Introduction to the Parton Model and Perturbative QCD

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I. The Parton Model and Deep-inelastic Scattering

II. From the Parton Model to QCD

III. Factorization and Evolution

In an historic era . . .

The Context of QCD: "Fundamental Interactions"

- Electromagnetic
- + Weak Interactions $\Rightarrow$ Electroweak
- + Strong Interactions (QCD) $\Rightarrow$ Standard Model
- + ... = Gravity and the rest?
- QCD: A theory "off to a good start". Think of ...
  - $\vec{F}_{12} = -GM_1M_2\hat{r}/R^2$ $\Rightarrow$ elliptical orbits
    ... 3-body problem ...
  - $L_{\rm QCD} = \bar{q}\ \not{D} q - (1/4)F^2$ $\Rightarrow$ asymptotic freedom
    ... confinement ...

I. The Parton Model and Deep-inelastic Scattering

IA. Nucleons to Quarks

IB. DIS: Structure Functions and Scaling

IC. Getting at the Quark Distributions

ID. Classic Parton Model Extensions:
Fragmentation and Drell-Yan

Introduce concepts and results that predate QCD, led to QCD and were incorporated and explained by QCD.

IA. From Nucleons to Quarks

- **The pattern: nucleons, pions and isospin:**

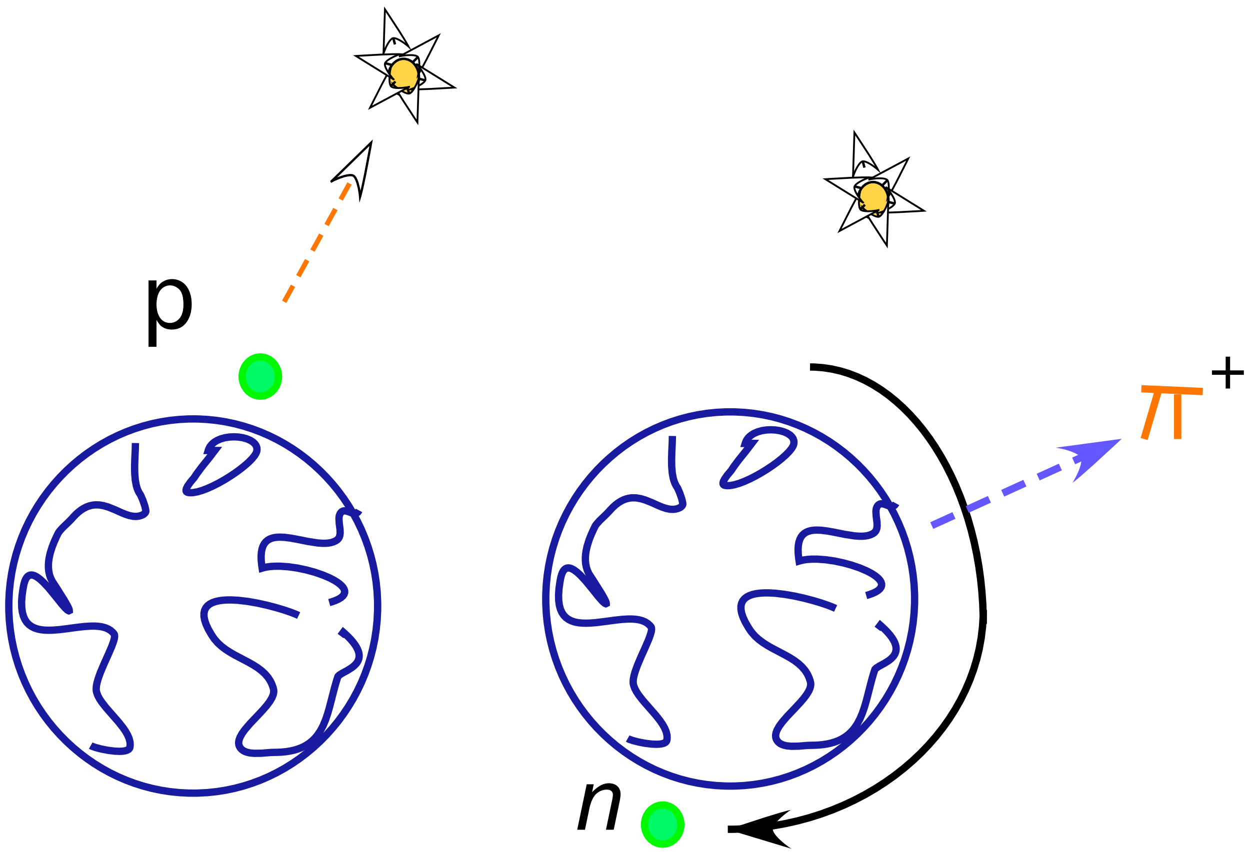
$$\begin{pmatrix} p \\ n \end{pmatrix}$$

  - **p: m=938.3 MeV,** $S = 1/2$**,** $I_3 = 1/2$
  - **n: m=939.6 MeV,** $S = 1/2$**,** $I_3 = -1/2$

$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

  - $\pi^\pm$**: m=139.6 MeV,** $S = 0$**,** $I_3 = \pm 1$
  - π^0**: m=135.0 MeV,** $S = 0$**,** $I_3 = 0$

- **Isospin space . . .**
- **Globe with a "north star" set by electroweak interactions:**



Analog: the rotation group (more specifically, $SU(2)$).

- **Explanation: π, N common substructure: quarks**
  (Gell Mann, Zweig 1964)

- **spin $S = 1/2$,**
  $I = 1/2$ (u, d) & $I = 0$ (s)
  with approximately equal masses (s heavier);

$$\begin{pmatrix} u \ (Q = 2e/3, I_3 = 1/2) \\ d \ (Q = -e/3, I_3 = -1/2) \\ s \ (Q = -e/3, I_3 = 0) \end{pmatrix}$$

$$\pi^+ = (u\bar{d}) \ , \quad \pi^- = (\bar{u}d) \ , \quad \pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) \ ,$$

$$p = (uud) \ , \quad n = (udd) \ , \quad K^+ = (u\bar{s}) \ldots$$

This is the quark model

- Quark model nucleon has symmetric spin/isospin wave function (return to this later)

- Early success: μ_p/μ_n = -3/2 (from $S = 1/2$, $I = 1/2$ uud, ddu wave functions; good to %)

- And now, six: 3 'light' (u, d, s), 3 'heavy': (c, b, t)

- Of these all but t form bound states of quark model type.

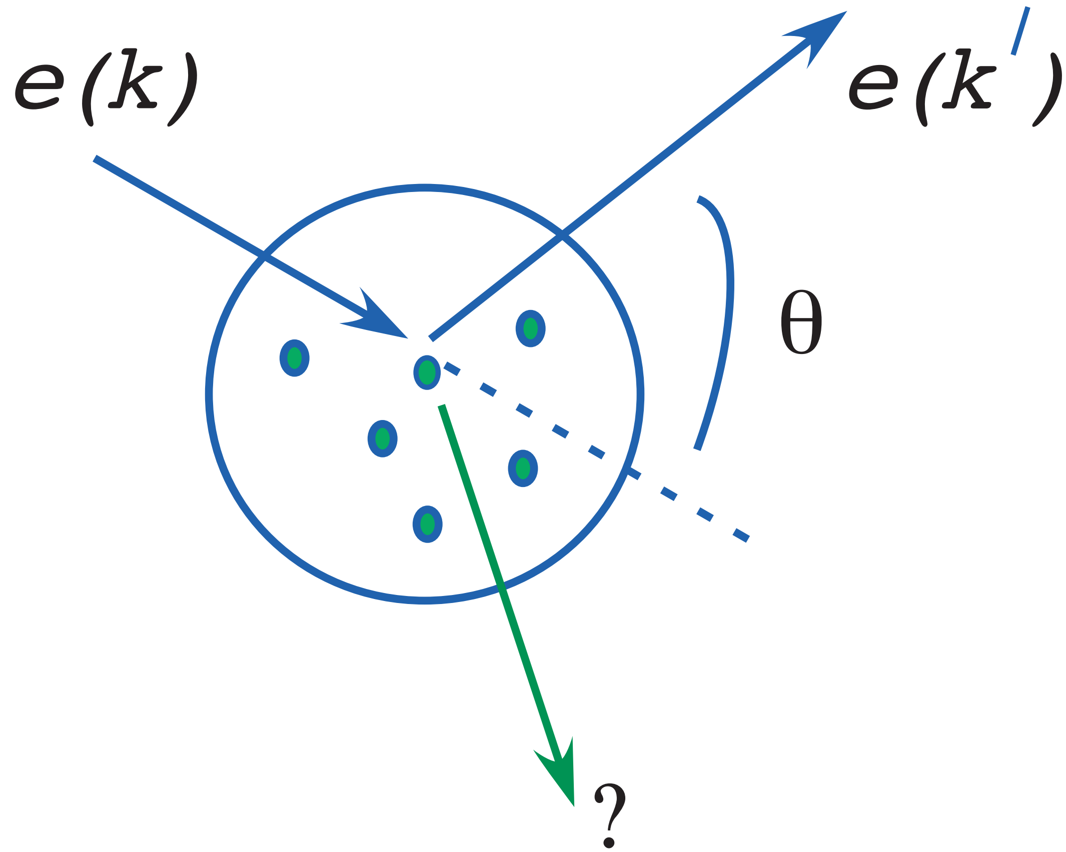
- **Quarks as Partons: “Seeing” Quarks.**

  No isolated fractional charges seen (“confinement.”)

