Introduction to heavy quarks – top-quarks –

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CTEQ - MCnet School 2016 *QCD & Electroweak Phenomenology and Monte Carlo Generators*, Hamburg, Jul 09, 2016

Plan

- Stability of the electro-weak vacuum
- Renormalization and the top-quark mass
- Top-quark mass measurements

Stability of the electro-weak vacuum

Fate of the universe

Higgs boson too light ? Are we doomed ?

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WORLD

Scientists studying so-called subatomic 'God particle' say there will be an universe-ending 'catastrophe'

The end of the universe won't come for tens of billions of years, but when it does happen it will destroy everything, according to researchers studying the Higgs boson particle. "If you use all the physics that we know now and you do what you think is a straightforward calculation, it's bad news," theoretical physicist Joseph Lykken said Monday.

Comments (24)

REUTERS

TUESDAY, FEBRUARY 19, 2013, 11:44 AM

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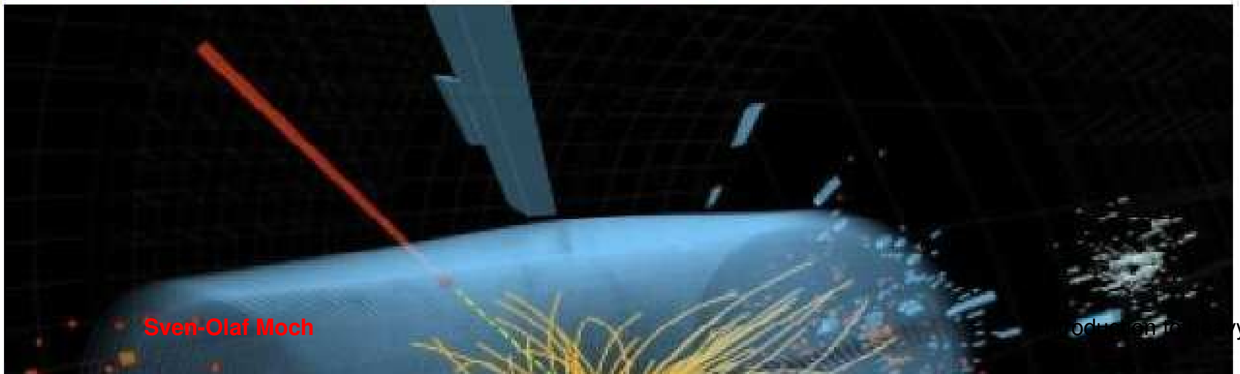
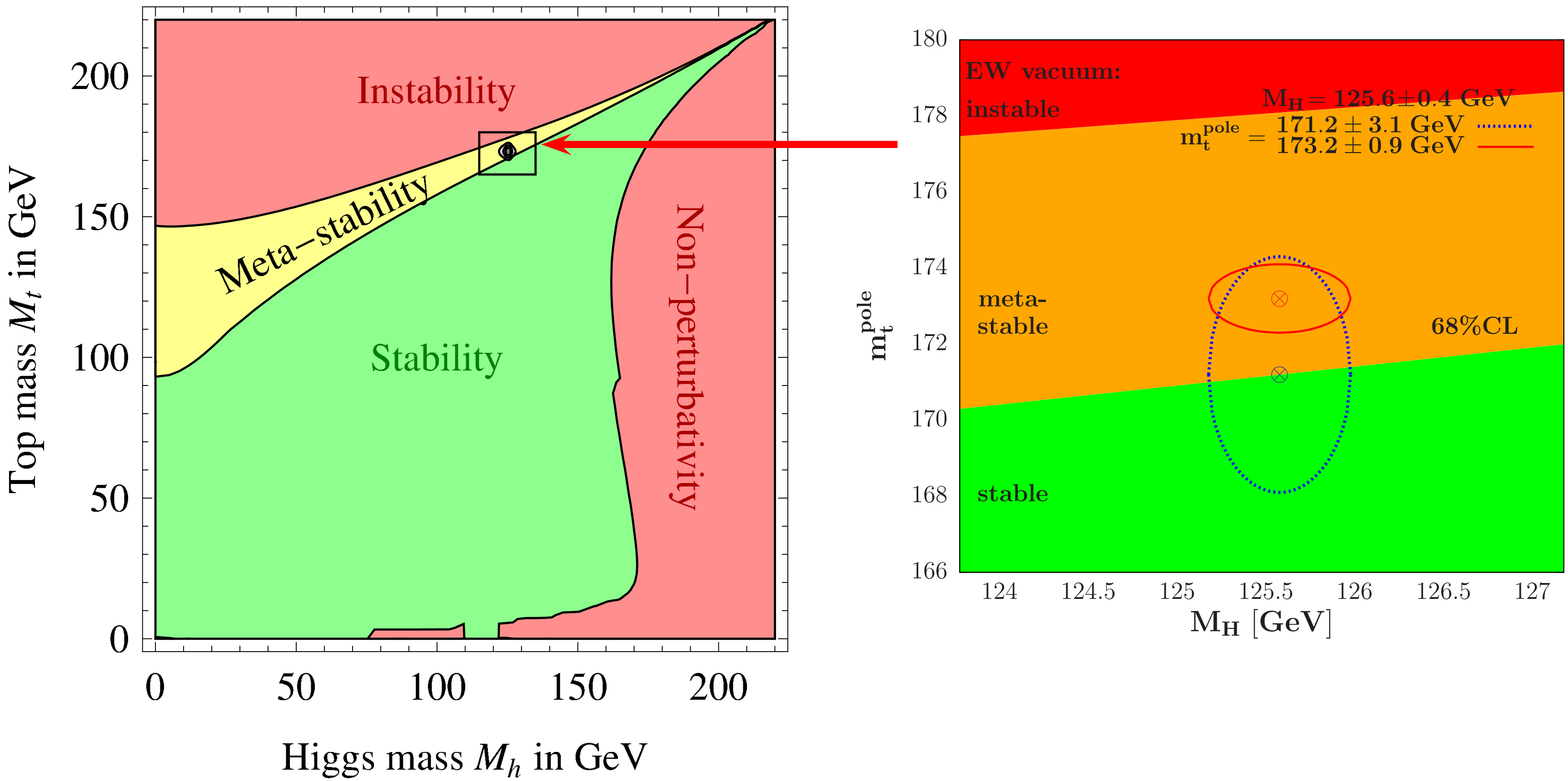


Fate of the universe



Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Alekhin, Djouadi, S.M. '12; Masina '12

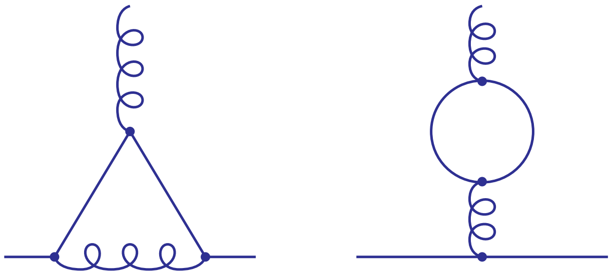
- Lower bound on Higgs mass

$$m_H \geq 129.6\text{GeV} + 2.0 \times (m_t - 173.2\text{GeV}) + \ldots$$

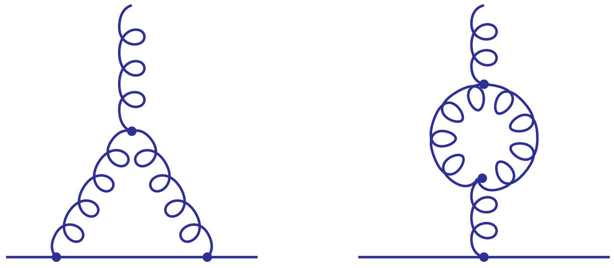
Quantum effects

Running coupling in QCD

- Effective coupling constant α_s depends on resolution
- QCD distinguished by self-interaction of gluons; e.g. vertex corrections



– screening (like in QED)



– anti-screening (color charge of g)

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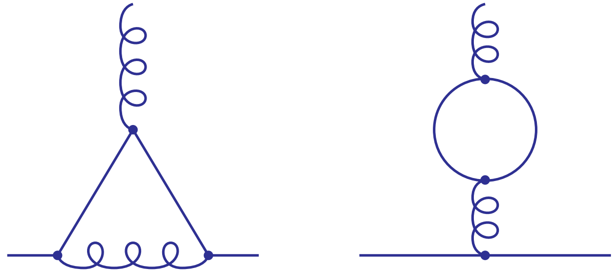
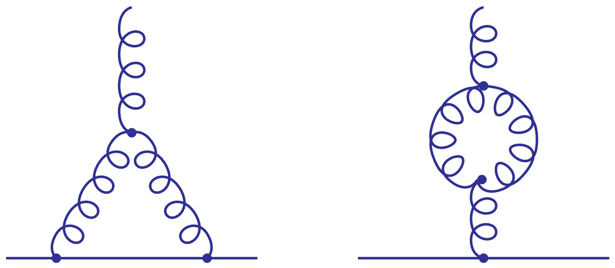




