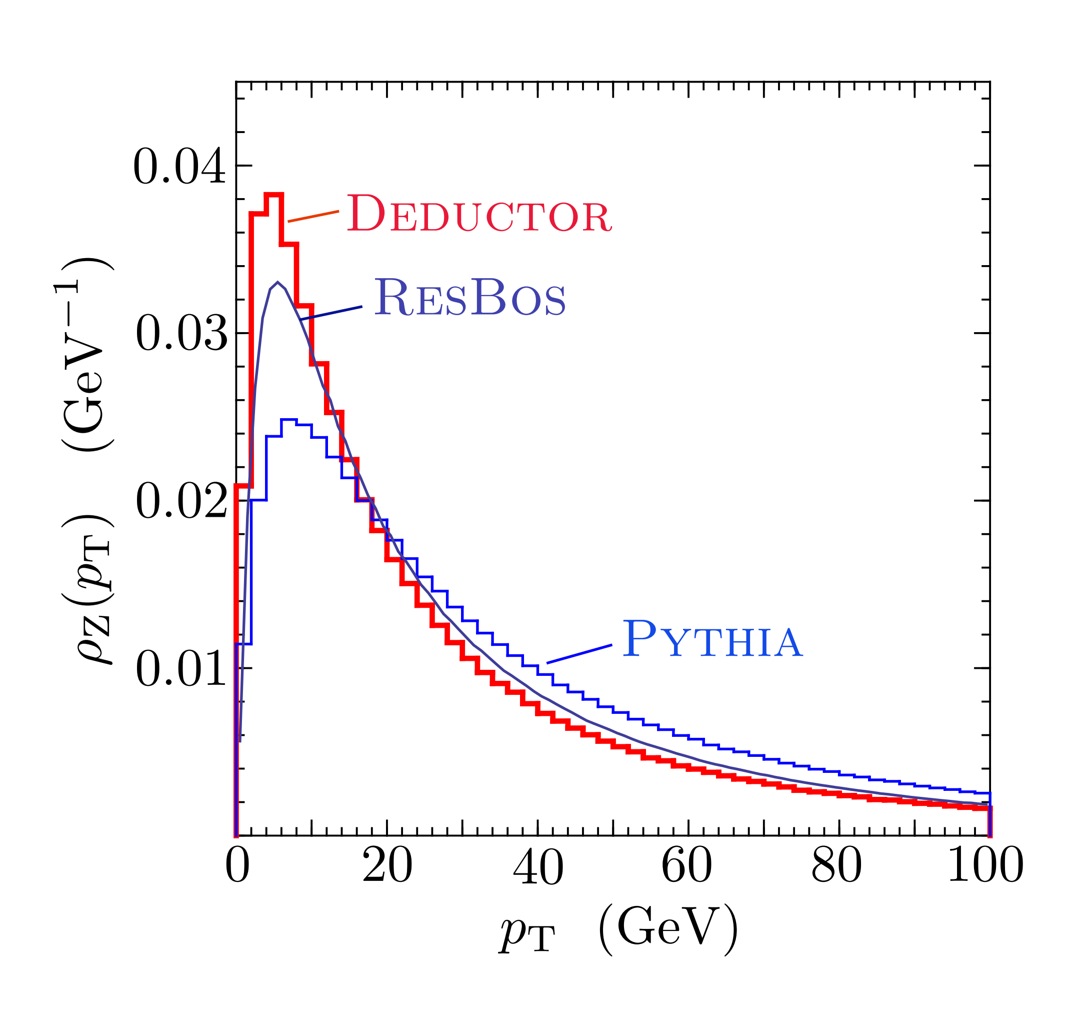
$$A(\alpha_s) = 2\,C_F\,\frac{\alpha_s}{2\pi} + 2\,C_F\,\left\{C_A\left[\frac{67}{18} - \frac{\pi^2}{6}\right] - \frac{5\,n_f}{9}\right\}\left(\frac{\alpha_s}{2\pi}\right)^2 + \cdots\quad,$$

$$B(\alpha_s) = -4\,\frac{\alpha_s}{2\pi} + \xcancel{\left[-\frac{197}{3} + \frac{34n_f}{9} + \frac{20\pi^2}{3} - \frac{8n_f\pi^2}{27} + \frac{8\zeta(3)}{3}\right]}\left(\frac{\alpha_s}{2\pi}\right)^2 + \cdots\quad,$$

$$C_{a'a}(z,\alpha_s) = \delta_{a'a}\delta(1-z) + \frac{\alpha_s}{2\pi}\xcancel{\left[\delta_{a'a}\left\{\frac{4}{3}(1-z) + \frac{2}{3}\delta(1-z)\left(\pi^2-8\right)\right\} + \delta_{ag}\,z(1-z)\right]}$$

Numerical comparison

- Compare with parton shower DEDUCTOR (Nagy-Soper 2014).
- Compare DEDUCTOR (no hadronization),PYTIA (hadronization turned off), & RESBOS (with non-perturbative functions).
- Look at distribution of P_{T} of $e^{+}e^{-}$ pairs with $M > 400$ GeV.
- $\int_0^{100\ \mathrm{GeV}} dp_{\mathrm{T}}\rho(p_{\mathrm{T}}) = 1.$
- A parton shower should get this right except for soft effects at $P_{\mathrm{T}} < 10$ GeV.



Other approaches

- There are many cases with two large logarithms per loop.
- Some can be summed to suitable accuracy.
- Some cannot.
- For the Drell-Yan $Q_\perp$ distribution, there are many variations.
- Some authors prefer to give results directly as functions of $Q_\perp^2$.

- Soft-collinear effective theory (SCET) provides a popular method of summing large logarithms.

- *Eg.*

  T. Becher and M. Neubert,
  "Drell-Yan Production at Small q_T, Transverse Parton Distributions and the Collinear Anomaly,"
  Eur. Phys. J. C **71**, 1665 (2011)
  [arXiv:1007.4005 [hep-ph]].

- Banfi, Salam, and Zanderighi (2002) have developed a fairly general method for summing large logarithms.

- For a review of direct QCD methods, see

  G. Luisoni and S. Marzani,
  "Resummation in QCD,"
  arXiv:1505.04084 [hep-ph].