  Could such a particle be detected?

  Look closer: do high energy electrons bounce off anything hard? (SLAC 1969 – ‘Rutherford-prime’)

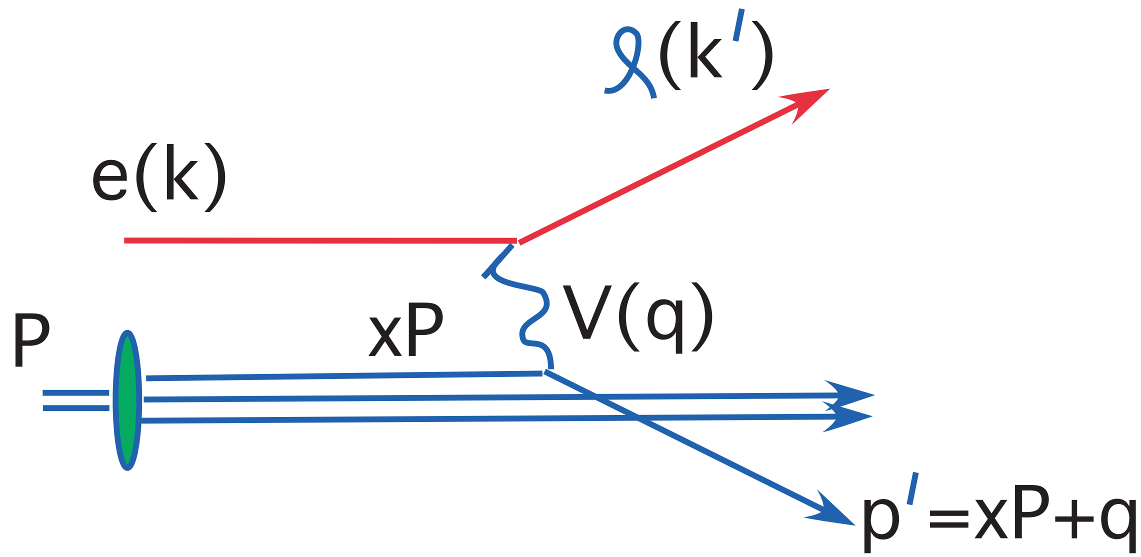
- **So look for:**



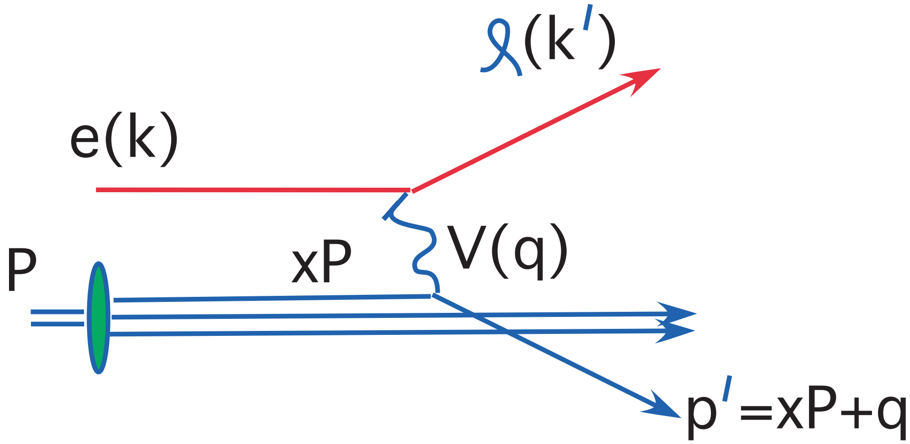
“Point-like’ constituents.

The angular distribution gives information on the constituents.

Kinematics ($e + N(P) \to \ell + X$)



- $V = \gamma, Z_0 \Rightarrow \ell = e,\ \mu,$ **"neutral current" (NC).**
- $V = W^-(e^-, \nu_e)$, $V = W^+(e^+, \bar{\nu}_e)$, **also** (μ, ν_μ) **reactions. "charged current" (CC).**
- $W^2 \equiv (p+q)^2 \gg m^2_{\text{proton}}$: **Deep-inelastic scattering (DIS)**



$Q^2 = -q^2 = -(k-k')^2$ **momentum transfer.**

$x \equiv \frac{Q^2}{2p\cdot q}$ **momentum fraction (from** $p'^2 = (xp+q)^2 = 0$**).**

$y = \frac{p\cdot q}{p\cdot k}$ **fractional energy transfer in** p **rest frame.**

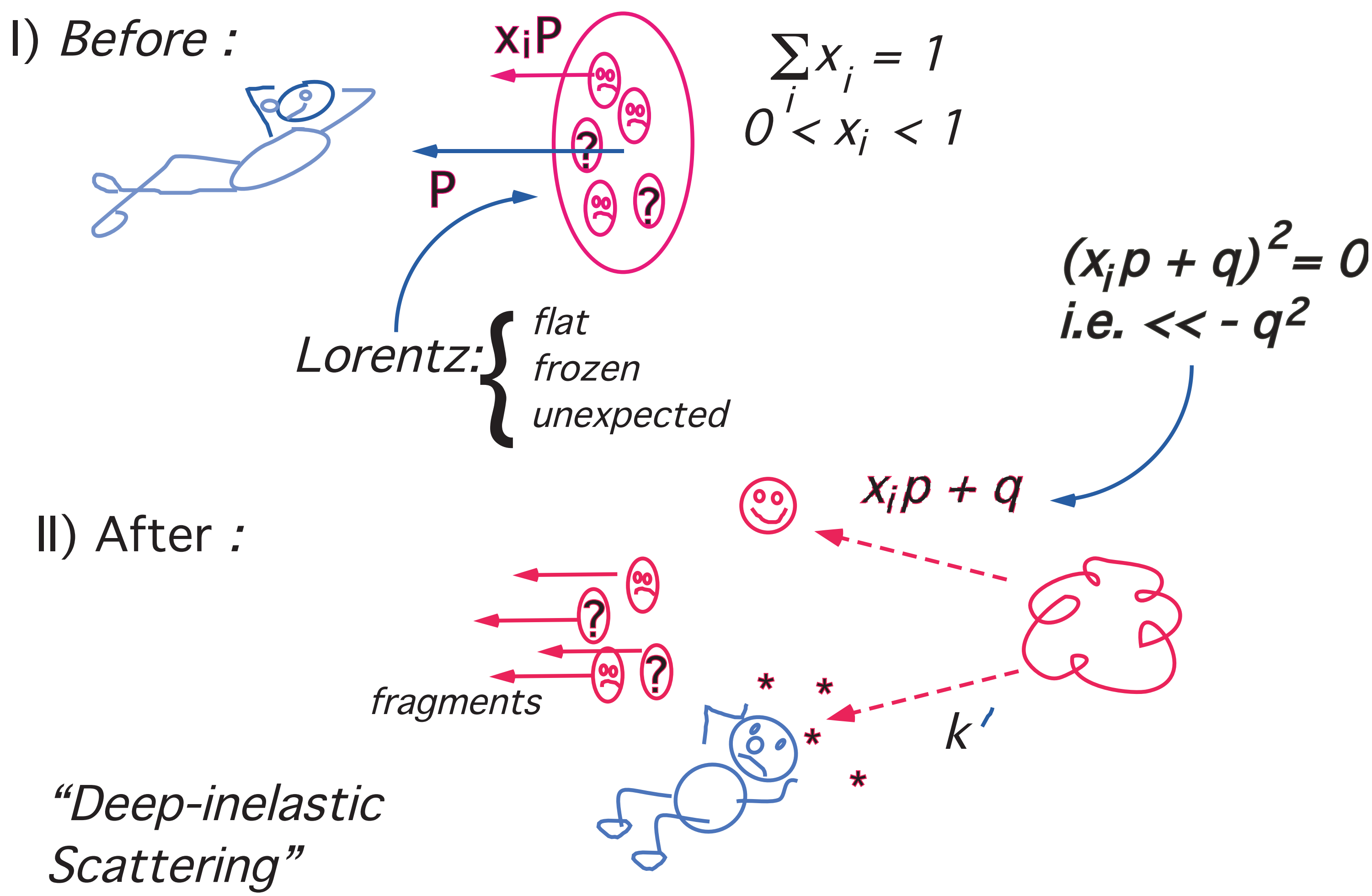
$W^2 = (p+q)^2 = \frac{Q^2}{x}(1-x)$ **squared invariant mass of final-state hadrons.**

A useful identity:

$$xy = \frac{Q^2}{S}$$

From CTEQ Summer School 1992:

Parton Interpretation (Feynman 1969, 72)
Look in the electron's rest frame . . .