  – screening (like in QED) – anti-screening (color charge of g)

- Scale dependence governed by β-function of QCD

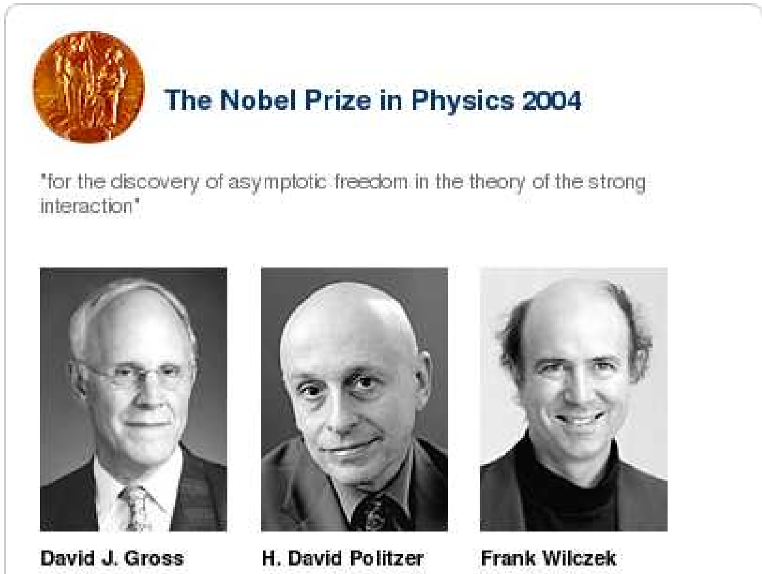
$$\mu^2 \frac{d}{d\mu^2}\alpha_s(\mu) \;=\; \beta\left(\alpha_s\right) \;=\; -\beta_0\alpha_s^2 - \beta_1\alpha_s^3 - \beta_2\alpha_s^4 - \beta_3\alpha_s^5 - \ldots$$

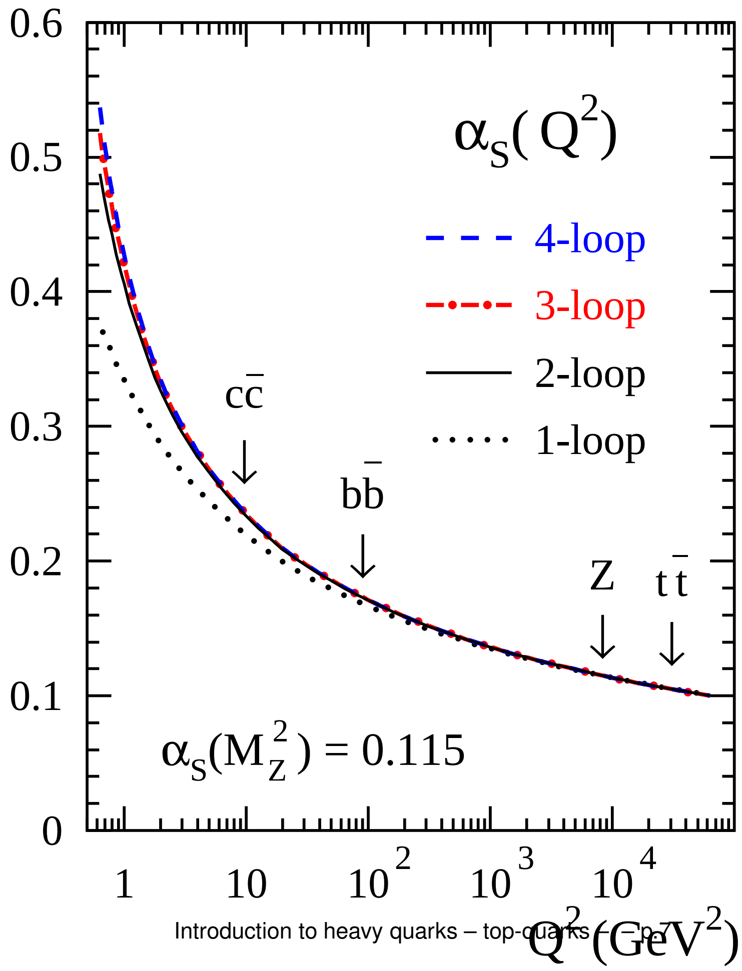
  - QCD β-function has negative sign
  - perturbative expansion with coefficients $\beta_0, \beta_1, \beta_2, \ldots$

$$\beta_0 \;=\; \frac{1}{4\pi}\left(\frac{11}{3}C_A - \frac{2}{3}n_f\right) \;=\; \frac{1}{4\pi}\left(7\right) \qquad (\text{for } n_f = 6)$$

Asymptotic freedom

- Solution of QCD β-function
  - perturbative expansion to four loops van Ritbergen, Vermaseren, Larin '97
  - very good convergence of perturbative series even at low scales (but $\alpha_s \gg \alpha_{\rm QED}$)



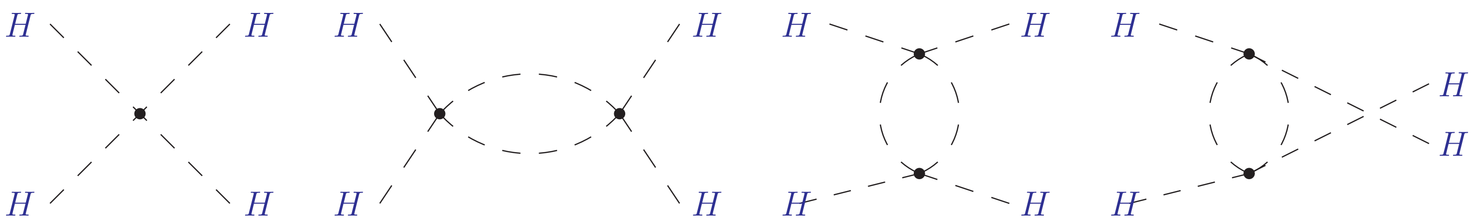


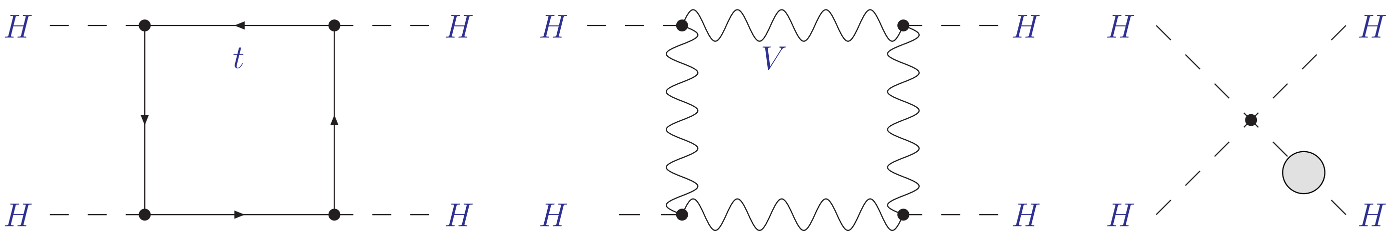
Higgs potential

Renormalization group equation

- Quantum corrections to Higgs Lagrangian
  $\mathcal{L}_{\rm cl} = (D^\mu \Phi^\dagger)(D_\mu \Phi) + \lambda \left| \Phi^\dagger \Phi - \frac{v}{2} \right|^2$
- Radiative corrections to Higgs self-coupling λ
  - electro-weak couplings g and g' of $SU(2)$ and $U(1)$
  - top-Yukawa coupling y_t

$$16\pi^2 \frac{d\lambda}{dQ} = 24\lambda^2 - \left(3g'^2 + 9g^2 - 12y_t^2\right)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \dots$$





Higgs potential

Triviality

- Large mass implies large λ
  - renormalization group equation dominated by first term

$$16\pi^2 \frac{d\lambda}{dQ} \simeq 24\lambda^2 \qquad \longrightarrow \qquad \lambda(Q) = \frac{m_H^2}{2v^2 - \frac{3}{2\pi^2} m_H^2 \ln(Q/v)}$$

- $\lambda(Q)$ increases with Q
- Landau pole implies cut-off Λ
  - scale of new physics smaller than Λ to restore stability
  - upper bound on m_H for fixed Λ

$$\Lambda \leq v \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right)$$

- Triviality for $\Lambda \to \infty$
  - vanishing self-coupling $\lambda \to 0$ (no interaction)