– Basic Parton Model Relation

$$\sigma_{\rm eh}(p,q) \;=\; \sum_{\text{partons } a} \int_0^1 d\xi\, \hat{\sigma}^{\rm el}_{ea}(\xi p, q)\, \phi_{a/h}(\xi)\,,$$

– where: $\sigma_{eh}(p,q)$ is the inclusive cross section $e(k) + h(p) \to e(k' = k - q) + X(p+q)$

– and: $\hat{\sigma}^{\rm el}_{ea}(\xi p, q)$ is the elastic cross section $e(k) + a(\xi p) \to e(k' - q) + a(\xi p + q)$ which sets $(\xi p + q)^2 = 0 \to \xi = -q^2/2p \cdot q \equiv x$.

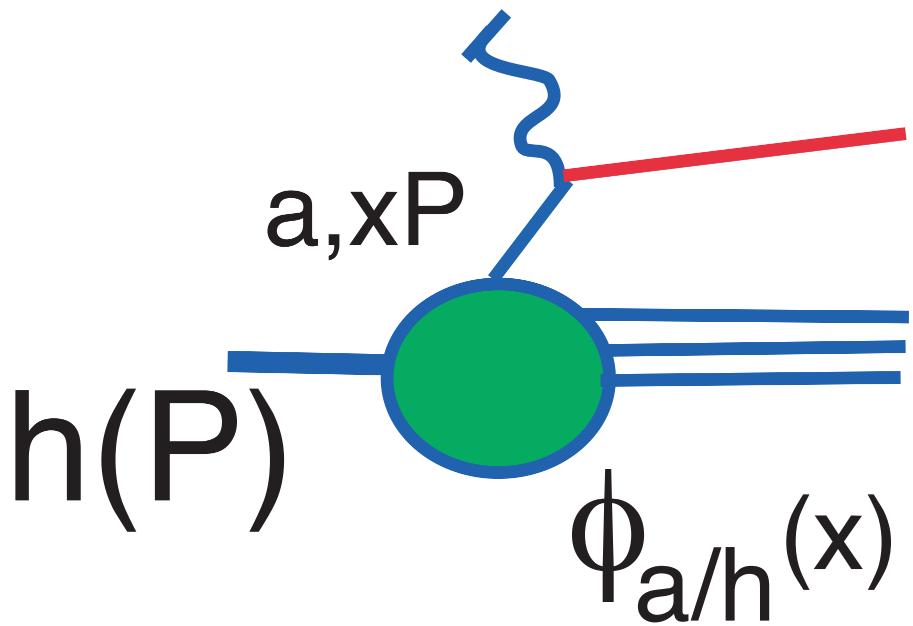
– and: $\phi_{a/h}(\xi)$ is the distribution of parton a in hadron h, the “probability for a parton of type a to have momentum ξp”. It is independent of the details of the hard scattering – the hallmark of factorization.

– **in words: Hadronic INELASTIC cross section is the sum of convolutions of partonic ELASTIC cross sections with the hadron's parton distributions.**

– **The nontrivial assertion: quantum mechanical incoherence of large-q scattering and the partonic distributions. Multiply probabilities without adding amplitudes.**

– **Heuristic justification: the binding of the nucleon involves long-time processes that do not interfere with the short-distance scattering. Later we'll see how this works in QCD.**

– **The familiar picture**



– **“QM incoherence” $\Leftrightarrow$ no interactions between the outgoing scattered quark and the rest.**

- **A contemporary set of parton distributions "at different scales": see "evolution" (CTEQ 2015: 1506.07443):**

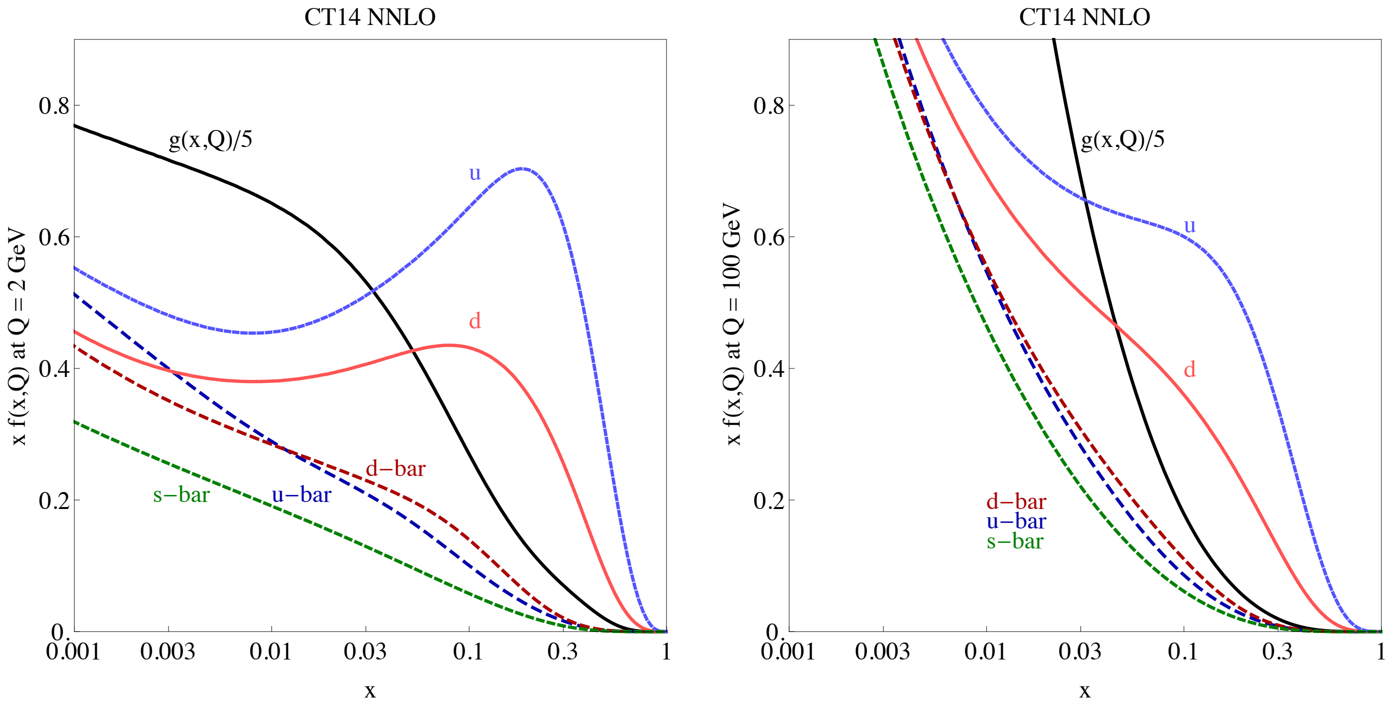
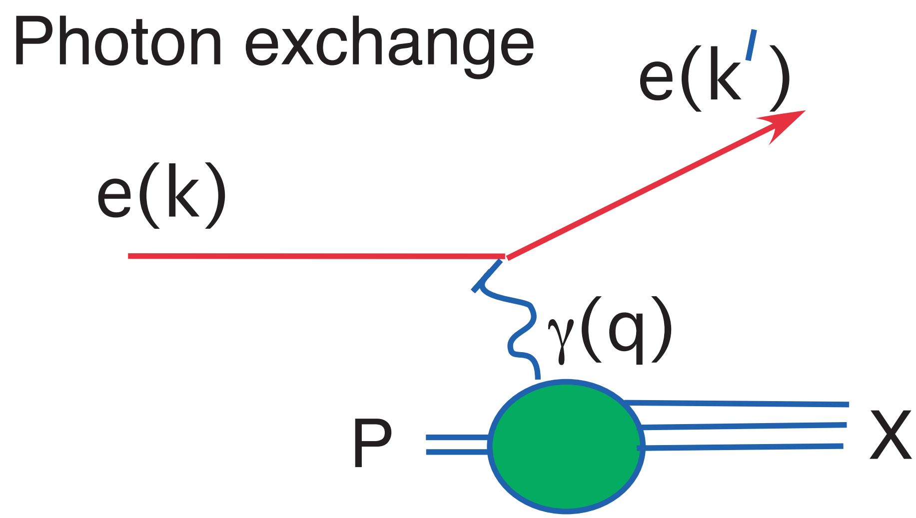


FIG. 4: The CT14 parton distribution functions at $Q = 2$ GeV and $Q = 100$ GeV for $u, \overline{u}, d, \overline{d}, s = \overline{s}$, and g.

- **It's convenient to let the distributions change with q – we'll see where this comes from.**

IB. DIS: Structure Functions and Scaling



$$A_{e+N\to e+X}(\lambda,\lambda',\sigma;q) = \bar{u}_{\lambda'}(k')(-ie\gamma_\mu)u_\lambda(k) \times \frac{-ig^{\mu\mu'}}{q^2} \times \langle X|\, eJ^{\rm EM}_{\mu'}(0)\,|p,\sigma\rangle$$

- **Historically an assuption that the photon couples to hadrons by point-like current operator. Now, built into the Standard Model.**

- **The cross section:**

$$d\sigma_{\mathrm{DIS}} = \frac{1}{2^2}\frac{1}{2s}\frac{d^3k'}{(2\pi)^3 2\omega_{k'}} \sum_X \sum_{\lambda,\lambda',\sigma} |A|^2$$
$$\times\ (2\pi)^4\, \delta^4(p_X + k' - p - k)$$

In $|A|^2$, separate the known leptonic part from the "unknown" hadronic part: $\sum |A|^2 \delta^4(\cdots) \equiv L^{\mu\nu} W_{\mu\nu}$.

- **The leptonic tensor:**

$$L^{\mu\nu} = \frac{e^2}{8\pi^2} \sum_{\lambda,\lambda'} \left(\bar{u}_{\lambda'}(k')\gamma^\mu u_\lambda(k)\right)^* \left(\bar{u}_{\lambda'}(k')\gamma^\nu u_\lambda(k)\right)$$
$$= \frac{e^2}{2\pi^2}\left(k^\mu k'^{\,\nu} + k'^{\,\mu} k^\nu - g^{\mu\nu} k\cdot k' \right)$$

- Leaves us with the hadronic tensor:

$$W_{\mu\nu} = \frac{1}{8\pi} \sum_{\sigma,X} \langle X|J_\mu|p,\sigma\rangle^* \langle X|J_\nu|p,\sigma\rangle \times (2\pi)^4 \delta^4(p_X - p - q)$$

- And the cross section becomes:

$$2\omega_{k'} \frac{d\sigma}{d^3k'} = \frac{1}{s(q^2)^2} L^{\mu\nu} W_{\mu\nu}$$

- $W_{\mu\nu}$ has sixteen components,
  but known properties of the strong interactions constrain $W_{\mu\nu}$ ...

- **An example: current conservation,**

$$
\begin{aligned}
&\partial^\mu J_\mu^{\mathrm{EM}}(x) = 0 \\
&\qquad \Rightarrow \langle X|\, \partial^\mu J_\mu^{\mathrm{EM}}(x)\, |p\rangle = 0 \\
&\qquad \Rightarrow (p_X - p)^\mu \langle X|\, J_\mu^{\mathrm{EM}}(x)\, |p\rangle = 0 \\
&\qquad \Rightarrow q^\mu W_{\mu\nu} = 0
\end{aligned}
$$

- **With parity, time-reversal, etc . . .**

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(x, Q^2) + \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) W_2(x, Q^2)$$

- **Often given in terms of the dimensionless structure functions,**

$$F_1 = W_1 \qquad F_2 = p \cdot q W_2$$

- **Note that if there is no other mass scale, the F's cannot depend on Q except indirectly through x.**

- **Structure functions in the Parton Model: The Callan-Gross Relation**

From the "basic parton model formula":

$$\frac{d\sigma_{eh}}{d^3k'} = \sum_{\text{quarks } f} \int d\xi \, \frac{d\sigma_{ef}^{\text{el}}(\xi p)}{d^3k'} \, \phi_{f/h}(\xi) \tag{1}$$

Can treat a quark of "flavor" f just like any hadron and get

$$\omega_{k'} \, \frac{d\sigma_{ef}^{\text{el}}(\xi p)}{d^3k'} = \frac{1}{2(\xi s)Q^4} \, L^{\mu\nu} \, W_{\mu\nu}^{ef}(k + \xi p \to k' + p')$$

Let the charge of f be e_f.

Exercise 1: Compute $W_{\mu\nu}^{ef}$ to find:

$$W_{\mu\nu}^{ef} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\,\delta\left(1-\frac{x}{\xi}\right)\frac{e_f^2}{2}$$
$$+\left(\xi p_\mu - q_\mu\,\frac{\xi p\cdot q}{q^2}\right)\,\left(\xi p_\nu - q_\nu\,\frac{\xi p\cdot q}{q^2}\right)\,\delta\left(1-\frac{x}{\xi}\right)\frac{e_f^2}{\xi p\cdot q}$$

Ex. 2: by substituting in (1), find the Callan-Gross relation,

$$F_2(x) = \sum_{\text{quarks} f} e_f^2 x\,\phi_{f/p}(x) = 2xF_1(x)$$

And Ex. 3: that this relation is quite different for scalar quarks.

- **The Callan-Gross relation shows the compatibility of the quark and parton models.**

- **In addition: parton model structure functions are independent of Q^2, a property called “scaling”.**

- **With massless partons, there is no other massive scale. Then the F’s must be Q-independent; see above.**

- **Approximate properties of the kinematic region explored by SLAC in late 1960’s – early 1970’s.**

- **QCD “evolution” gives corrections to this picture.**

The F's, W's from DIS: (CTEQ "Handbook")

$$\frac{d\sigma^{(lh)}}{dx\,dy} = \frac{E_{k'}y}{m} N^{(lV)} \Bigg[2W_1^{(Vh)}(x,q^2)\sin^2(\theta/2) + W_2^{(Vh)}(x,q^2)\cos^2(\theta/2) \pm W_3^{(Vh)}(x,q^2)\frac{E+E'}{m_h}\sin^2(\theta/2) \Bigg], \tag{3.23}$$

where the $\pm$ corresponds to $V = W^{\pm}$, and where

$$N^{(l^{\pm}\gamma)} = 8\pi\alpha^2 \frac{m_h E}{Q^4}, \qquad N^{(\nu W^+)} = N^{(\bar{\nu} W^-)} = \pi\alpha^2 \frac{m_h E}{2\sin^4(\theta_W)(Q^2+M_W^2)^2}. \tag{3.24}$$

Here θ_W is the weak mixing angle, and $\pi\alpha^2/(2M_W^4\sin^4\theta_W) = G_F^2/\pi$, with G_F the Fermi constant.

Other useful expressions for this cross section are given directly in terms of y,

$$\frac{d\sigma^{(lh)}}{dx\,dy} = N^{lV}\left[\frac{y^2}{2}2xF_1^{(Vh)} + \left(1-y-\frac{m_h xy}{2E}\right)F_2^{(Vh)} + \delta_V\left(y-\frac{y^2}{2}\right)xF_3^{(Vh)}\right], \tag{3.25}$$

where δ_V is $+1$ for $V=W^+$ (neutrino beam), -1 for $V=W^-$ (antineutrino beam), and zero for the photon, while m_h is the target mass.

IC. Getting at the Quark Distributions

- **Relating the parton distributions to experiment**
- **<u>Simplifying</u> assumptions that illustrate the general approach.**

$$\phi_{u/p} = \phi_{d/n} \qquad \phi_{d/p} = \phi_{u/n} \qquad \text{isospin}$$

$$\phi_{\bar{u}/p} = \phi_{\bar{u}/n} = \phi_{\bar{d}/p} = \phi_{\bar{d}/n} \qquad \text{symmetric sea}$$

$$\phi_{c/p} = \phi_{b/N} = \phi_{t/N} = 0 \qquad \text{no heavy quarks}$$

- **Adequate to early experiments, but no longer.**

- **With assumptions above,** **the quark-parton model gives for e, ν (W^+) and $\bar{\nu}$ (W^-) DIS (see appendix)**

$$F_2^{(eN)}(x) = 2xF_1^{(eN)}(x) = \sum_{f=u,d,s} e_F^2 x \phi_{f/N}(x)$$

$$F_2^{(\nu N)} = 2x \left(\sum_{D=d,s,b} \phi_{D/N}(x) + \sum_{U=u,c,t} \phi_{\bar{U}/N}(x) \right)$$

$$F_2^{(\bar{\nu} N)} = 2x \left(\sum_{D} \phi_{\bar{D}/N}(x) + \sum_{U} \phi_{U/N}(x) \right)$$

$$F_3^{(\nu N)} = 2 \left(\sum_{D} \phi_{D/N}(x) - \sum_{U} \phi_{\bar{U}/N}(x) \right)$$

$$F_3^{(\bar{\nu} N)} = 2 \left(- \sum_{D} \phi_{\bar{D}/N}(x) + \sum_{U} \phi_{U/N}(x) \right)$$

- **Ex: Trace the relative minus sign between quarks and anti-quarks in F_3 back to the Born diagrams.**

- The distributions are actually overdetermined with these assumptions, which checks the consistency of the picture.
- Further consistency checks: Sum Rules.

$$N_{u/p} = \int_0^1 dx \left[\phi_{u/p}(x) - \phi_{\bar{u}/p}(x) \right] = 2$$

etc. for $N_{d/p} = 1$.