Higgs potential

Vacuum stability

- Small mass
  - renormalization group equation dominated by y_t

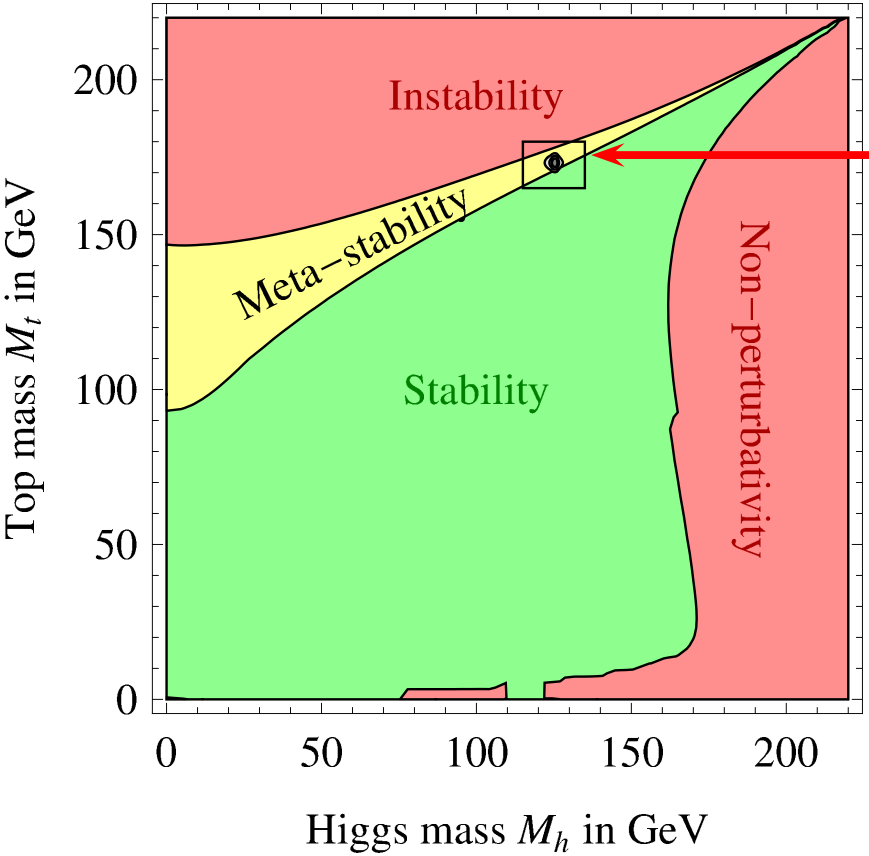
$$16\pi^2 \frac{d\lambda}{dQ} \simeq -6y_t^4 \qquad \longrightarrow \qquad \lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)}{1 - \frac{9}{16\pi^2} y_0^2 \ln(Q/Q_0)}$$

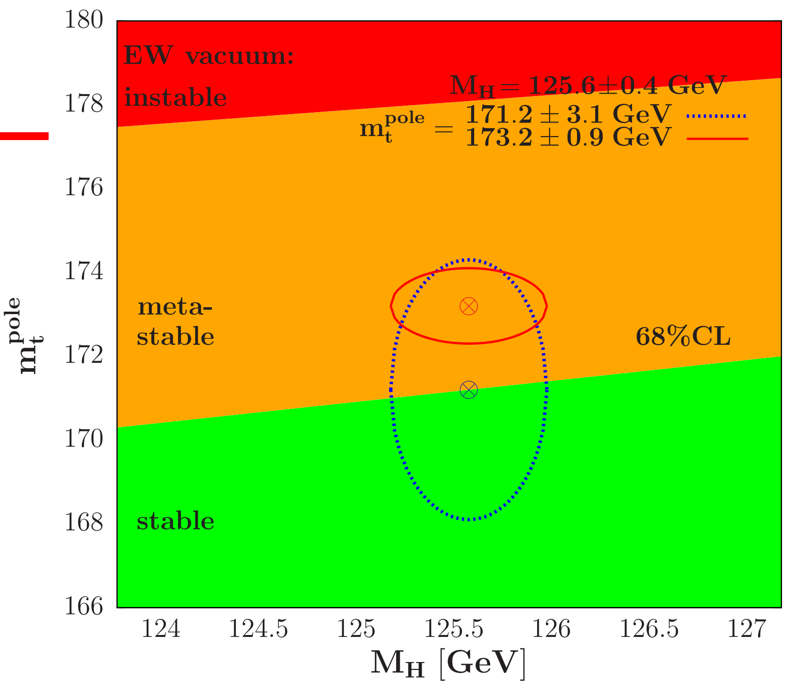
- $\lambda(Q)$ decreases with Q
- Higgs potential unbounded from below for $\lambda < 0$
- $\lambda = 0$ for $\lambda_0 \simeq \frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)$
- Vacuum stability

$$\Lambda \leq v \exp\left(\frac{4\pi^2 m_H^2}{3y_t^4 v^2}\right)$$

  - scale of new physics smaller than Λ to ensure vacuum stability
  - lower bound on m_H for fixed Λ

Fate of the universe still undecided





Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Alekhin, Djouadi, S.M. '12; Masina '12

- Uncertainty in Higgs bound relaxes $m_H \geq 125.3 \pm 6.2$ GeV
  - $\overline{MS}$ mass $m_t^{\overline{\rm MS}}(m_t) = 162.3 \pm 2.3 \pm 0.7$ GeV implies pole mass $m_t^{\rm pole} = 171.2 \pm 2.4 \pm 0.7$ GeV

Renormalization and the top-quark mass

Quantum field theory

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu}_{b} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i} \not{D} - m_q \right)_{ij} q_j$$

  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a \right)_{ij} A^a_\mu$

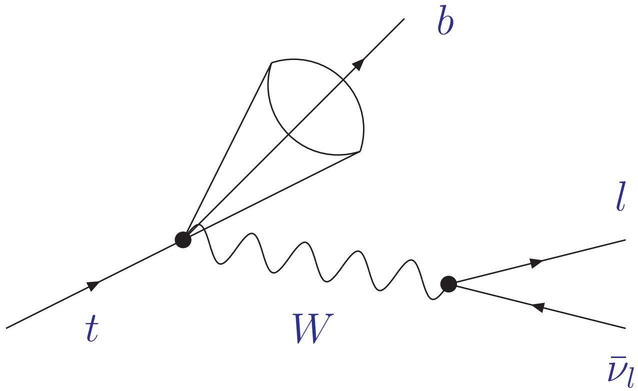
- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q
- Parameters of Lagrangian have no unique physical interpretation
  - radiative corrections require definition of renormalization scheme

Challenge

- Suitable observables for measurements of α_s, m_q, . . .
  - comparison of theory predictions and experimental data

Top-quark mass from kinematic reconstruction

- Top-quark decays on shell (e.g. leptonic decay $t \to bW \to bl\bar{\nu}_l$)
- Top-quark mass from based on reconstructed physics objects
  - jets, identified charged leptons, missing transverse energy
  - $m_t^2 = (p_{W-\mathrm{boson}} + p_{b-\mathrm{jet}})^2$



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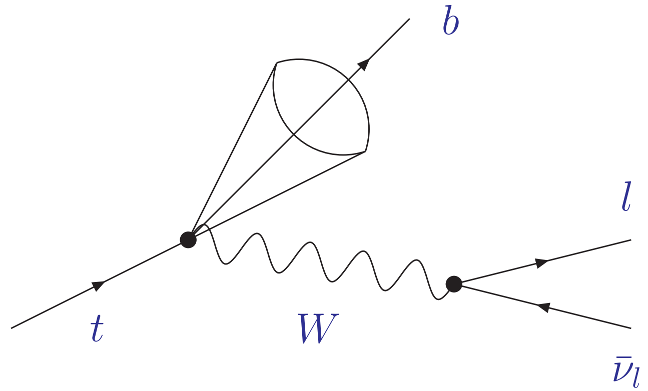
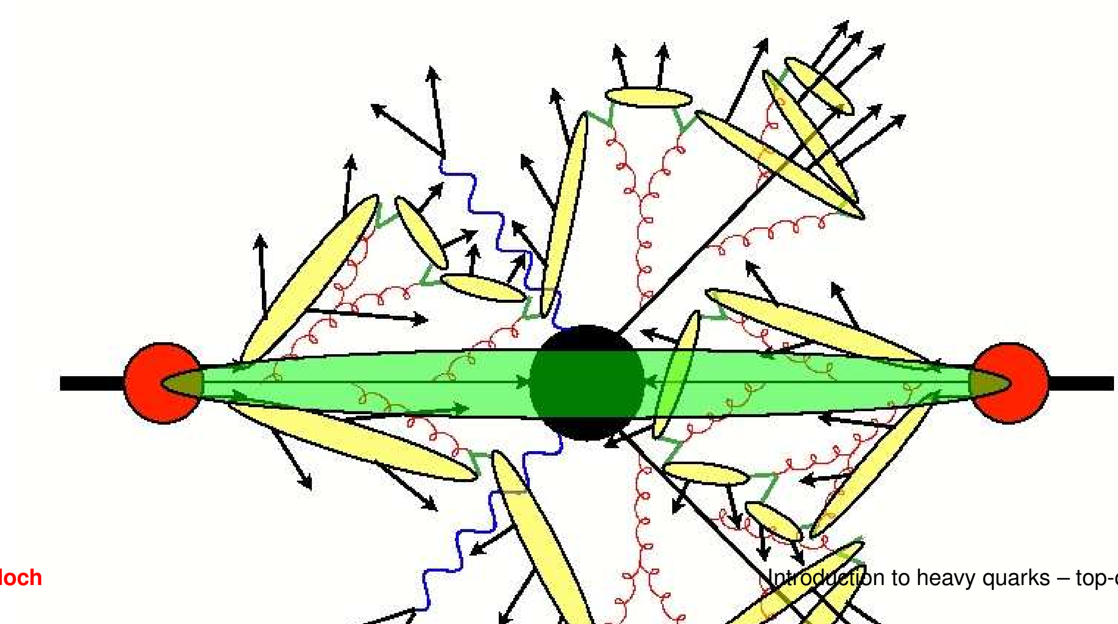


- However, hard interaction and parton emission in QCD followed by hadro



Renormalization

Physics picture

- Parameters of Lagrangian in quantum field theory have no unique physical interpretation
- Generic quantity R depends on
  - hard scale Q, mass m_q
  - in perturbative study on coupling constant α_s
- Radiative corrections
  - resolve quantum fluctuations at given resolution length $a \sim 1/\mu$
  - induce dependence of R on scale μ

Renormalization

Physics picture

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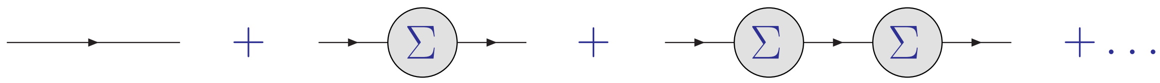
- Radiative corrections
  - resolve quantum fluctuations at given resolution length $a \sim 1/\mu$
  - induce dependence of R on scale μ
- Renormalization "group" governed by QCD describes changes R with respect to μ (differential equation of first order)

$$\left\{\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} - \gamma_m(\alpha_s)\, m_q \frac{\partial}{\partial m_q}\right\} R\left(\frac{Q^2}{\mu^2}, \alpha_s, \frac{m_q^2}{Q^2}\right) = 0$$

  - partial derivatives $\beta(\alpha_s) = \frac{\partial}{\partial \mu^2}\alpha_s$ and $\gamma_m(\alpha_s)\, m_q = \frac{\partial}{\partial \mu^2} m_q$
  - solution of differential equation requires initial conditions $\longrightarrow$ definition of renormalization scheme

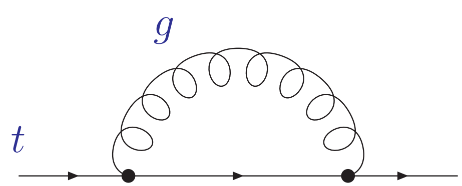
Quark mass renormalization

- Heavy-quark self-energy $\Sigma(p, m_q)$



$$= \frac{\mathrm{i}}{\not{p} - m_q - \Sigma(p, m_q)}$$

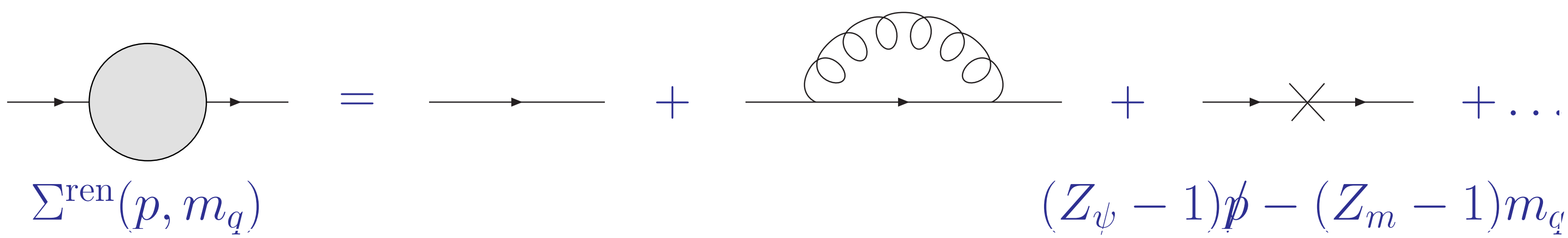
QCD



- QCD corrections to self-energy $\Sigma(p, m_q)$
  - dimensional regularization $D = 4 - 2\epsilon$
  - one-loop: UV divergence $1/\epsilon$ (Laurent expansion)

$$\Sigma^{(1),\mathrm{bare}}(p, m_q) = \frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{m_q^2}\right)^{\epsilon}\left\{(\not{p} - m_q)\left(-C_F\,\frac{1}{\epsilon} + \text{fin.}\right) + m_q\left(3C_F\,\frac{1}{\epsilon} + \text{fin.}\right)\right\}$$

- Relate bare and renormalized mass parameter $m_q^{\mathrm{bare}} = m_q^{\mathrm{ren}} + \delta m_q$



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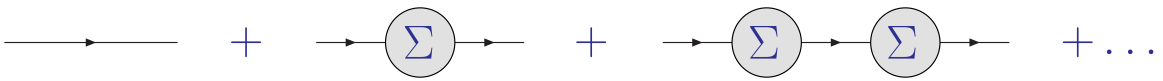
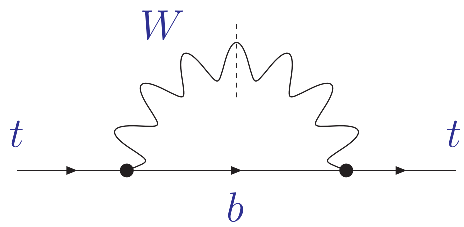


$$= \frac{\mathrm{i}}{\not{p} - m_q - \Sigma(p, m_q)}$$

EW sector



- EW corrections to top-quark self-energy
  - on-shell intermediate (virtual) W-boson
  - m_t complex parameter with imaginary part $\Gamma_t = 2.0 \pm 0.7$ GeV
  - $\Gamma_t > 1$ GeV: top-quark decays before it hadronizes

Mass renormalization scheme

Pole mass

- Based on (unphysical) concept of top-quark being a free parton
  - $m_q^{\rm ren}$ coincides with pole of propagator at each order

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{\not{p}=m_q} \to \not{p} - m_q^{\rm pole}$$

- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Bounds:
  - lattice QCD $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV Bauer, Bali, Pineda '11
  - perturbative QCD: $\Delta m_q \simeq 70$ MeV Beneke, Marquard, Nason, Steinhauser '16

$\overline{\rm MS}$ *scheme*

- $\overline{\rm MS}$ mass definition: for example one-loop minimal subtraction

$$\delta m_q^{(1)} = m_q \frac{\alpha_s}{4\pi} 3 C_F \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right)$$