The most famous ones make predictions for structure functions . . .

- **The Adler Sum Rule:**

$$\int_0^1 dx\, \frac{1}{2x} \left[F_2^{(\nu n)} - F_2^{(\nu p)} \right]$$

$$= \int_0^1 dx \left[\sum_D \phi_{D/n}(x) + \sum_U \phi_{\bar{U}/n}(x) \right]$$

$$- \int_0^1 dx \left[\sum_D \phi_{D/p}(x) + \sum_U \phi_{\bar{U}/p}(x) \right]$$

$$= \int_0^1 dx \left[\phi_{d/n}(x) - \phi_{\bar{u}/p}(x) - \left(\phi_{d/p}(x) - \phi_{\bar{u}/n}(x) \right) \right]$$

$$= N_{u/p} - N_{d/p}$$

$$= 1$$

In the 2nd equality, all the extra terms from heavy quarks $D = s, b$, $U = c, t$ cancel between proton and neutron. In the 3rd, we've used isospin invariance.

- **And similarly, the Gross-Llewellyn-Smith Sum Rule:**

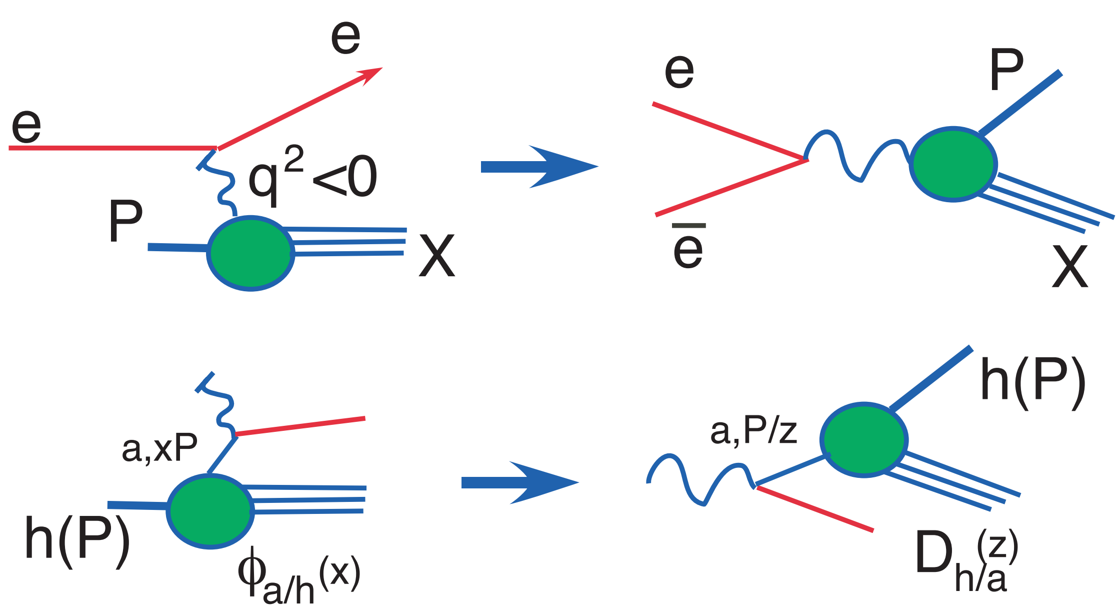
$$\begin{aligned} 3 &= N_{u/p} + N_{d/p} \\ &= \int_0^1 dx\, \frac{1}{2x} \left[xF_3^{(\nu n)} + xF_3^{(\nu p)} - \left(xF_3^{(\bar{\nu} n)} + xF_3^{(\bar{\nu} p)} \right) \right] \end{aligned}$$

Ex: work this one out from the relations of structure functions to quark and antiquark distributions.

ID. Classic Parton Model Extensions: Fragmentation and Drell Yan

- **Fragmentation functions**
- **"Crossing" applied to DIS: "Single-particle inclusive" (1PI) From scattering to pair annihilation.**

Parton distributions become "fragmentation functions".



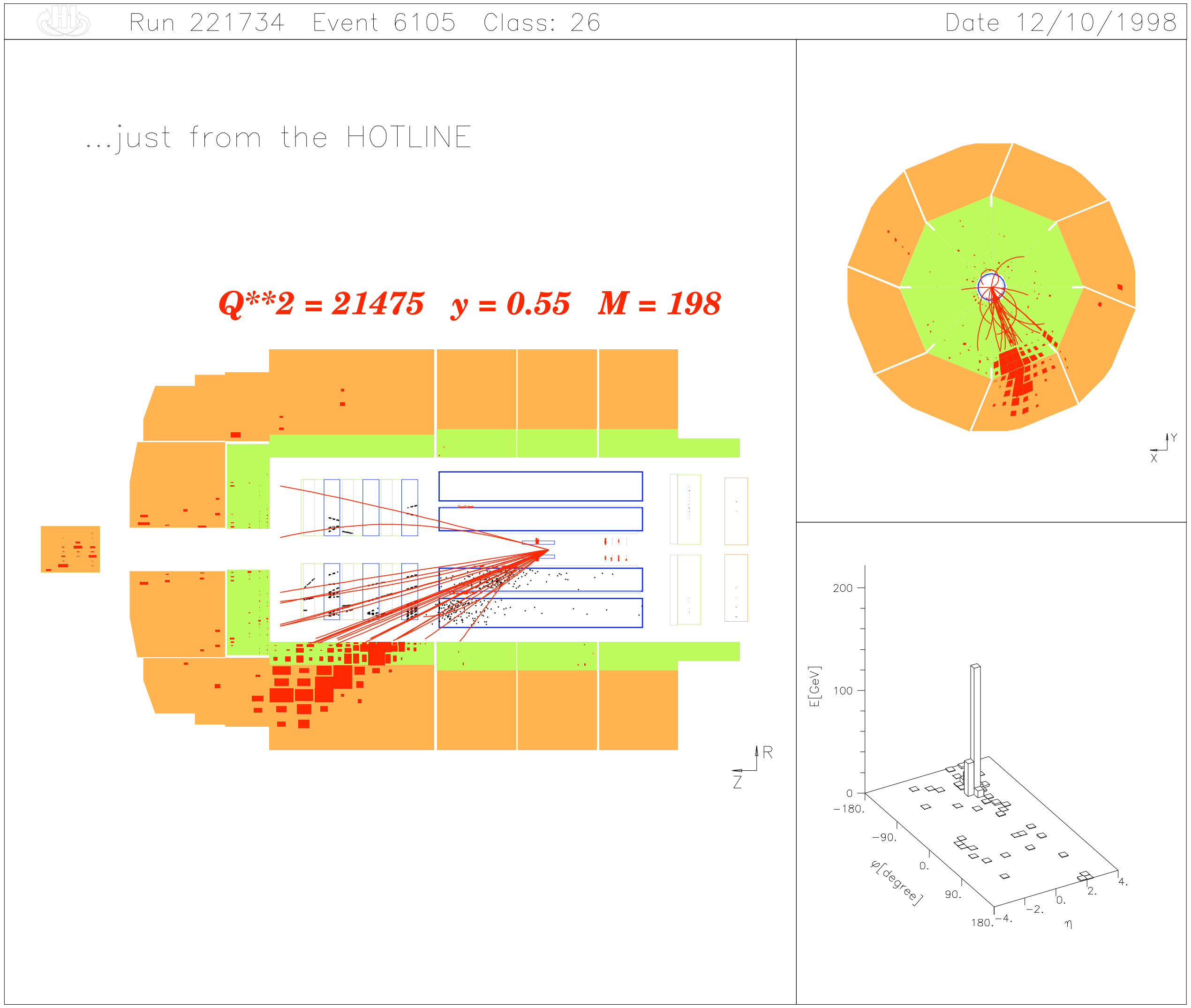
- Parton model relation for 1PI cross sections

$$\sigma_h(P,q) = \sum_a \int_0^1 dz\ \hat{\sigma}_a(P/z,q)\ D_{h/a}(z)$$

- Heuristic justification: Formation of hadron C from parton a takes a time τ_0 in the rest frame of a, but much longer in the CM frame – this "fragmentation" thus decouples from $\hat{\sigma}_a$, and is independent of q (scaling).