- $\overline{\rm MS}$ scheme induces scale dependence: $m(\mu)$

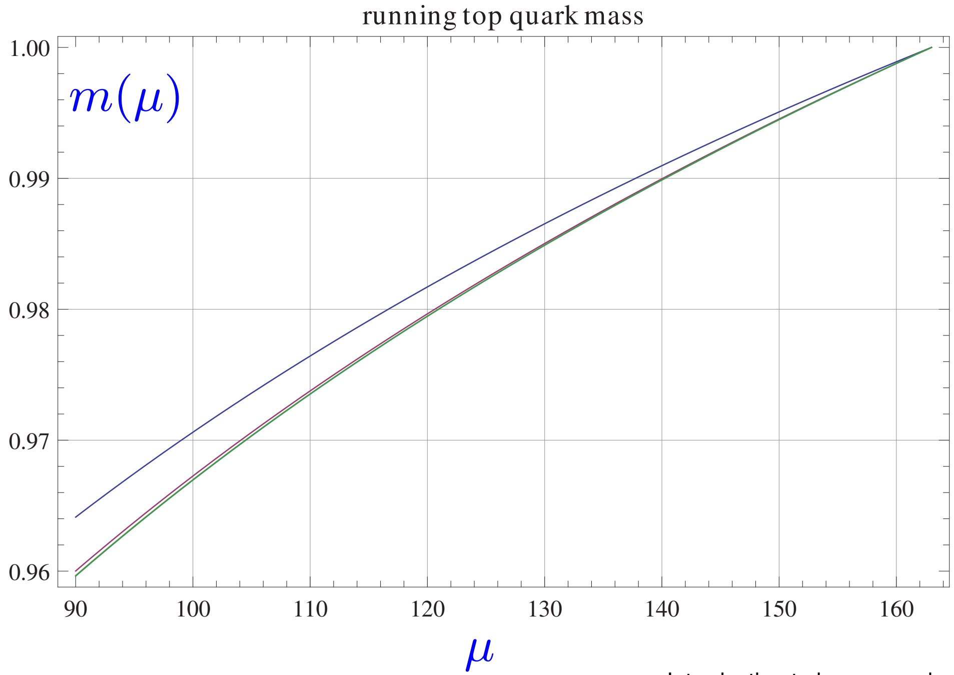
Running quark mass

Scale dependence

- Renormalization group equation for scale dependence
  - mass anomalous dimension γ known to four loops
    Chetyrkin '97; Larin, van Ritbergen, Vermaseren '97

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s}\right) m(\mu) = \gamma(\alpha_s) m(\mu)$$

- Plot mass ratio $m_t(163\text{GeV})/m_t(\mu)$



Scheme transformations

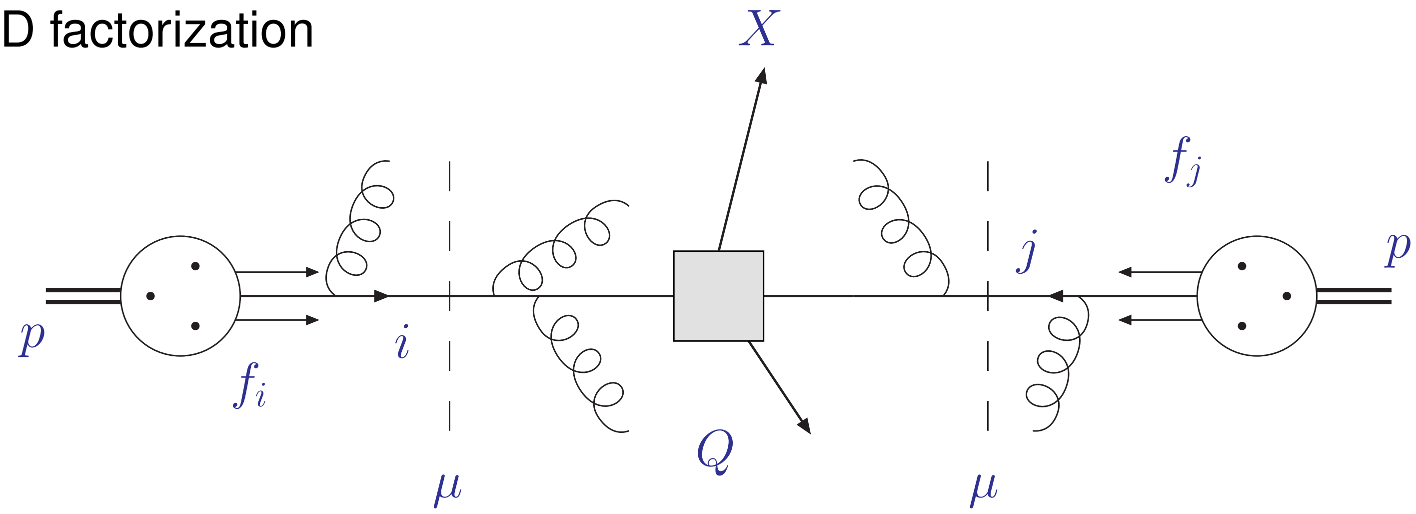
- Conversion between different renormalization schemes possible in perturbation theory
- Relation for pole mass and $\overline{\mathrm{MS}}$ mass
  - known to four loops in QCD Gray, Broadhurst, Gräfe, Schilcher '90; Chetyrkin, Steinhauser '99; Melnikov, v. Ritbergen '99; Marquard, Smirnov, Smirnov, Steinhauser '15
  - EW sector known to $\mathcal{O}(\alpha_{\mathrm{EW}}\alpha_{\mathrm{s}})$ Jegerlehner, Kalmykov '04; Eiras, Steinhauser '06
  - example: one-loop QCD

$$m^{\mathrm{pole}} = m(\mu)\left\{1 + \frac{\alpha_s(\mu)}{4\pi}\left(\frac{4}{3} + \ln\left(\frac{\mu^2}{m(\mu)^2}\right)\right) + \dots\right\}$$

Top-quark mass measurements

Top mass from cross sections

- QCD factorization



$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Joint dependence on non-perturbative parameters:
  parton distribution functions f_i, strong coupling α_s, masses m_X

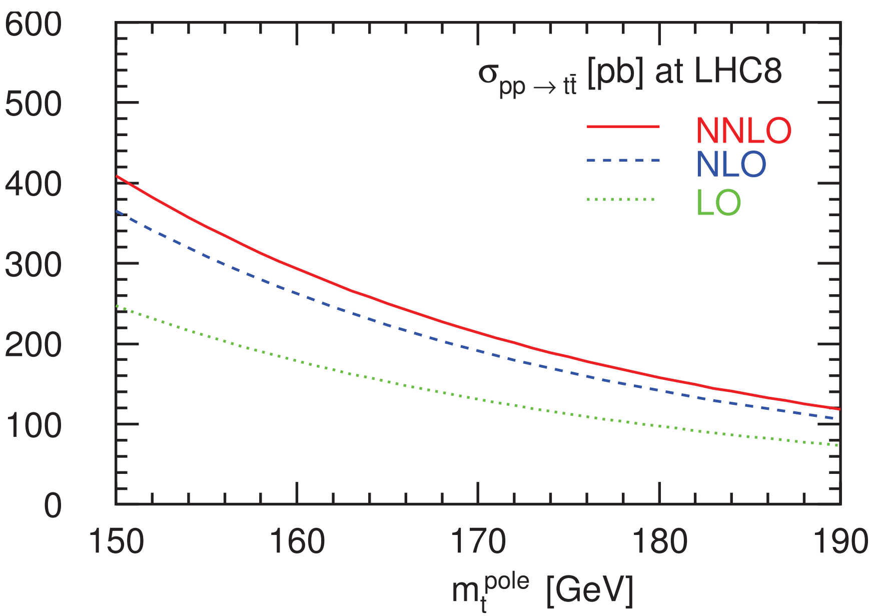
- Total cross section: intrinsic limitation in through sensitivity $\mathcal{S} \simeq 5$

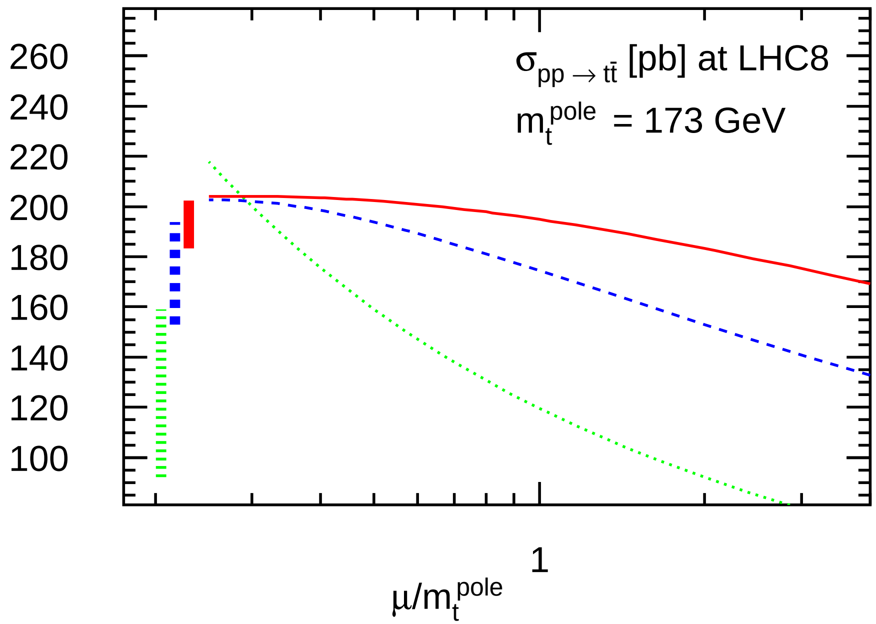
$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq \mathcal{S} \times \left|\frac{\Delta m_t}{m_t}\right|$$

Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov ‘13



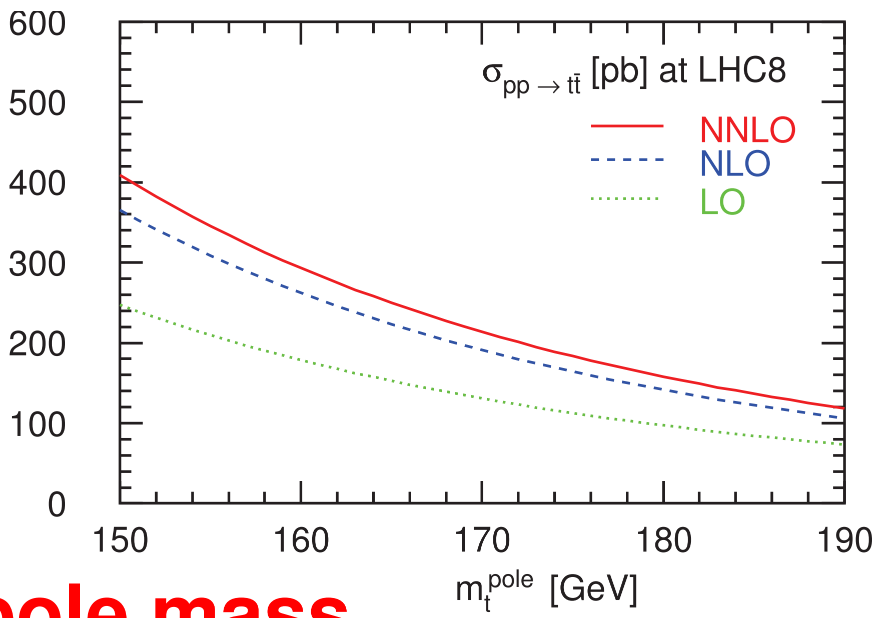


- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO $\to$ NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

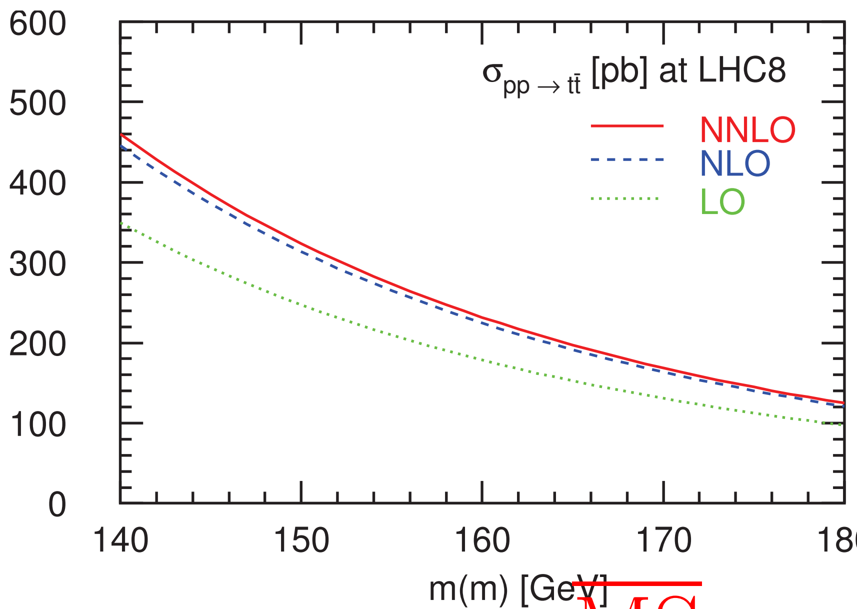
Total cross section with running mass

Comparison pole mass vs. $\overline{\text{MS}}$ mass (I)

Dowling, S.M. ‘13



pole mass



$\overline{\text{MS}}$ mass

- NNLO cross section with running mass significantly improved
  - good apparent convergence of perturbative expansion
  - small theoretical uncertainity from scale variation

Total cross section with running mass

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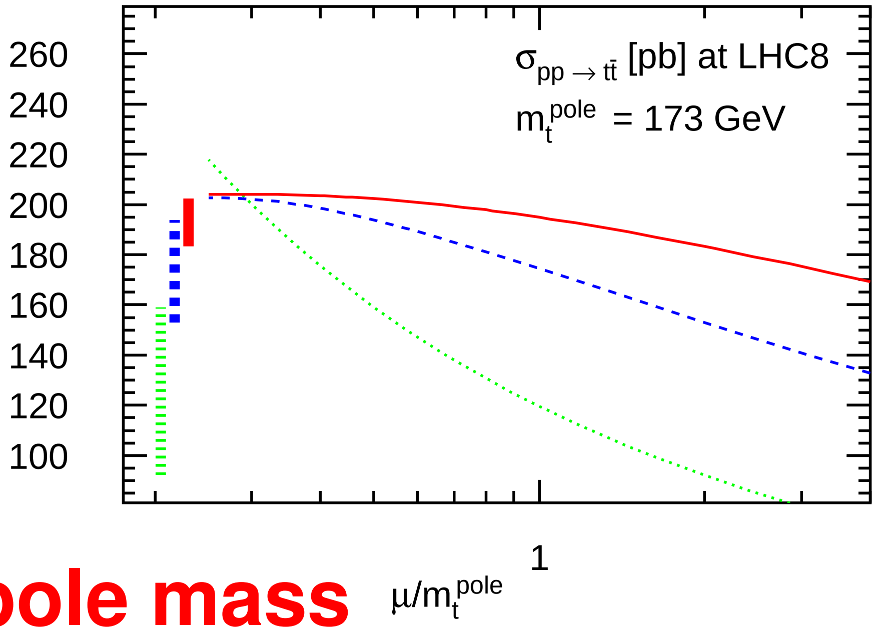
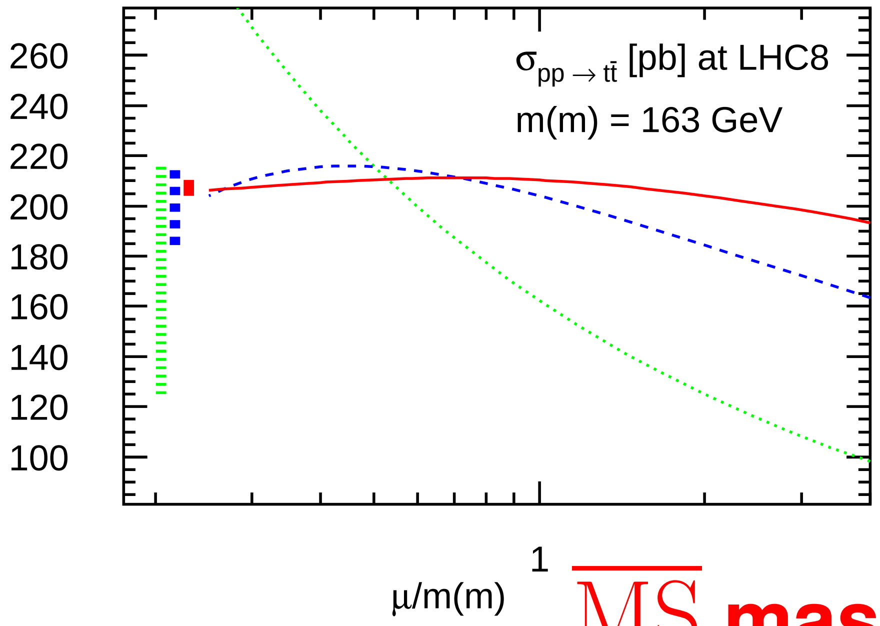


pole mass



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Top-quark mass determination