- Fragmentation picture suggests that hadrons are aligned along parton direction $\Rightarrow$ jets. And this is what happens.

- **For DIS:**



Run 221734 Event 6105 Class: 26
Date 12/10/1998
...just from the HOTLINE
Q**2 = 21475 y = 0.55 M = 198
R
Z
Y
X
E[GeV]
200
100
0
−180.
−90.
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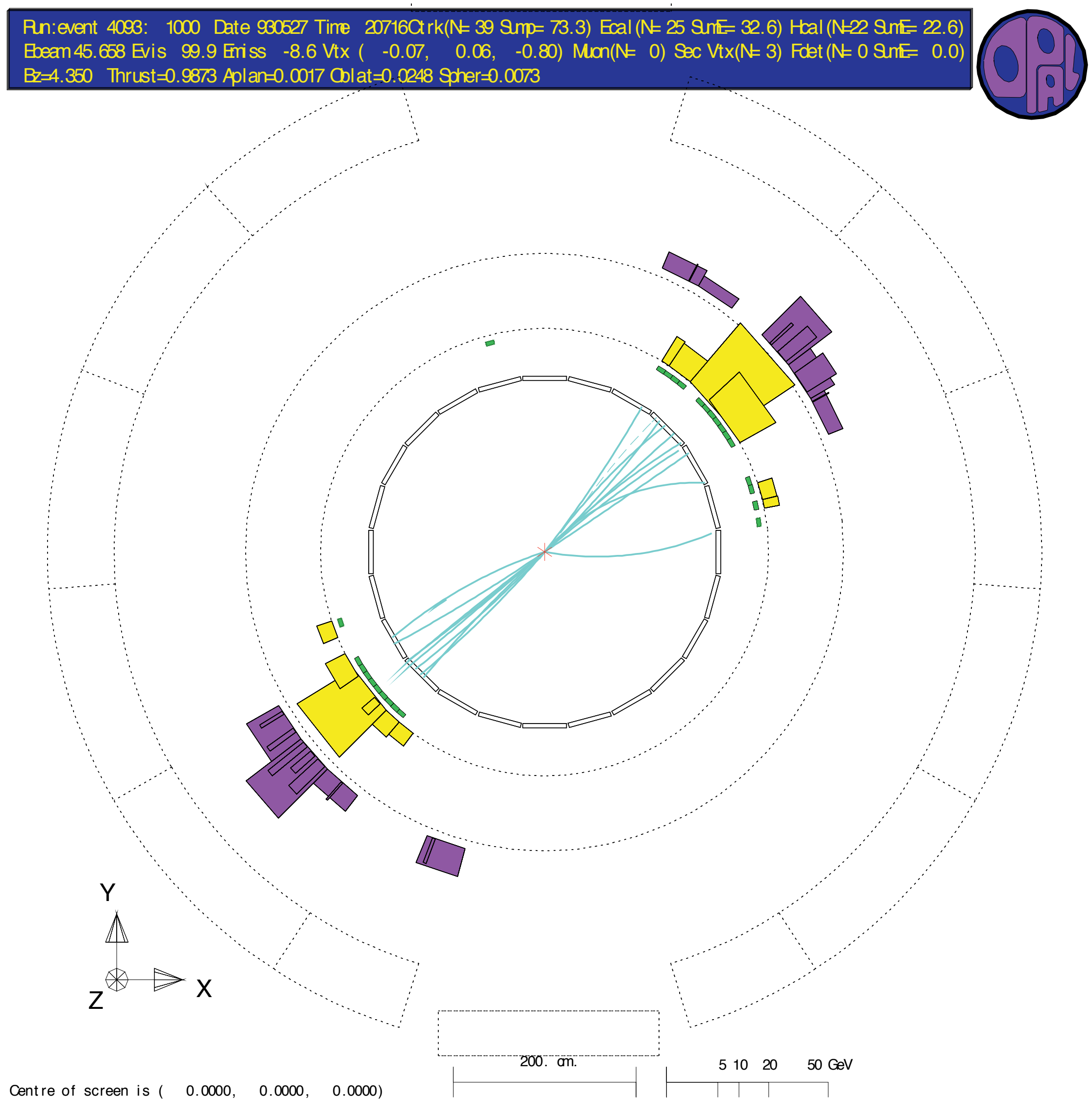
η

• **For** e^+e^-**:**



Run:event 4093: 1000 Date 930527 Time 20716 Ctrk(N= 39 Sump= 73.3) Ecal(N= 25 SumE= 32.6) Hcal(N=22 SumE= 22.6)
Ebeam 45.658 Evis 99.9 Emiss -8.6 Vtx (-0.07, 0.06, -0.80) Muon(N= 0) Sec Vtx(N= 3) Fdet(N= 0 SumE= 0.0)
Bz=4.350 Thrust=0.9873 Aplan=0.0017 Oblat=0.0248 Spher=0.0073
Y
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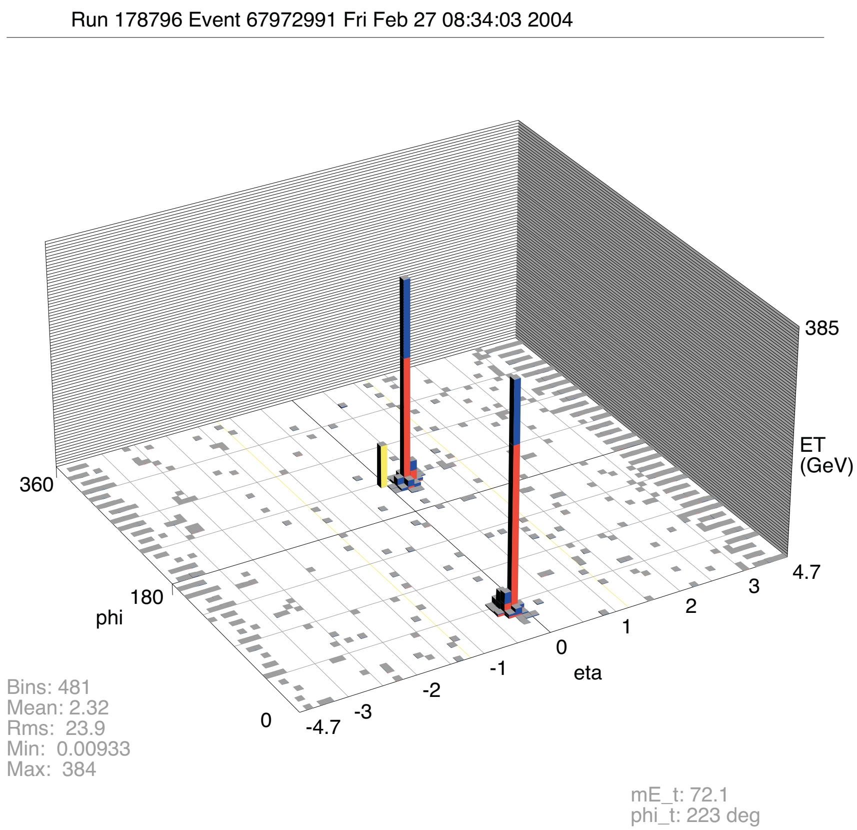
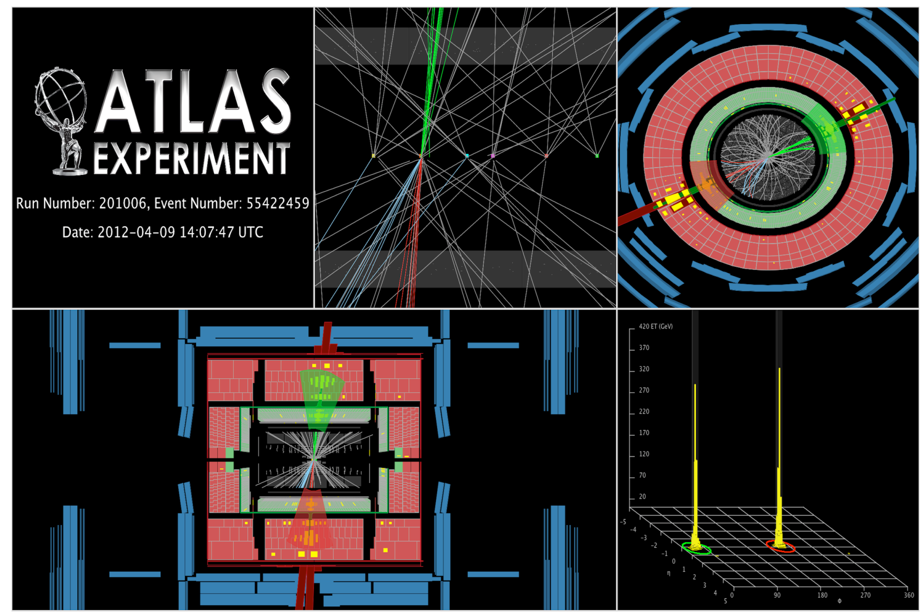
5 10 20 50 GeV
Centre of screen is (0.0000, 0.0000, 0.0000)