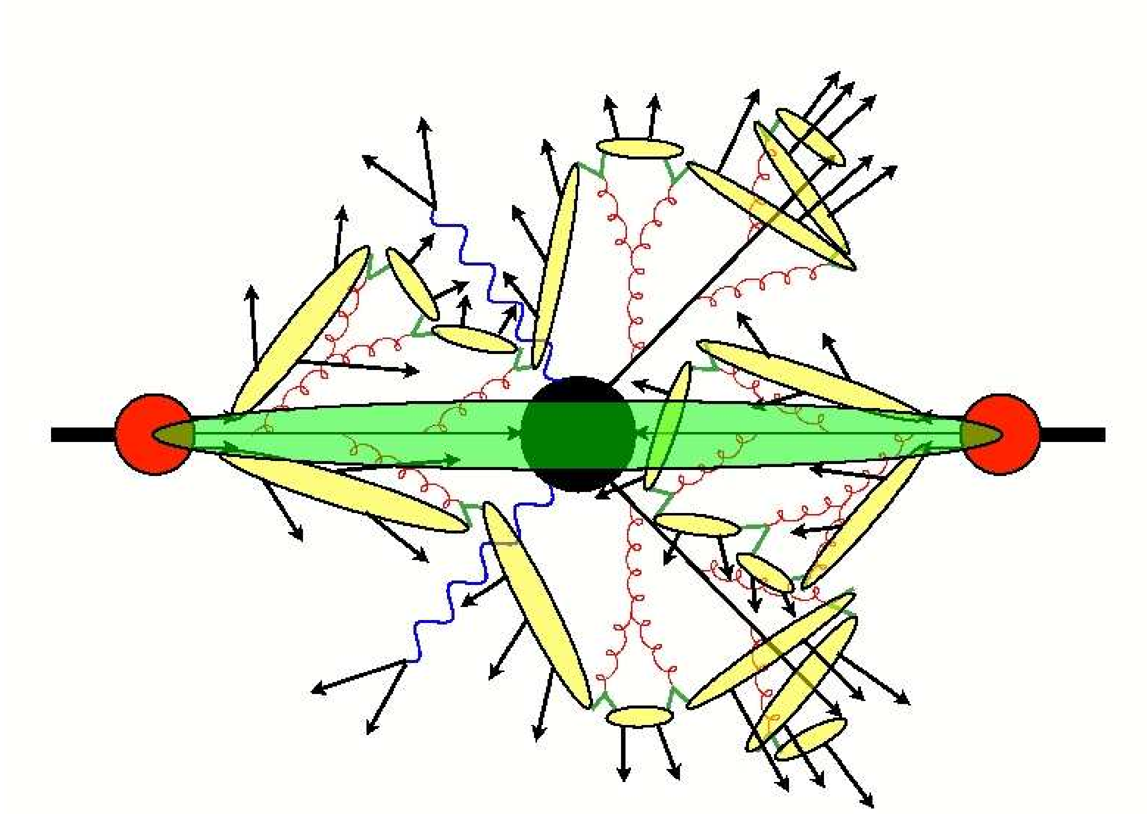
- Cross section measurement ATLAS arXiv:1406.5375

$$\sigma_{t\bar{t}} = 242.4 \pm 9.5 \text{ pb}$$

	$m^{\text{pole}} + \Delta^{\text{exp}} + \Delta^{\text{th+PDF}}$	$m(m) + \Delta^{\text{exp}} + \Delta^{\text{th+PDF}}$	$m^{\text{pl}}_{1\text{lp}}$	$m^{\text{pl}}_{2\text{lp}}$	$m^{\text{pl}}_{3\text{lp}}$
ABM12	$166.4 \pm 1.3 \pm 2.1$	$159.1 \pm 1.2 \pm 1.2$	166.2	167.8	168.4
CT14	$173.8 \pm 1.3 \pm 2.2$	$165.9 \pm 1.3 \pm 1.3$	173.5	175.4	176.0
MMHT	$173.7 \pm 1.3 \pm 2.0$	$165.8 \pm 1.3 \pm 1.0$	173.4	175.2	175.9
NNPDF3.0	$173.5 \pm 1.3 \pm 2.0$	$165.6 \pm 1.3 \pm 1.0$	173.2	175.0	175.7

- m_t from total cross section sensitive to PDFs
  - pole mass from $\overline{MS}$ mass $m_t(m_t)$ gives spread
    $m^{\text{pole}} = 168.4 \ldots 176.0$ GeV
- Scale uncertainty from range $m_t/2 \leq \mu_r, \mu_f \leq 2m_t$ GeV

Monte Carlo mass



[picture by B.Webber]

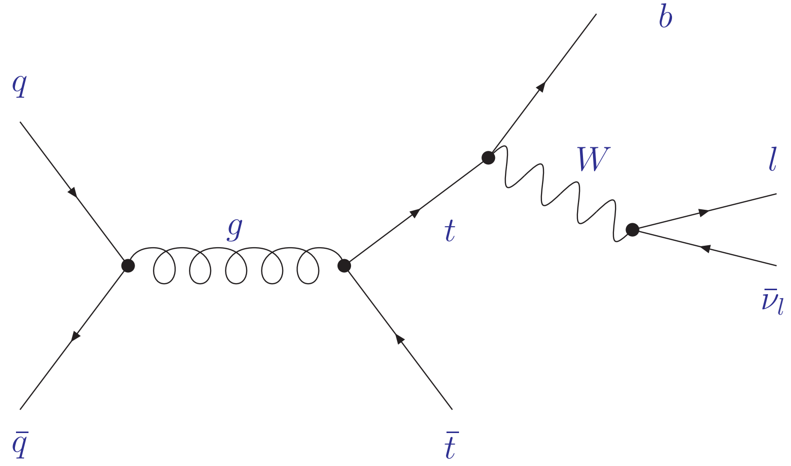
- Intuition: Monte Carlo mass identified with pole mass due to kinematics

$$m_q^2 = E_q^2 - p^2$$

- Caveat: heavy quarks in QCD interact with potential due to gluon field

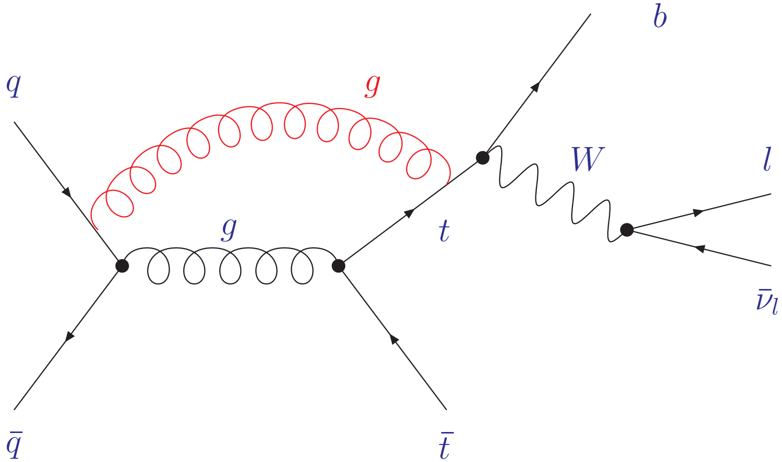
Hard scattering process

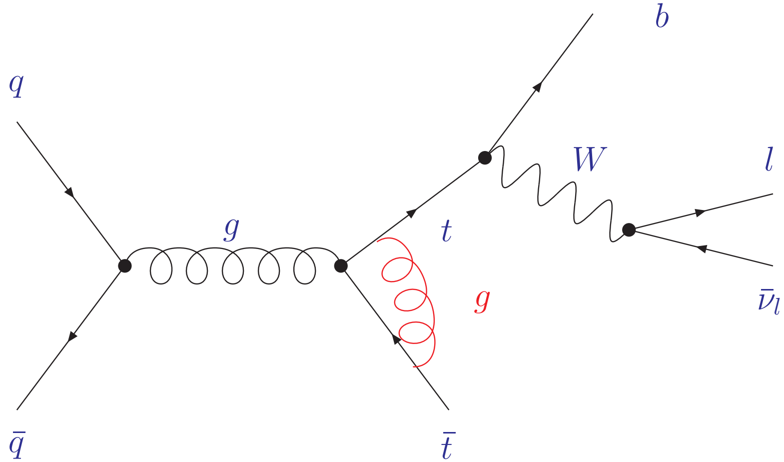
- Born process ($q\bar{q}$-channel) with leptonic decay $t \to bl\bar{\nu}_l$



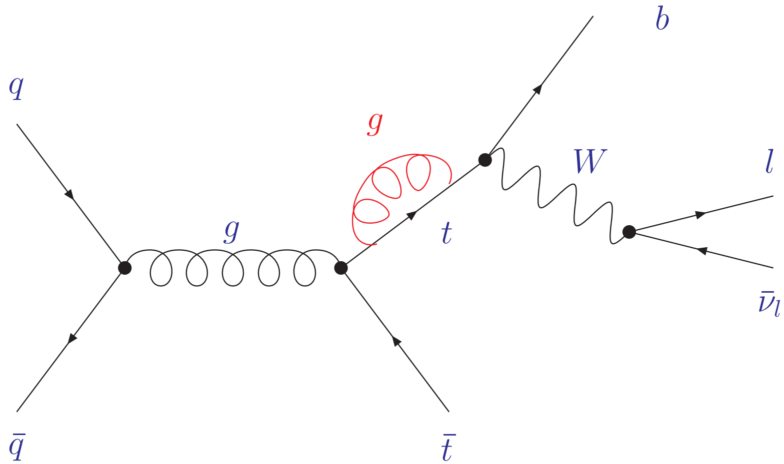
Radiative corrections

- Virtual corrections (examples): gluon exchange
  - box diagram (left) and vertex corrections (right)
  - infrared divergences cancel against real emission contributions



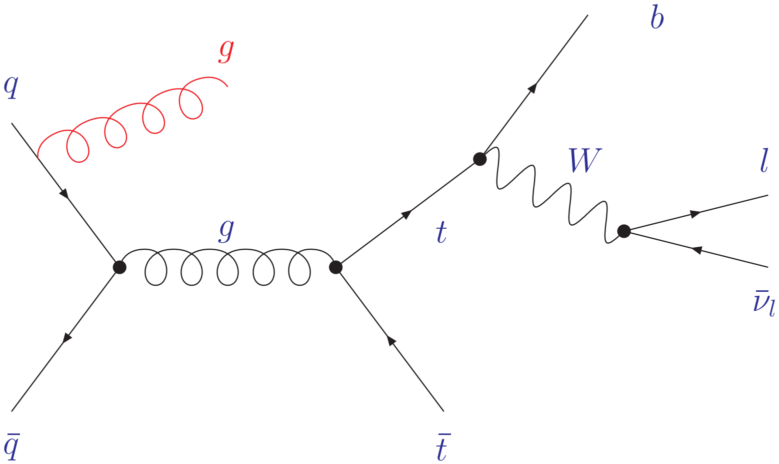


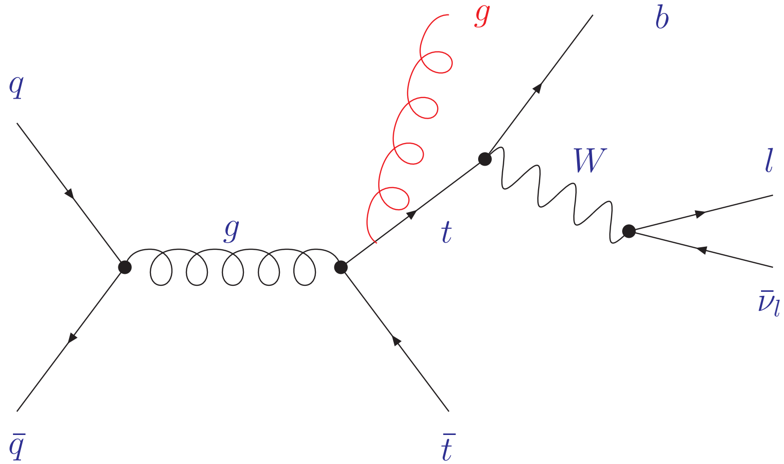
- Mass renormalization from self-energy corrections to top-quark



Radiative corrections

- Real corrections (examples): gluon emission
  - phase space integration $\to$ infrared divergences (soft/collinear singularities)





- Parton shower MC
  - emission probability modeled by Sudakov exponential with cut-off Q_0
  - leading logarithmic accuracy

$$\Delta\left(Q^2, Q_0^2\right) = \exp\left(-C_F \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)\right)$$

  - subtraction of IR contributions at hadronization scale $Q_0 \simeq \mathcal{O}(1)$GeV

Top-quark mass from kinematic reconstruction

Experiment: ATLAS, CDF, CMS & D0 coll. 1403.4427

$$m_t = 173.76 \pm 0.76\,\mathrm{GeV}$$

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That is, we can state as the final result for the likely relation between the top quark mass measured using a given Monte Carlo event generator ("MC") and the pole mass as

$$m_{\text{pole}} = m_{\text{MC}} + Q_0 \left[\alpha_s(Q_0) c_1 + \ldots\right]$$

where $Q_0 \sim 1$ GeV and c_1 is unknown, but presumed to be of order 1 and, according to the argument above, presumed to be positive.

A. Buckley et al. arXiv:1101.2599

Calibration of Monte-Carlo Mass (I)

Idea Kieseler, Lipka, S.M. '15

- Simultaneous fit of $m^{\rm MC}$ and observable $\sigma(m_t)$ sensitive to m_t, e.g., total cross section, differential distributions, ...
- Observable σ does not rely on any prior assumptions about relation between m_t and $m^{\rm MC}$
- Extraction of m_t from $\sigma(m_t)$ calibration of $m^{\rm MC}$, e.g. pole mass

$$\Delta_m = m_t^{\rm pole} - m^{\rm MC}$$

Implementation [J. Kieseler, DESY-THESIS-2015-054]

- Confront N^d reconstructed events to N^p simulated ones
  - model parameters $\vec{\lambda}$

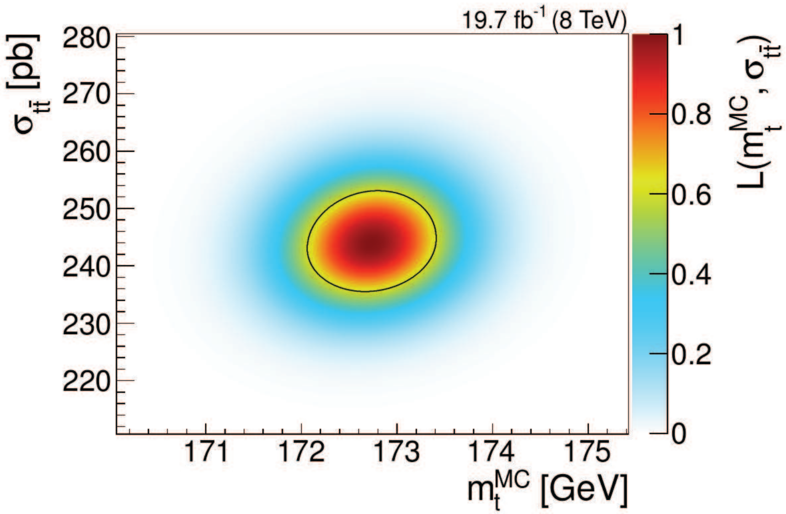
$$N^p = \mathcal{L} \cdot \underbrace{\epsilon(m^{\rm MC}, \vec{\lambda})}_{\text{efficiency}} \cdot \underbrace{\sigma}_{\text{observable}} \cdot \underbrace{n^p(m^{\rm MC}, \vec{\lambda})}_{\text{predicted shape contribution}} + \underbrace{N^{bg}(\vec{\lambda})}_{\text{background}}$$

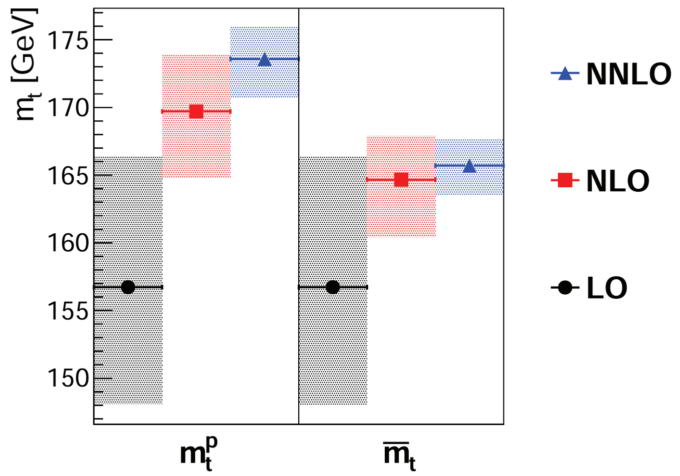
  - shape of distribution constrains $m^{\rm MC}$, normalization determines σ

Top-Quark Monte-Carlo Mass (II)

Likelihood fit [J. Kieseler, DESY-THESIS-2015-054]

- Correlations between $m^{\rm MC}$ and σ present in $\epsilon(m^{\rm MC}, \vec{\lambda})$
  - minimize in $m^{\rm MC}$ dependence in efficiency
- Reduce contribution of $m^{\rm MC}$ to total uncertainty of σ
  - constrain $m^{\rm MC}$ in predicted events $n^p(m^{\rm MC}, \vec{\lambda})$





- Cross section measurement CMS at $\sqrt{s} = 8$ TeV: $\sigma_{t\bar{t}} = 243.9 \pm 9.3$ pb
  J. Kieseler, DESY-THESIS-2015-054
- Calibration of $m^{\rm MC}$ with uncertainty of approximately 2 GeV on $\Delta_m = m_t^{\rm pole} - m^{\rm MC}$ possible

Summary

Top-quark mass

- Top-quark mass is parameter of Standard Model Lagrangian
- Measurements of m_t require careful definition of observable
- Quality of perturbative expansion depends on scheme for top-quark mass
- Monte-Carlo mass $m^{\rm MC}$ needs calibration with data
  - current calibration of $m^{\rm MC}$ with uncertainty of approximately 2 GeV

Future of our universe

- Challenge to precision of theory computations and measurements