• And in nucleon-nucleon collisions:





- Finally: the Drell-Yan process

- **In the parton model (1970).**
  Drell and Yan: look for the annihilation of quark pairs into virtual photons of mass Q . . . any electroweak boson in NN scattering.

$$\frac{d\sigma_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\ldots} \sim$$

$$\int d\xi_1 d\xi_2 \sum_{a=\mathrm{q}\bar{\mathrm{q}}} \frac{d\sigma^{\mathrm{EW,\,Born}}_{\mathrm{a\bar{a}}\to\mu\bar{\mu}}(Q,\xi_1 p_1,\xi_2 p_2)}{dQ^2 d\ldots}$$

$$\times(\text{probability to find parton a}(\xi_1)\text{ in }N)$$

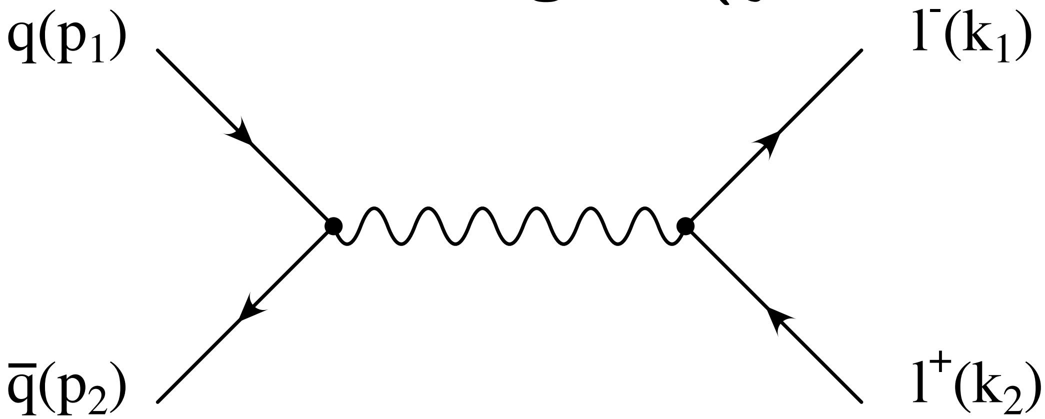
$$\times(\text{probability to find parton }\bar{\mathrm{a}}(\xi_2)\text{ in }N)$$

The probabilities are $\phi_{q/N}(\xi_i)$'s from DIS!

How it works (with colored quarks) ...

- **The Born cross section**

$\boldsymbol{\sigma}^{\mathrm{EW,Born}}$ **is all from this diagram (ξ's set to unity):**



With this matrix element:

$$M \;=\; e_q \frac{e^2}{Q^2}\, \overline{u}(k_1)\gamma_\mu v(k_2)\; \overline{v}(p_2)\gamma^\mu u(p_1)$$

- **First square and sum/average M. Then evaluate phase space.**

- **Total cross section at pair mass Q**

$$\sigma^{\mathrm{EW,\,Born}}_{q\bar{q}\to\mu\bar{\mu}}(x_1p_1, x_2p_2) = \frac{1}{2\hat{s}}\int \frac{d\Omega}{32\pi^2}\frac{e_q^2 e^4}{3}(1+\cos^2\theta)$$

$$= \frac{4\pi\alpha^2}{9Q^2}\sum_q e_q^2$$

With Q the pair mass and 3 for color average.

- **And measured rapidity:**

  Pair mass (Q) and rapidity

$$\eta \equiv (1/2)\ln\left(\frac{Q^+}{Q^-}\right) = (1/2)\ln\left(\frac{Q^0+Q^3}{Q^0-Q^3}\right)$$

- **ξ's are overdetermined $\to$ delta functions in the Born cross section**

$$\frac{d\sigma^{(PM)}_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\eta} =$$

$$\int_{\xi_1,\xi_2} \sum_{a=q\bar{q}} \sigma^{\mathrm{EW,\,Born}}_{a\bar{a}\to\mu\bar{\mu}}(\xi_1 p_1, \xi_2 p_2)$$
$$\times\delta\left(Q^2-\xi_1\xi_2 S\right)\, \delta\left(\eta - \frac{1}{2}\ln\left(\frac{\xi_1}{\xi_2}\right)\right)$$
$$\times\phi_{a/N}(\xi_1)\,\phi_{\bar{a}/N}(\xi_2)$$

- and integrating over rapidity, back to $d\sigma/dQ^2$,

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha_{\rm EM}^2}{9Q^4}\right) \int_0^1 d\xi_1\, d\xi_2\, \delta\left(\xi_1\xi_2 - \tau\right) \\ \times \sum_a \lambda_a^2\, \phi_{a/N}(\xi_1)\, \phi_{\bar{a}/N}(\xi_s)$$

Found by Drell and Yan in 1970 (aside from 1/3 for color).

Analog of DIS scaling in x is DY scaling in $\tau = Q^2/S$.

- Template for all hard hadron-hadron scattering
- Next, the quantum field theory of all this . . . QCD

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- **Appendix I: Quarks in the Standard Model**
  Electroweak interactions of quarks: SU(2)$_L$ $\times$ U(1). Their non-QCD interactions.

- **Quark and lepton fields: L(eft) and R(ight)**
  - $\psi = \psi^{(L)} + \psi^{(R)} = \frac{1}{2}(1 - \gamma_5)\psi + \frac{1}{2}(1 + \gamma_5)\psi$; $\psi = q, \ell$
  - **Helicity: spin along $\vec{p}$ (R=right handed) or opposite (L=left handed) in solutions to Dirac equation**
  - $\psi^{(L)}$: **expanded only in L particle solutions to Dirac eqn. R antiparticle solutions**
  - $\psi^{(R)}$: **only R particle solutions, L antiparticle**
  - **An essential feature: L and R have different interactions in general!**

– L quarks come in "weak $SU(2)$" = "weak isospin" pairs:

$$q_i^{(L)} = \begin{pmatrix} u_i \\ d'_i = V_{ij} dj \end{pmatrix} \qquad u_i^{(R)},\ d_i^{(R)}$$

$$(u, d') \qquad (c, s') \qquad (t, b')$$

$$\ell_i^{(L)} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \qquad e_i^{(R)},\ \nu_i^{(R)}$$

$$(\nu_e, e) \qquad (\nu_\mu, \mu) \qquad (\nu_\tau, \tau)$$

(We've neglected neutrino masses.)

– V_{ij} is the "CKM" matrix.

– The electroweak interactions distinguish L and R.

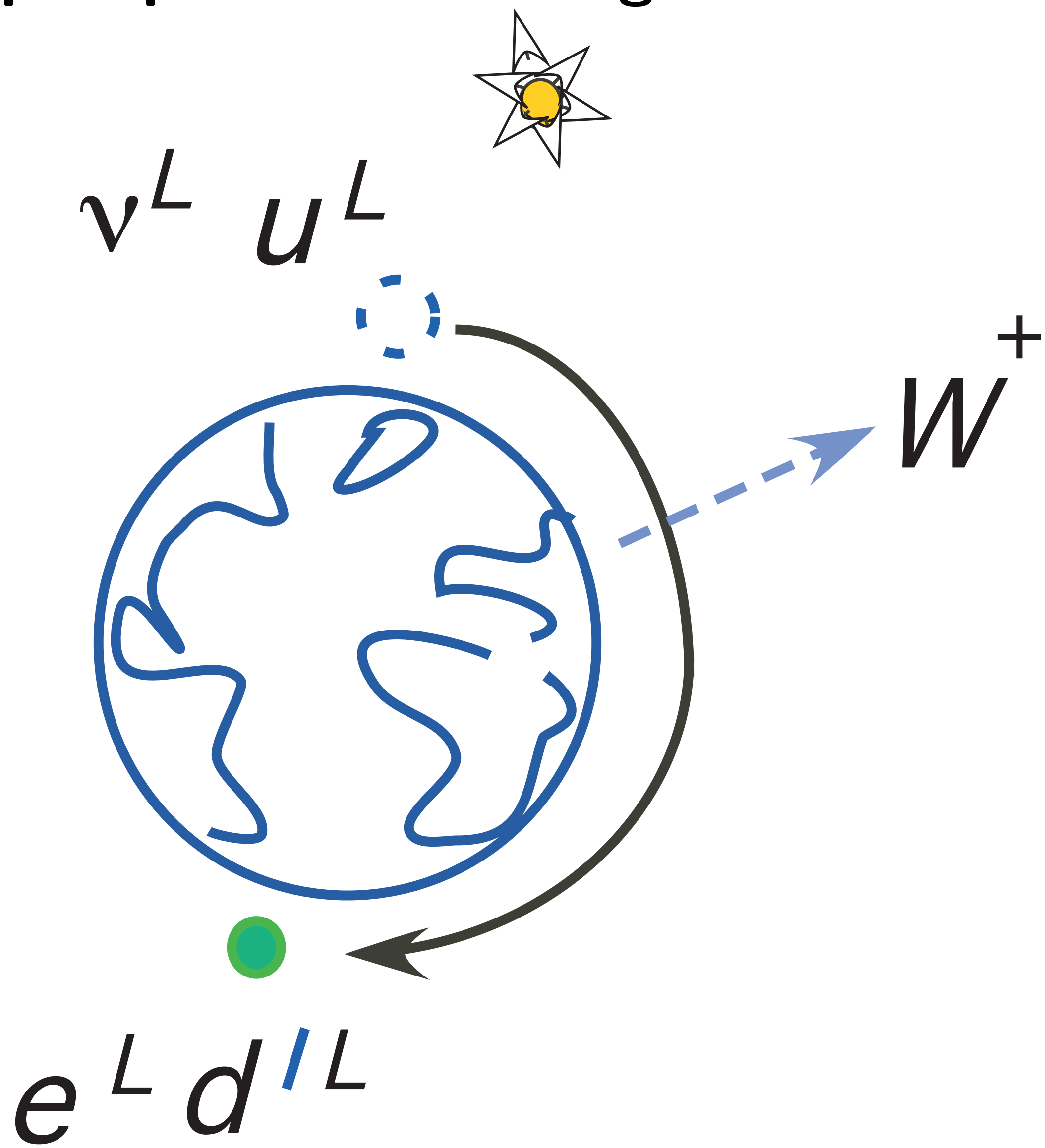
- **Weak vector bosons: electroweak gauge groups**
  - **SU(2): three vector bosons B_i with coupling g**
  - **U(1); one vector boson C with coupling g'**
  - **The physical bosons:**

$$W^{\pm} = B_1 \pm iB_2$$
$$Z = -C\sin\theta_W + B_3\cos\theta_W$$
$$\gamma \equiv A = C\cos\theta_W + B_3\sin\theta_W$$

$$\sin\theta_W = g'/\sqrt{g^2+g'^2} \qquad M_W = M_Z/\cos\theta_W$$

$$e = gg'/\sqrt{g^2+g'^2} \qquad M_W \sim g/\sqrt{G_F}$$

- **Weak isospin space: connecting u with d'**



- **Only left handed fields move around this globe.**

– **The interactions of quarks and leptons with the photon, W, Z**

$$\mathcal{L}_{\mathrm{EW}}^{(fermion)} = \sum_{\text{all }\psi} \bar{\psi}\left(i\not{\partial} - e\lambda_\psi \not{A} - (g m_\psi 2 M_W) h\right)\psi$$
$$-(g/\sqrt{2}) \sum_{q_i, e_i} \bar{\psi}^{(L)} \left(\sigma^+ \not{W}^+ + \sigma^- \not{W}^-\right) \psi^{(L)}$$
$$-(g/2\cos\theta_W) \sum_{\text{all }\psi} \bar{\psi}\left(v_f - a_f \gamma_5\right) \not{Z}\, \psi$$

– **Interactions with W are through ψ_L's only.**

– **Neutrino Z exchange depends on $\sin^2\theta_W$ even at low energy.**

– **This observation made it clear by early 1970's that $M_W \sim g/\sqrt{G_F}$ is large $\rightarrow$ a need for colliders.**

– **Coupling to the Higgs $h \propto$ mass (special status of t).**

- **Symmetry violations in the standard model:**

  – W's interact through $\psi^{(L)}$ only, $\psi = q, \ell$.

  – These are left-handed quarks & leptons; right-handed antiquarks, antileptons.

  – Parity (P) exchanges L/R; Charge conjugation (C) exchanges particles, antiparticles.

  – CP combination OK ($L \rightarrow_P R \rightarrow_C L$) if all else equal, but it's not (quite) . . .

  Complex phases in CKM V result in CP violation.

- **Appendix II: Structure Functions and Photon Polarizations**

In the P rest frame can take

$$q^\mu = \left(\nu; 0, 0, \sqrt{Q^2 + \nu^2}\right) , \qquad \nu \equiv \frac{p \cdot q}{m_p}$$

In this frame, the possible photon polarizations ($\epsilon \cdot q = 0$):

$$\epsilon_R(q) = \frac{1}{\sqrt{2}} (0; 1, -i, 0)$$
$$\epsilon_L(q) = \frac{1}{\sqrt{2}} (0; 1, i, 0)$$
$$\epsilon_{\text{long}}(q) = \frac{1}{Q} \left(\sqrt{Q^2 + \nu^2}, 0, 0, \nu\right)$$

- **Alternative Expansion**

$$W^{\mu\nu} = \sum_{\lambda=L,R,long} \epsilon_\lambda^{\mu *}(q)\epsilon_\lambda^{\nu}(q)\, F_\lambda(x,Q^2)$$

- **For photon exchange (Exercise 4):**

$$F_{L,R}^{\gamma e} = F_1$$
$$F_{\mathrm{long}} = \frac{F_2}{2x} - F_1$$

- **So F_{long} vanishes in the parton model by the C-G relation.**

- **Generalizations: neutrinos and polarization**

- **Neutrinos: flavor of the "struck" quark is changed when a $W^{\pm}$ is exchanged. For W^{+}, a d is transformed into a linear combination of u, c, t, determined by CKM matrix (and momentum conservation).**

- **Z exchange leaves flavor unchanged but still violates parity.**

• The Vh structure functions for $= W^+, W^-, Z$:

$$\begin{aligned} W_{\mu\nu}^{(Vh)} - & \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^{(Vh)}(x, Q^2) \\ & + \left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) \frac{1}{m_h^2} W_2(x, Q^2) \\ & - i\epsilon_{\mu\nu\lambda\sigma} p^\lambda q^\sigma \frac{1}{m_h^2} W_3^{(Vh)}(x, Q^2) \end{aligned}$$

• with dimensionless structure functions:

$$F_1 = W_1\,, \qquad F_2 = \frac{p\cdot q}{m_h^2} W_2\,, \qquad F_3 = \frac{p\cdot q}{m_h^2} W_3$$

• $F_i^{(\nu h)}$ gives $W^+ h$ scattering, $F_i^{(\bar{\nu} h)}$ gives $W^- h$

• **And with spin (for the photon).**

$$W^{\mu\nu} = \frac{1}{4\pi} \int d^4z\, e^{iq\cdot z}\ \langle h(P,S) \mid J^\mu(z) J^\nu(0) \mid h(P,S)\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\ F_1(x,Q^2)$$

$$+ \left(P^\mu - q^\mu \frac{P\cdot q}{q^2}\right)\left(P^\nu - q^\nu \frac{P\cdot q}{q^2}\right) F_2(x,Q^2)$$

$$+ \, i m_h\, \epsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{P\cdot q} g_1(x,Q^2) + \frac{S_\sigma (P\cdot q) - P_\sigma (S\cdot q)}{(P\cdot q)^2} g_2(x,Q^2)\right]$$

- **Parton model structure functions:**

$$F_2^{(eh)}(x) = \sum_f e_f^2 \, x \, \phi_{f/h}(x)$$

$$g_1^{(eh)}(x) = \frac{1}{2} \sum_f e_f^2 \left(\Delta\phi_{f/n}(x) + \Delta\bar{\phi}_{f/h}(x)\right)$$

- **Notation:** $\Delta\phi_{f/h} = \phi^{+}_{f/h} \; - \; \phi^{-}_{f/h}$ **with** $\phi^{\pm}_{f/h}(x)$ **probability for struck quark** f **to have momentum fraction** x **and helicity with (+) or against (−)** h**'s helicity.**